# A Fuzzy Multi-Attribute Decision Making Approach for Multi-Objective Thermal Power Dispatch

Saksorn Chalermchaiarbha and Weerakorn Ongsakul

***Abstract***— *This paper proposes a fuzzy multi-attribute decision making approach for solving multi-objective thermal power dispatch problem. The fuzzy multi-attribute decision considering the maximum of minimum membership function value of each non-dominated solution could well trade-off the contradicting objective functions consisting of fuel cost, $NO_X$, $SO_2$ and $CO_2$ emission. The weighting aggregation method is employed to generate the non-dominated Newton-Raphson based solutions. Test results on 3 and 6 generating systems indicate that the obtained best optimal solution having more compromise characteristics than the ones derived from fuzzy cardinal priority ranking normalized approach (FCPRN) and technique for order preference by similarity to ideal solution (TOPSIS) when taking account of percentage total deviation from the ideal solution as well as fulfilling preferred zones condition.*

***Keywords***— **Best Compromise Solution, Fuzzy Multi Attribute Decision Making, Membership Function, Non-Dominated Solutions.**

## 1. INTRODUCTION

The optimal economic load dispatch in electric power systems has currently gained increasing importance since not only the generation cost keeps on increasing but also pollutant emission level caused by thermal power plants is not allowed to exceed the quantities imposed by environmental laws. In generating electricity, combustion of fossil fuel emits several gaseous pollutant into atmosphere such as nitrogen oxides ($NO_x$) , sulfur dioxide ($SO_2$) and carbon dioxide ($CO_2$). As a result, the allocation of power generation to different thermal power units is to minimize both operating cost and pollutant emission level subject to diverse equality and inequality constraints of the system such as covering power load demand and loss, generating capacities, etc [1]-[5]. Multi-objectives formulation is then implemented to solve for the optimal strategy for electric power generation. The main problem of multi-objective optimization, however, is that such objectives are mostly contradicting one another, where improvements in one objective may lead to an exacerbation in another objective. Trade-off, therefore, exists between such conflicting objectives. Consequently, there are more than one optimal solution for multi-objective problem which is different to the single objective one. Identifying a set of feasible solutions is therefore important for the decision maker to select a compromise solution satisfying the objectives as best possible. Such solutions are referred to as non-dominated solutions [6]-[7].

Traditional techniques which are used for solving multi-objective problem, for example, are goal programming, the, the ε-constraint method, and weighting method, etc [7].

For goal programming, the decision maker has to assign targets or goals that wish to achieve for each objective. These values are included in the problem as additional constraints. The objective function then tries to minimize the absolute deviation from the targets to the objectives

The ε-constraint method is based on optimization of the most preferred objective while considering the other objectives as constraint bounded by some allowable levels (ε). Such levels are altered to generate the non-dominated solutions.

The weighting method uses the concept of combing different objectives through the weighted sum method to convert the multi-objective problem into single objective one. This method generates the non-dominated solution by varying the weight combination.

In addition, the meta-heuristic approaches, such as, evolutionary algorithms and swarm intelligence, is an alternative to aforementioned techniques. A non-dominated sorting genetic algorithm (NSGA) is used to solve for environmental/economic power dispatch (EED). Likewise, a modified multi-objective particle swarm optimization algorithm (MOPSO) and multi-objective evolutionary algorithm (MOEA) is also presented to handle the EED problem [2]-[5]. Importantly, whatever optimization methods are applied to produce the feasible solutions, finally, there is simply one solution chosen as the best that maximizes the satisfaction of all objectives to decision maker [7].

A way that is widely used to extract the best optimal solution in several papers pertaining to electric power generation planning is a fuzzy cardinal priority ranking normalized approach (FCPRN) [2], [3], [5], [8]-[9].

In this paper, a fuzzy multi-attribute decision making (FMADM) approach is proposed to be an alternative decision process for extracting the best compromise solution of multi-objective thermal power dispatch

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problem. The Newton-Raphson algorithm is utilized, in optimization process, to produce the non-dominated solutions through weighting aggregation method. Test results are demonstrated on 3 and 6 generating systems. The best compromise solution obtained is compared to the ones derived from FCPRN approach as well as an approach that use the technique for order preference by similarity to ideal solution (TOPSIS) [4] in terms of percentage total deviation from the ideal (minimum) solution including checking preferred zones conditions.

## 2. PROBLEM FORMULATION

In this section, the multi-objective problem of thermal power dispatch with equality and inequality constraints are described. The important objectives considered here are operating fuel cost, $NO_X$ emission, $SO_2$ emission and $CO_2$ emission. These objectives are competing one another owing to contradiction characteristics.

### 2.1 Fuel Cost

The first objective function to be minimized is the total fuel cost for thermal generating units in the system which can be approximately modeled by a quadratic function of generator power output $P_i$ [10], [11]

$$F_1 = \sum_{i=1}^{NG}(a_i P_i^2 + b_i P_i + c_i) \tag{1}$$

where

$a_i$, $b_i$, $c_i$ are the fuel cost coefficients.

NG is the total number of generating units.

### 2.2 Gaseous Pollutant Emission

As thermal power plant uses fossil fuel for power generation, it therefore releases the polluting gases into atmosphere. The most important emission considered in gernerating electric power that effects on the environment are $NO_x$, $SO_2$ and $CO_2$. These emissions can be approximately modeled through a quadratic function in terms of active power generation [10].

The $NO_x$ emission objective can be defined as

$$F_2 = \sum_{i=1}^{NG}(d_{1i} P_i^2 + e_{1i} P_i + f_{1i}) \tag{2}$$

where

$d_{1i}$, $e_{1i}$, $f_{1i}$ are the $NO_x$ emission coefficients.

NG is the total number of generating units.

In a similar fashion, $SO_2$ and $CO_2$ emission objectives can be defined below:

$$F_3 = \sum_{i=1}^{NG}(d_{2i} P_i^2 + e_{2i} P_i + f_{2i}) \tag{3}$$

where $d_{2i}$, $e_{2i}$, $f_{2i}$ are the $SO_2$ emission coefficients.

$$F_4 = \sum_{i=1}^{NG}(d_{3i} P_i^2 + e_{3i} P_i + f_{3i}) \tag{4}$$

where $d_{3i}$, $e_{3i}$, $f_{3i}$ are the $CO_2$ emission coefficients.

### 2.3 Equality and Inequality Constraints

The total power generation must cover the total load demand and real power loss in the transmission system. For the fixed network configuration, the equality constraint is represented by the power balance equation stated as:

$$\sum_{i=1}^{NG} P_i = P_D + P_L \tag{5}$$

where $P_D$ and $P_L$ is total load demand and transmission loss, respectively.

The power output limit are imposed as

$$P_i^{\min} \le P_i \le P_i^{\max},\ i = 1, 2, 3, \dots, NG \tag{6}$$

### 2.4 Transmission Loss

One common practice for calculating the effect of transmission losses is to express the total transmission loss as a quadratic function of the generator power outputs. The simplest quadratic form is [11]:

$$P_L = \sum_{i=1}^{NG}\sum_{j=1}^{NG} P_i B_{ij} P_j \tag{7}$$

The coefficients $B_{ij}$ are called loss coefficients or B-coefficients and assumed constant.

Thus, the problem formulation is to minimize all objective functions simultaneously, while satisfying both equality and inequality constraints which can be expressed as follows:

$$\text{Minimize } [F_1, F_2, F_3, F_4]^T \tag{8}$$

subject to

$$\sum_{i=1}^{NG} P_i = P_D + P_L \tag{9}$$

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{10}$$

where $F_1$, $F_2$, $F_3$, and $F_4$ are the objective functions to be minimized over the set of admissible decision variables $P_i$.

## 3. METHODOLOGY

### 3.1 Optimization Method.

The weighting aggregation method is employed to generate the non-dominated solutions through Newton-Raphson algorithm. This method defines an aggregate objective function as a weighted sum of the objectives. Hence, the multi-objective optimization problem is redefined as [6], [7]:

$$\text{Minimize } \sum_{j=1}^{m} w_j F_j(P_i) \tag{11}$$

subject to

$$\sum_{i=1}^{NG} P_i = P_D + P_L \tag{12}$$

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{13}$$

$$\sum_{j=1}^{m} w_j = 1, \; w_j \ge 0, \tag{14}$$

where $w_j$ are the weighting coefficients. In this study, the value of weighting coefficients vary in the range of 0 to 1 in steps of 0.1 and the weight of fuel cost, $w_1$, is not allowed to be zero except in the case of determining the minimum value of other objectives i.e. $F_2$, $F_3$, and $F_4$; $m$ is the total number of objectives.

To solve the scalar optimization problem, the Lagrangian function is defined as

$$L(P_i, \lambda) = \sum_{j=1}^{m} w_j F_j + \lambda\left( P_D + P_L - \sum_{i=1}^{NG} P_i \right) \tag{15}$$

where λ is Lagrangian multiplier and $m$ is the number of objective functions.

The necessary conditions to minimize the unconstrained Lagrangian function are:

$$\frac{\partial L}{\partial P_i} = 0 \text{ and } \frac{\partial L}{\partial \lambda} = 0 \tag{16}$$

To implement the Newton-Raphson method, the following equation is solved iteratively until having no further improvement in decision variables.

$$\begin{bmatrix} \nabla_{PP} & \nabla_{P\lambda} \\ \nabla_{\lambda P} & \nabla_{\lambda\lambda} \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} -\nabla_P \\ -\nabla_\lambda \end{bmatrix} \tag{17}$$

The steps for Newton-Raphson algorithm to produce the admissible solutions can be explained below [10].

***Newton-Raphson Algorithm***

1. Read data, viz. cost coefficients, emission coefficients and *B*-coefficients, demand, Error (convergence tolerance) and ITMAX (maximum allowed iterations), *M* (number of objectives), NG (number of generators) and *K* (number of non-dominated solutions).
2. Set iteration for non-dominated solutions, $k = 1$.
3. If ($k \ge K$) GOTO step 15
4. Feed weights combination, $w_j : j = 1, 2, \ldots, m$
5. Compute the initial value of $P_i$ ( $i$ = 1, 2, …, NG) and λ by assuming that $P_L = 0$. The value of λ and $P_i$ can be calculated using the Eq. (18) and (19).

$$P_i = \frac{\lambda - b_i}{2a_i}, \; (i=1, 2, 3, \ldots, NG) \tag{18}$$

$$\lambda = \frac{P_D + \sum_{i=1}^{NG} \frac{b_i}{2a_i}}{\sum_{i=1}^{NG} \frac{1}{2a_i}} \tag{19}$$

Assume that no generator has been fixed either at lower limit or at upper limit at this step.

6. Set iteration counter, IT = 1.
7. Compute Hessian and Jacobian matrix elements in Eq.(17). Deactivate row and column of Hessian matrix and row of Jacobian matrix representing the generator whose generation is fixed either at lower limit or upper limit in order that those fixed generators cannot participate in allocation.
8. Find $\Delta P_i$ ($i$ = 1, 2, …, $R$) and $\Delta\lambda$ using Gauss elimination method. Here, R is the number of generators that can participate in allocation.
9. Modify control variables,

$$P_i^{new} = P_i + \Delta P_i \qquad (i = 1, 2, \ldots, R)$$

$$\lambda^{new} = \lambda + \Delta\lambda$$

10. Update old control variable values with new values.

$$P_i = P_i^{new} \qquad (i = 1, 2, \ldots, R),$$

$$\lambda = \lambda^{new} \qquad \text{and GOTO Step 8 and repeat.}$$

11. Check the inequality constraint of generators from the following conditions.

If $P_i < P_i^{\min}$ then $P_i = P_i^{\min}$

If $P_i > P_i^{\max}$ then $P_i = P_i^{\max}$

12. Check convergence tolerance condition from $\sqrt{\sum_{i=1}^{R} (\Delta P_i)^2 + (\Delta\lambda)^2} \le \varepsilon$

If convergence condition is satisfied or IT ≥ ITMAX then GOTO Step 13 otherwise update iteration counter, IT = IT+1 and GOTO Step 7.

13. Record the obtained non-dominated solution. Compute $F_j$ (*j=1, 2, …, m*) and transmission loss.
14. Increment count of non-dominated solutions, $k = k+1$ and GOTO step 3.
15. Stop.

### *3.2 Membership Function*

Optimization of multi-objective problem yields a set of non-dominated solutions. However, only one solution would finally be selected as the best that well trades-off the all conflicting objectives.

Typically, it is natural to assume that decision maker may have fuzzy or imprecise goals for each objective function. The membership function based upon fuzzy sets theory, therefore, are introduced to represent the goals of each objective function. The membership function value describes the degree of minimum value attainment of each objective function using values from 0 to 1. The membership value of zero indicates incompatibility with the sets, while one means complete compatibility. Thus, the membership function is a strictly momotonic decreasing and continuous function which is defined as [12]:

$$\mu_j = \begin{cases} 1 & ; \quad F_j \le F_j^{\min} \\ \dfrac{F_j^{\max} - F_j}{F_j^{\max} - F_j^{\min}} & ; \quad F_j^{\min} \le F_j \le F_j^{\max} \\ 0 & ; \quad F_j \ge F_j^{\max} \end{cases} \tag{20}$$

Where $\mu_j$ is membership function of objective $F_j$ and $F_j^{\min}, F_j^{\max}$ are minimum and maximum values of the $j$-th objective, respectively.

### *3.3 Evaluation of Optimal Solution using the Proposed Fuzzy Multi-attribute Decision Making Approach*

The proposed FMADM approach is utilized to elicit the best compromise solution out of a set of non-dominated ones. The concept of FMADM approach can be described as follows. [13]-[16].

Let $X=\{x_1,\ldots, x_n\}$ be a set of optimal solutions. The importance (weight) of the $j$-th objective is expressed by $w_j$. The attainment of objective $F_j$ with respect to $F_j^{\min}$ by solution $x_i$ is expressed by the degree of membership $\mu_j(x_i)^{w_j}$.

The procedure for determining the objective weights and the best optimal solution can be described below:

1. Establish by pair-wise comparison the relative importance, $\alpha_j$, of the fuzzy objectives amongst themselves. Arrange the $\alpha_j$ in a matrix M.

$$M = \begin{bmatrix} \frac{\alpha_1}{\alpha_1} & \frac{\alpha_1}{\alpha_2} & \cdots & \cdots & \frac{\alpha_1}{\alpha_m} \\ \frac{\alpha_2}{\alpha_1} & & & & \\ & & & & \vdots \\ & & & & \vdots \\ \frac{\alpha_m}{\alpha_1} & & & & \frac{\alpha_m}{\alpha_m} \end{bmatrix} \tag{21}$$

2. Determine consistent weights $w_j$ for each objective through approximated Saaty's eigenvector method by normalizing the geometric mean in each row. Thus, the summation of all weights is equal to $m$, $\sum_{j=1}^{m} w_j = m$.
3. Weight the degrees of objective attainment, $\mu_j(x_i)$ exponentially by the respective $w_j$. The resulting fuzzy sets are $\mu_j(x_i)^{w_j}$
4. Determine the value of attainment in all objectives of solution $x_i$ via intersection of all $\mu_j(x_i)^{w_j}$:

$$\tilde{D}(x_i) = \mu_1(x_i)^{w_1} \cap \mu_2(x_i)^{w_2} \cap \ldots \cap \mu_m(x_i)^{w_m} \tag{22}$$

5. The best optimal solution, $\tilde{D}(x^*)$, is defined as that achieving the largest degree of membership in $\tilde{D}(x_i)$.

$$\tilde{D}(x^*) = \max\{\, \tilde{D}(x_i) \mid i = 1,2,\ldots,n\} \tag{23}$$

### *3.4 Evaluation of Optimal Solution using Fuzzy Cardinal Priority Ranking Normalized Approach*

For this approach, the accomplishment of each non-dominated solution is considered with respect to all the $n$ non-dominated solutions by normalizing its accomplishment in all objectives over the sum of the accomplishments of $n$ non-dominated solutions as follows [2], [3], [5], [8]-[9]:

$$\mu_{\tilde{D}}(x_i) = \frac{\sum_{j=1}^{m} \mu_j(x_i)}{\sum_{i=1}^{n}\sum_{j=1}^{m} \mu_j(x_i)} \tag{24}$$

where $\mu_{\tilde{D}}(x_i)$ represents the normalized fuzzy membership function of the the $i$-th solution. The solution that attains the maximum membership constitutes the best one, $\mu_{\tilde{D}}(x^*)$, owing to having highest cardinal priority ranking. Hence, the best optimal solution is obtained from

$$\mu_{\tilde{D}}(x^*) = \max\{\, \mu_{\tilde{D}}(x_i) \mid i = 1,2,\ldots,n\} \tag{25}$$

### *3.5 Evaluation of Optimal Solution through Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)*

The concept of TOPSIS is that the most preferred solution should have the shortest distance from the positive ideal solution and, in the meantime, also have the longest distance from the negative ideal solution [4],[17].

The entropy measure of importance is used to score the contrast intensity of the $j$-th objective. Specifically, the larger entropy is, the less information is transmitted by the $j$-th objective leading to being removed from further consideration.

Let $R=\{R_{ij}|\ i=1,\ \ldots,\ n;\ j=1,\ \ldots,\ m\}$ be performance rating matrix of the $i$-th solution with respect to the $j$-th objective where each element represents the degrees of closeness of $F_{ij}$ to $F_j^{\min}$.

$$R = \begin{bmatrix} R_{11} & R_{12} & ... & R_{1m} \\ R_{21} & R_{22} & ... & R_{2m} \\ ... & ... & ... & ... \\ R_{n1} & R_{n2} & ... & R_{nm} \end{bmatrix} \quad (26)$$

Then, normalizing the performance rating matrix in Eq.(26) for each objective as

$$p_{ij} = \frac{R_{ij}}{\sum_{k=1}^{n} R_{kj}} \quad (27)$$

The contrast intensity of the $j$-th objective can now be measured to consider the amount of decision information contained in and transmitted by the objective by means of Shannon's entropy measure ($e_j$) which is defined as.

$$e_j = -\frac{1}{\ln n} \sum_{i=1}^{n} p_{ij} \ln p_{ij} \quad (28)$$

Thus, a total entropy is

$$E = \sum_{j=1}^{m} e_j \quad (29)$$

The objective weight ($\tilde{\lambda}_j$), therefore, is given by

$$\tilde{\lambda}_j = \frac{1}{m-E}(1-e_j) \quad (30)$$

The normalized performance rating element in Eq.(27) is now weighted as

$$v_{ij} = \tilde{\lambda}_j p_{ij}\ ,\ j = 1, 2,\dots, m:\ i=1, 2,\dots, n \quad (31)$$

The next step is to find the set of positive ideal solution ($A^+$) and negative ideal solution ($A^-$) which are defined by

$$A^+ = (\max(v_{i1}), \max(v_{i2}),..., \max(v_{im})) = (v_1^+, v_2^+,..., v_m^+) \quad (32)$$

$$A^- = (\min(v_{i1}), \min(v_{i2}),..., \min(v_{im})) = (v_1^-, v_2^-,..., v_m^-) \quad (33)$$

Hence, the distance of each solution from the positive ideal solution is given as

$$d_i^+ = \sqrt{\sum_{j=1}^{m} (v_{ij}^+ - v_{ij})^2} \quad (34)$$

Likewise, the distance of each solution from the negative ideal solution is given as

$$d_i^- = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_{ij}^-)^2} \quad (35)$$

The relative closeness to the positive ideal solution of the $i$-th solution is defined as

$$c_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad (36)$$

The most preferred solution is the solution having the highest $c_i$ value.

### 3.6 Comparing FMADM Approach to FCPRN Approach and TOPSIS Approach

The objective of comparing the proposed FMADM approach to FCPRN approach and TOPSIS approach is to demonstrate that the best optimal solution obtained from the proposed FMADM approach having more compromise characteristics than both FCPRN and TOPSIS approach in terms of both percentage total deviation from the ideal solution and fulfilling the preferred zones condition.

As the ideal solution that one wishes to attain in multi-objective problem is the solution consisting of minimum value in all objectives, therefore, it can be represented as $F^{ideal} = \{F_1^{\min}, F_2^{\min},..., F_m^{\min}\}$ when let $F^{ideal}$ be a set of the ideal solution. Such a solution, however, is impossible one due to all objectives having conflicting characteristics one another. Nonetheless, there is an attempt to elicit the best optimal solution which has the values in the vicinity of the minimum in all objectives as best possible

Accordingly, two measures below are utilized to compare the solution qualities obtained from FMADM, FCPRN and TOPSIS approach.

3.6.1 Percentage Total Deviation from the Ideal Solution $(\varepsilon_i)$. This measure uses the concept of Euclidean distance between two points in $n$-dimensions. Hence, the percentage total deviation between the chosen optimal solution and the ideal solution can be defined as.

$$\varepsilon_i = 100 * \sqrt{\sum_{j=1}^{m} \left( \frac{F_j^* - F_j^{\min}}{F_j^{\min}} \right)^2} \quad (37)$$

where

$F_j^*$ is the optimal value of objective $F_j$ of the chosen solution.

$F_j^{\min}$ is the minimum value of objective $F_j$

$m$ is the number of objectives.

3.6.2 Preferred Zones Condition ($PZ_j$) Here is the condition to check whether the objective values of the obtained solution is in the preferred zones by vetting from

$$PZ_j = \left( \frac{F_j^{\max} + F_j^{\min}}{2} \right) - F_j^* \geq 0 \quad (38)$$

Specifically, this condition help check whether the chosen solution has a well-balanced characteristics in all objectives. Only the criterion of percentage total deviation from the ideal solution $(\varepsilon_i)$ is not enough to indicate that the solution with lower value of $\varepsilon_i$ is the better one since certain objectives may have the dominant tendency towards the minimum value whereas the others probably rather away from the their respective minimum. As a result, it would be more favorable should all objectives of the chosen solution is in the preferred zones. In other words, they all have a good tendency towards the minimum value. The term in the bracket of Eq.(38) is referred to as threshold value.

## 4. NUMERICAL RESULTS

In this section, two numerical examples are provided to demonstrate the main features of the proposed approach. The simulation program of the proposed approach is written in MATLAB and run on 1.6 GHz Pentium M processor with 512 MB of RAM.

In this study, all objective functions are assumed to have an equal importance. That means all $w_j$ of Eq.(22) having the value of one in order that the obtained results are compared basing upon the same underlying hypotheses as above-mentioned FCPRN and TOPSIS approach. Also, it should be noted that such weight combinations do not involve the weight used in Eq.(11) since those weight combinations are simply employed to produce the different non-dominated solutions. They, therefore, do not involve in decision making for extracting the best optimal solution. The input information for the first test system is given below.

***Test System 1: Three Generators with Four Objectives***

*Fuel Cost Characteristics of Thermal Plants($/h)*

$$F_{11} = 5.25x10^{-3} P_1^2 + 8.6625P_1 + 328.125$$

$$F_{12} = 6.085x10^{-3} P_2^2 + 10.0403P_2 + 136.9125$$

$$F_{13} = 5.9155x10^{-3} P_3^2 + 9.7606P_3 + 59.155$$

*$NO_X$ Emission Characteristics of Thermal Plants (kg/h)*

$$F_{21} = 0.006323P_{g1}^2 - 0.38128P_{g1} + 80.9019$$

$$F_{22} = 0.006483P_{g2}^2 - 0.79027P_{g2} + 28.8249$$

$$F_{23} = 0.003174P_{g3}^2 - 1.36061P_{g3} + 324.1775$$

*$SO_2$ Emission Characteristics of Thermal Plants (kg/h)*

$$F_{31} = 0.001206P_{g1}^2 + 5.05928P_{g1} + 51.3778$$

$$F_{32} = 0.002320P_{g2}^2 + 3.84624P_{g2} + 182.2605$$

$$F_{33} = 0.001284P_{g3}^2 + 4.45647P_{g3} + 508.5207$$

*$CO_2$ Emission Characteristics of Thermal Plants (kg/h)*

$$F_{41} = 0.265110P_{g1}^2 - 61.01945P_{g1} + 5080.148$$

$$F_{42} = 0.140053P_{g2}^2 - 29.95221P_{g2} + 3824.770$$

$$F_{43} = 0.105929P_{g3}^2 - 9.552794P_{g3} + 1342.851$$

*The B-Coefficients ($MW^1$)*

$$B_{ij} = \begin{bmatrix} 1.36255x10^{-4} & 1.753x10^{-5} & 1.8394x10^{-4} \\ 1.754x10^{-5} & 1.5448x10^{-4} & 2.82765x10^{-4} \\ 1.8394x10^{-4} & 2.82765x10^{-4} & 1.6147x10^{-3} \end{bmatrix}$$

*Power output constraint*

$$50 \le P_1 \le 250 \text{ MW}$$

$$5 \le P_2 \le 150 \text{ MW}$$

$$15 \le P_3 \le 100 \text{ MW}$$

The load demand is 190 MW

**Table 1. Comparison of the Best Compromise Solution derived from FMADM , FCPRN and TOPSIS Approach (3 Generators, 4 Objectives)**

| Approach | | F1 | F2 | F3 | F4 |
|---|---|---|---|---|---|
| FCPRN | Value | 2,450.84 | <u>392.09</u> | 1,637.92 | 5,286.81 |
| | PZ | + | - | + | + |
| | Weight | 0.3 | 0.3 | 0.4 | 0.0 |
| TOPSIS | Value | <u>2,657.82</u> | 302.26 | <u>1,706.41</u> | <u>6,637.65</u> |
| | PZ | - | + | - | - |
| | Weight | 0.0 | 1.0 | 0.0 | 0.0 |
| FMADM | Value | 2,487.43 | 369.18 | 1,635.50 | 5,556.68 |
| | PZ | + | + | + | + |
| | Weight | 0.2 | 0.4 | 0.4 | 0.0 |
| MDS | Value | 2,509.35 | 343.46 | <u>1,674.95</u> | 5,642.26 |
| | PZ | + | + | - | + |
| | Weight | 0.3 | 0.7 | 0.0 | 0.0 |
| Minimum Value | | 2,393.91 | 302.26 | 1,604.01 | 5,183.75 |
| Maximum Value | | 2,657.82 | 475.01 | 1,706.73 | 6,637.76 |
| Threshold Value | | 2,525.87 | 388.64 | 1,655.37 | 5,910.76 |

The 224 non-dominated solutions are produced considering all the objectives concurrently through 224 different weight combinations using Newton-Raphson algorithm. The running time for eliciting the best compromise solution from the proposed FMADM approach takes 872.30 s.

Table 1 illustrates the comparison of the best compromise solutions obtained from the FMADM, FCPRN and TOPSIS approach in terms of objective value including checking the preferred zones condition. The percentage total deviation from the minimum of the solution obtained from FMADM, FCPRN and TOPSIS approach are shown in Table 2.

As seen from Table 1, the FMADM approach is the only one approach which is satisfied with preferred zones condition since all $PZ_j$ have positive sign. That means the obtained solution having a good balance towards the minimum in all objectives.

Minimum deviation solution (MDS) shown in Table 1 is the solution yielding the lowest percentage total deviation from the ideal solution which can be seen in Table 2. It is noted that despite MDS having the lowest percentage total deviation, it still have certain objectives having the value higher than the threshold value. In other words, some objectives of MDS have the values further away from the ideal solution than the best optimal solution derived from FMADM approach.

**Table 2. Percentage Total Deviation from the Ideal Solution (3 Generators, 4 Objectives)**

| Approach | Deviation from the Ideal Value (%) | | | | Total Deviation (%) |
|---|---|---|---|---|---|
| | F1 | F2 | F3 | F4 | |
| FCPRN | 2.38 | 29.72 | 2.11 | 1.99 | 29.96 |
| TOPSIS | 11.02 | 0.00 | 6.38 | 28.05 | 30.80 |
| FMADM | 3.91 | 22.14 | 1.96 | 7.19 | 23.69 |
| MDS | 4.82 | 13.63 | 4.42 | 8.85 | 17.52 |

As seen from Table 2, the FMADM approach yields the solution having percentage total deviation from the ideal solution lower than both FCPRN and TOPSIS approach. Also, it should be noted that TOPSIS approach does not yield the lowest percentage total deviation despite the fact that it uses the concept of selecting the solution which has the shortest distance from the positive ideal solution, meanwhile, also has the longest distance from the negative ideal solution. This is because the weight of second objective ($\tilde{\lambda}_2$), for decision making according to Eq.(30), has the value much higher than the others especially when compared to $\tilde{\lambda}_1$ and $\tilde{\lambda}_3$ as shown below

$$\tilde{\lambda}_1 = 0.035891 \qquad \tilde{\lambda}_2 = 0.636456$$

$$\tilde{\lambda}_3 = 0.011373 \qquad \tilde{\lambda}_4 = 0.316280$$

This is resulted from objective F2 having high contrast intensity when compared to the objective F1 and F3

TOPSIS's mechanism for selecting the most preferred solution, therefore, boils down to attempting to elicit the solution which has the value of objective F2 approaching its minimum as nearest as possible. From calculating the value of relative closeness to the positive ideal solution ($c_i$) using Eq.(36), the solution that produces the highest $c_i$ is the solution yielding the minimum value of objective F2 thereby making the percentage total deviation of objective F2 zero.

Importantly, it should be noticed that the best optimal solution obtained from TOPSIS approach is the one which also provides the maximum value of objective F1, in the meantime, the value of both objective F3 and F4 is approaching their respective maximum. Hence, it is considered as an extreme solution thereby making it rather difficult to be chosen in real-world application.

***Test System 2: Six Generators with Three Objectives***

The information for the second test system is given below.

*Fuel Cost Characteristics of Thermal Plants($/h)*

$$F_{11} = 0.002035P_{g1}^2 + 8.43205P_{g1} + 85.6348$$

$$F_{12} = 0.003866P_{g2}^2 + 6.41031P_{g2} + 303.7780$$

$$F_{13} = 0.002182P_{g3}^2 + 7.42890P_{g3} + 847.1484$$

$$F_{14} = 0.001345P_{g4}^2 + 8.30154P_{g4} + 274.2241$$

$$F_{15} = 0.002182P_{g5}^2 + 7.42890P_{g5} + 847.1484$$

$$F_{16} = 0.005963P_{g6}^2 + 6.91559P_{g6} + 202.0258$$

*$NO_X$ Emission Characteristics of Thermal Plants (kg/h)*

$$F_{21} = 0.006323P_{g1}^2 - 0.38128P_{g1} + 80.9019$$

$$F_{22} = 0.006483P_{g2}^2 - 0.79027P_{g2} + 28.8249$$

$$F_{23} = 0.003174P_{g3}^2 - 1.36061P_{g3} + 324.1775$$

$$F_{24} = 0.006732P_{g4}^2 - 2.39928P_{g4} + 610.2535$$

$$F_{25} = 0.003174P_{g5}^2 - 1.36061P_{g5} + 324.1775$$

$$F_{26} = 0.006181P_{g6}^2 - 0.39077P_{g6} + 50.3808$$

*$SO_2$ Emission Characteristics of Thermal Plants (kg/h)*

$$F_{31} = 0.001206P_{g1}^2 + 5.05928P_{g1} + 51.3778$$

$$F_{32} = 0.002320P_{g2}^2 + 3.84624P_{g2} + 182.2605$$

$$F_{33} = 0.001284P_{g3}^2 + 4.45647P_{g3} + 508.5207$$

$$F_{34} = 0.000813P_{g4}^2 + 4.97641P_{g4} + 165.3433$$

$$F_{35} = 0.001284P_{g5}^2 + 4.45647P_{g5} + 508.5207$$

$$F_{36} = 0.003578P_{g6}^2 + 4.14938P_{g6} + 121.2133$$

*The B-Coefficients ($MW^{1}$)*

$$B_{ij} = \begin{bmatrix} 20 & 1 & 1.5 & 0.5 & 0 & -3 \\ 1 & 30 & -2 & 0.1 & 1.2 & 1 \\ 1.5 & -2 & 10 & -1 & 1 & 0.8 \\ 0.5 & 0.1 & -1 & 15 & 0.6 & 5 \\ 0 & 1.2 & 1 & 0.6 & 25 & 2 \\ -3 & 1 & 0.8 & 5 & 2 & 21 \end{bmatrix} \text{x}10^{-5}$$

*Power output constraint*

$$90 \le P_1 \le 350\,\text{MW}$$

$$100 \le P_2 \le 500\,\text{MW}$$

$$200 \le P_3 \le 800\,\text{MW}$$

$$100 \le P_4 \le 500\,\text{MW}$$

$$150 \le P_5 \le 600\,\text{MW}$$

$$100 \le P_6 \le 500\,\text{MW}$$

The load demand is 1800 MW.

The different 58 weight combinations are used to generate the 58 non-dominated solutions. The running time for this test system takes 676.52 s to derive the best compromise solution from the proposed approach.

**Table 3. Comparison of the Best Compromise Solution obtained from FMADM, FCPRN and TOPSIS Approach (6 Generators, 3 Objectives)**

| Approach | | F1 | F2 | F3 |
|---|---|---|---|---|
| FCPRN | Value | 18,745.90 | 2,165.21 | 11,236.45 |
| | PZ | + | + | + |
| | Weight | 0.4 | 0.3 | 0.3 |
| TOPSIS | Value | <u>*18,950.86*</u> | 2,070.13 | <u>*11,356.50*</u> |
| | PZ | - | + | - |
| | Weight | 0.0 | 1.0 | 0.0 |
| FMADM | Value | 18,778.75 | 2,122.26 | 11,255.47 |
| | PZ | + | + | + |
| | Weight | 0.4 | 0.5 | 0.1 |
| MDS | Value | <u>*18,862.17*</u> | 2,079.83 | <u>*11,304.31*</u> |
| | PZ | - | + | - |
| | Weight | 0.2 | 0.8 | 0.0 |
| Minimum Value | | 18,721.38 | 2,070.13 | 11,222.94 |
| Maximum Value | | 18,950.86 | 2,282.97 | 11,356.50 |
| Threshold Value | | 18,836.12 | 2,176.55 | 11,289.72 |

The comparison of objective value and checking the preferred zones condition of the best optimal solution obtained from the FMADM, FCPRN and TOPSIS approach are given in Table 3. Percentage total deviation from the ideal solution is shown in Table 4. It can be seen in Table 3 that both the FMADM and FCPRN approach yields all objective values satisfied with preferred zones condition due to all $PZ_j$ having positive sign. On the contrary, TOPSIS and MDS approach has certain objectives unsatisfied with the preferred zones condition which is likewise the above result in test system 1.

**Table 4. Percentage Total Deviation from the Ideal Solution (6 Generators, 3 Objectives)**

| Approach | Deviation from the Ideal Value (%) | | | Total Deviation (%) |
|---|---|---|---|---|
| | F1 | F2 | F3 | |
| FCPRN | 0.13 | 4.59 | 0.12 | 4.60 |
| TOPSIS | 1.23 | 0.00 | 1.19 | 1.71 |
| FMADM | 0.31 | 2.52 | 0.29 | 2.55 |
| MDS | 0.75 | 0.47 | 0.73 | 1.14 |

Given the percentage total deviation, FCPRN approach still has the value higher than the proposed approach which is similar to the results of test system 1. The percentage deviation value of objective F2 derived from TOPSIS approach is zero since the objectives weight $\tilde{\lambda}_2$ is much more dominant than the others, namely $\tilde{\lambda}_2 =$ 0.985941 whereas $\tilde{\lambda}_1$ and $\tilde{\lambda}_3$ is 0.007258 and 0.006801 respectively. Thus, both $\tilde{\lambda}_1$ and $\tilde{\lambda}_3$ hardly effects decision making. Consequently, the mechanism for selecting the most preferred solution of TOPSIS approach is an attempt to choose the solution approaching F2 as closet as possible.

Incidentally, the percentage total deviation of TOPSIS approach becomes lower than the FMADM approach in this test system as the obtained solution yields the minimum value to the objective which has high contrast intensity, F2, meanwhile the rest of objectives having low contrast intensity have been decreased.

In addition, it should also be observed that the best optimal solution obtained from the TOPSIS approach gives the minimum value of objective F2, meanwhile, it also provides the maximum value for objective F1 and F3. It is obvious that the consequence is likewise the result in test system 1. Therefore, the solution obtained from TOPSIS approach is regarded as an extreme solution and is less compromise than the one derived from the FMADM approach. In practical way, despite the fact that the objectives for reducing environmental effects is increasingly concerned, the operating fuel cost objective (F1), however, is still the important one that always have to be considered and can not be neglected.

Thus, it is unfavorable to choose the best optimal solution yielding the maximum operating fuel cost or so since it is a key factor that determines viability of a utility. Consequently, it would be more favorable should the best optimal solution of multi-objective problem have well-balanced characteristics in all objectives towards their respective minimum.

## 5. CONCLUSION

The proposed fuzzy multi-attribute decision making approach was applied to help system operators extract the best compromise solution out of a set of non-dominated solutions of multi-objective thermal power dispatch problem. The obtained best compromise solution is compared to the ones derived from fuzzy cardinal priority ranking normalized approach (FCPRN) and the technique for order preference by similarity to ideal solution (TOPSIS) by means of two measures viz. percentage total deviation from the ideal solution and preferred zones condition

Given the percentage total deviation from the ideal solution, it is evident that the solution obtained from the FMADM approach is superior to the one derived form FCPRN approach. For TOPSIS approach, when the number of objectives has been increased, the value of percentage total deviation becomes larger, in the meantime, also higher than the value obtained from the proposed approach. Moreover, the solution obtained from TOPSIS approach is considered an extreme solution since it has an inclination to produce the minimum in an objective, meanwhile, they also yields the maximum or so for others.

For preferred zones condition, it helps check the quality of balance in all objectives towards the minimum. The simulation results demonstrate that the proposed FMADM approach is superior to TOPSIS approach. For the FCPRN approach, even it has fulfilled the condition in the case of the reducing the number of objective functions from four to three, however, its percentage total deviation still has the value higher than the proposed approach.

Thus, taking these two measures into account concurrently, the FMADM approach demonstrates that it can elicit the best optimal solution which has characteristics more compromise and favorable than its counterparts. The further research will be focused on the effects of the best compromise solution obtained from FMADM, FCPRN and TOPSIS approach when each objective has different important degree.

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