Optimal Capacitor Placement in Unbalanced Loading Distribution System with Nonlinear Loads by Adaptive Particle Swarm Technique

S. Auchariyamet and S. Sirisumrannukul

Abstract— *A capacitor placement problem in an unbalanced loading distribution system with the presence of nonlinear loads is formulated in this paper. The objective function of the problem is to minimize the total annual cost comprising the costs of energy loss, peak demand loss, and capacitor investment. This objective function is subjected to power flow equations, bus voltage limits, voltage distortion constraint, and also maximum capacitor kVAr to be installed at each bus. An adaptive particle swarm optimization (PSO) is proposed to search for an optimal or near-optimal solution. This adaptive technique appropriately activates the last obtained solution to seek for better solutions when a fixed number of iterations are reached or when no improvement solution is observed over the course of iterations. A radial distribution system of Provincial Electricity Authority (PEA), Thailand, which consists of 28 buses and 19 load points with a two-step load pattern, is studied to demonstrate the effectiveness of the proposed methodology. Test results indicate that the obtained optimal solutions give a saving in the total cost while satisfying all the specified constraints. In addition, the effects of unbalanced loading and nonlinearity of load on the optimal solutions are investigated. It is found that increases of these two factors introduce more total loss in the network and more total capacitor kVAr required for reactive power compensation, and therefore the saving in the total cost is decreased due to more investment cost for capacitors.*

Keywords— **Capacitor placement, Nonlinear loads, Particle swarm optimization, Unbalanced distribution system.**

1. INTRODUCTION

Electrical energy from power plants is transmitted to end-use customers by transmission and distribution systems. Around 13% of power generated is consumed as power loss at the distribution levels [1]. The power loss is determined as function of square of branch current which consists of real and reactive component. A portion of power loss in distribution systems, produced by reactive current, could be diminished by capacitor placement which is one of the most effective and useful methods.

With shunt capacitors, reactive power compensation is provided to reduce power and energy loss, to regulate bus voltages, to improve power quality, and to release feeders and system capacity. The extent of these advantages depends on how capacitors are placed in the distribution system. For this reason, it is necessary to solve capacitor placement problem to simultaneously determine the optimal locations, sizes, and types of capacitors.

Special attention should be paid to capacitor placement when nonlinear loads appear in the distribution system owing to widely used power electronics-base devices. This is because nonlinear loads behave as harmonic current sources. Sizes and locations of shunt capacitor are significant factors that response to harmonics. The combination of harmonic sources, system reactance, and capacitors introduce both series and parallel resonant frequencies to the networks. Due to the resonant conditions, the total harmonic distortion can be magnified greater than the permissible level. The capacitor placement problem, therefore, should take harmonic constraint into account to assure that the obtained optimal solution does not result in an excessive harmonic distortion.

The practical aspects of distribution system should also be considered when the capacitor placement problem is formulated. The actual distribution feeders are unbalanced because of the existence of single-phase and two-phase line segments as well as the three-phase unbalanced loads and the mutual coupling among phase conductors [2]. With unbalanced conditions, the generation and propagation of harmonics are more complicated [3]. Furthermore, the inclusion of system unbalances increases the dimension of the capacitor placement problem because all three phases have to be considered instead of the single phase balanced representation.

The capacitor placement problem is a zero-one decision with discrete step of standard capacitor bank size, each step of which has a different installation cost. Such zero-one decision and discrete steps make the problem as a nonlinear and non-differentiable mixed integer optimization problem. In addition, the solutions to be analyzed generally increase with the size of distribution system. The capacitor placement problem is, therefore, a hard and large-scale combinatorial problem, where most of conventional optimization tools find it difficult to search for the optimal solution.

One efficient method to solve the capacitor placement problem is particle swarm optimization (PSO). PSO is a heuristic search technique inspired by social interaction

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in herd of animals (e.g. bird flocks or fish schools). It was first introduced in 1995 [4]. Many advances in PSO development extend its abilities to handle difficult optimization problems in science and engineering fields. Key attractive advantages of PSO are its simplicity in concept and implementation, less computation time, and inexpensive memory for computer resource.

The capacitor placement problem is a combinatorial optimization problem that features a multimodal landscape of locally optimal solutions scattering throughout the search space. In some cases where the globally optimal solutions are slightly different from locally optimal solutions, the use of a conventional PSO algorithm would get trapped at local optimums. For this reason, the conventional PSO algorithm requires some variations to remedy this drawback. This paper presents an adaptive PSO technique to seek for an optimal or near-optimal solution at some other regions in the search space. The key of the approach is a modification on the last obtained solution with the proper number of activations during the computational process. The effectiveness of the proposed methodology is illustrated by test results of an 85-bus distribution system.

The objective function of capacitor placement problem in unbalanced loading distribution system connected to nonlinear loads is to minimize the total annual cost comprising the costs of energy loss, peak demand loss, and capacitor investment. This objective function is subjected to power flow equations, bus voltage limits, voltage distortion constraint, and number of discrete capacitors to be installed at each bus. A 28-bus distribution system with a two-step load pattern of Provincial Electricity Authority (PEA), Thailand, is used to investigate the effects of unbalanced loading and nonlinearity of load on the optimal solutions.

2. PROBLEM FORMULATION

The objective function of the capacitor placement problem is to minimize the total annual cost due to energy loss cost, peak power loss cost, and the investment cost of fixed and switched type capacitors. It can be stated as:

$$\min F = (\sum_{i=1}^{nl} k_{e,i} T_i P_i) + k_p P_{lp} + (\sum_{j\in SCf} k_{cf} Q_j^{cf}) + (\sum_{j\in SCs} k_{cs} Q_j^{cs}) \quad (1)$$

where F = total cost ($, Baht)
nl = number of load levels
$k_{e,i}$ = cost of energy loss for load level i ($/kWh, Baht/kWh)
T_i = time duration for load level i (hr)
P_i = power loss for load level i (kW)
k_p = cost of peak power loss ($/kW, Baht/kW)
P_{lp} = peak power loss (kW)
SC_f, SC_s = set of buses for fixed and switched type capacitor placement
k_{cf}, k_{cs} = investment cost for fixed and switched type capacitor ($/kVAr, Baht/kVAr)
Q_j^{cf}, Q_j^{cs} = fixed and switched type capacitor installed at bus j (kVAr)

Power loss, P_i, in Equation (1) is the summation of losses from all branches in the system being considered. Since branch loss is determined from loss in each phase of that branch. P_i, therefore, can be written as [5]:

$$P_i = \sum_{k=1}^{L} [\mathbf{I}_{k,i}]^T [\mathbf{R}_k][\mathbf{I}_{k,i}] \quad (2)$$

where P_i = power loss for load level i (kW)
L = number of branches in the system
$[\mathbf{I}_{k,i}]$ = three-phase rms current matrix in branch k for load level i (A)
$[\mathbf{R}_k]$ = three-phase resistance matrix of branch k (ohm)

$[\mathbf{I}_{k,i}]$ and $[\mathbf{R}_k]$ in Equation (2) are expressed as:

$$[\mathbf{I}_{k,i}] = \begin{bmatrix} \sqrt{\sum_{h=1}^{nh} \left|I_{k,i}^{A,h}\right|^2} \\ \sqrt{\sum_{h=1}^{nh} \left|I_{k,i}^{B,h}\right|^2} \\ \sqrt{\sum_{h=1}^{nh} \left|I_{k,i}^{C,h}\right|^2} \end{bmatrix} \quad (3)$$

$$[\mathbf{R}_k] = \begin{bmatrix} r_k^{AA} & r_k^{AB} & r_k^{AC} \\ r_k^{BA} & r_k^{BB} & r_k^{BC} \\ r_k^{CA} & r_k^{CB} & r_k^{CC} \end{bmatrix} \quad (4)$$

where nh = maximum harmonic order
$\left|I_{k,i}^{p,h}\right|$ = current magnitude of phase p in branch k for load level i at harmonic order h
p = $\{A, B, C\}$
$r_k^{AA}, r_k^{BB}, r_k^{CC}$ = self resistance of conductor in phase A, B, C of branch k (ohm)
$r_k^{AB}, r_k^{AC}, r_k^{BA}, r_k^{BC}, r_k^{CA}, r_k^{CB}$ = mutual coupling resistance between phase conductor of branch k (ohm)

The objective function is subjected to the following operational constraints.

- Three-phase power balance equations at fundamental frequency.
- Bus voltage and voltage distortion contraints:

$$V_{\min} \leq \sqrt{\left|V_{j,i}^{p,1}\right|^2 + \sum_{h \neq 1}^{nh} \left|V_{j,i}^{p,h}\right|^2} \leq V_{\max} \tag{5}$$

$$THD_{j,i}^{p}(\%) = 100 \times \frac{\sqrt{\sum_{h \neq 1}^{nh} \left|V_{j,i}^{p,h}\right|^2}}{\left|V_{j,i}^{p,1}\right|} \leq THD_{\max} \tag{6}$$

where

$\left\|V_{j,i}^{p,1}\right\|$	=	voltage magnitude in phase p of bus j for load level i at fundamental frequency
$\left\|V_{j,i}^{p,h}\right\|$	=	voltage magnitude in phase p of bus j for load level i at harmonic order h
nh	=	maximum harmonic order
$V_{\min}$	=	minimum limit of bus voltage
$V_{\max}$	=	maximum limit of bus voltage
$THD_{j,i}^{p}$	=	total harmonic distortion of voltage in phase p at bus j for load level i
$THD_{\max}$	=	maximum permissible limit for total harmonic distortion of voltage

- Limitation of capacitor kVAr to be placed at each bus:

$$0 \leq Q_{j,i}^{c} \leq Q_{\max}^{c} \tag{7}$$

where

$Q_{j,i}^{c}$	=	capacitor to be placed at bus j for load level i (kVAr)
$Q_{\max}^{c}$	=	maximum capacitor to be placed at any bus (kVAr)

3. THE CONVENTIONAL PSO ALGORITHMS

The PSO technique conducts the searching process using a population of particles. Each particle represents a potential solution in n-dimensional search space of the problem being considered. Particles change their positions, from current iteration to next iteration, based on their velocities to locate a good optimum. In general, the implementation of a conventional PSO algorithm consists of three main steps, namely, 1) generate initial particle's positions and velocities, 2) evaluate fitness value of each particle, and 3) update velocity and position of all particles [6].

First, the positions and velocities of the initial swarm of particles are randomly generated to allow all particles randomly distributed across the search space. The fitness value of each particle is evaluated in the second step to determine the best position of each particle and also to reveal the particle that has the best global fitness value in the current swarm. Next, the velocities of all particles are updated by using the information from the current velocity, the best solution of each particle, and the best solution found by the best particle in swarm. The velocity update from iteration t to $(t+1)$ can be expressed as [7]:

$$v_{id}(t+1) = wv_{id}(t) + c_1 r_{1d}(t)[y_{id}(t) - x_{id}(t)] + c_2 r_{2d}(t)[\hat{y}_d(t) - x_{id}(t)] \tag{8}$$

where

x	=	position of particle
v	=	velocity of particle
w	=	inertia weight
c_1, c_2	=	positive acceleration constants
r_{1d}, r_{2d}	=	uniformly distributed random values in the range [0,1]
y	=	personal best position
$\hat{y}$	=	global best position
i	=	i^{th} particle
d	=	d^{th} dimension
id	=	particle i in dimension d

Position update is the last step. The new position of each particle is calculated by [7]:

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \tag{9}$$

The step of velocity and position update is repeated and will terminate when a stopping criterion is met, e.g. maximum number of iteration is reached or the change in solution is smaller than the pre-specified tolerance.

4. THE PROPOSED ADAPTIVE PSO FOR CAPACITOR PLACEMENT PROBLEM

When a conventional PSO algorithm is applied to solve the capacitor placement problem, each particle will represent the size of capacitor to be placed at each bus for all load levels. Power flow is employed to calculate bus voltages and power losses, while the total cost is defined as a fitness value.

Since the capacitor placement problem has multimodal of locally optimal solutions [8], if solutions during the iterative process of conventional PSO are detected no improvement, it may result from convergence to an optimal solution or from being trapped at any local optimums. For the later case, a conventional PSO search algorithm should be tried to seek for better solutions at some other regions in the search space. This could be done by activating the position of particles (i.e., randomly generate with a uniform probability distribution) at the prespecified iteration instead of updating them by Equation (9). After activation, the process of position update in the conventional PSO algorithm resumes as normal. In this work, the PSO algorithm combined with the activation of particles is called adaptive PSO.

The key point of the adaptive PSO technique is to identify the appropriate numbers of activation. Two methods are suggested as follows:

1) Activate the solution when a fixed number of iterations are reached.

2) Activate the solution when no improvement solution is observed over the course of iterations.

The numerical results from 300 independent experimental runs with 10-, 15-, 23-, 34-, 69- and 85-bus systems indicate that when the proposed adaptive PSO is implemented with the proper numbers of activation, it can introduce an improvement in optimal solution which results in the reduction of total cost about 5-10% compared with that given by the conventional PSO. However, there is a probability about 0.3 that this technique will offer worse solutions compared with those without the activation. For this reason, the solution before activation should be stored and compared with the solution after the activation. The one with a better total cost will be selected as the final solution.

To demonstrate the performance of the proposed adaptive PSO, the test results of three cases from one experiment with an 85-bus system are presented in Table 1.

Table 1. Summary of test results from an 85-bus system

	Case A	Case B	Case C
Loss at light load (kW)	124.37	104.39	92.13
Loss at medium load (kW)	154.21	146.12	146.54
Loss at peak load (kW)	261.43	254.21	252.23
Total capacitor (kVAr)	7,350	6,150	5,700
Peak power loss cost ($)	31,372	30,505	30,268
Energy loss cost ($)	70,668	65,619	63,979
Capacitor cost ($)	2,595	2,198	2,018
Total cost ($)	104,635	98,322	96,265
Reduction in total cost (%)	-	6.03	7.99

The solution in case A is obtained from a conventional PSO algorithm while the solutions in cases B and C are calculated from the adaptive PSO. For case B, the activation point, identified by method 1, is set at iteration 150, whereas the activation points in case C are defined by method 2; that is, if the total cost remains unchanged for 30 iterations, the solution will be activated. The convergence characteristics for the three cases are given in Figures 1 to 3.

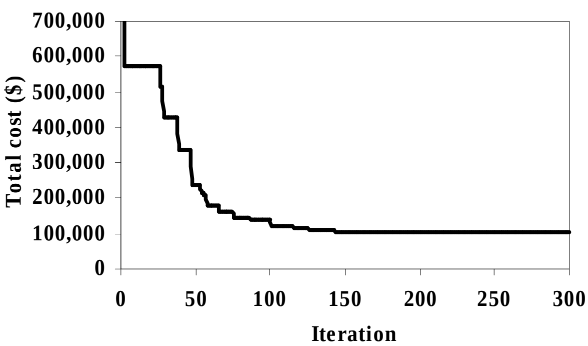

Fig. 1. Convergence characteristic of total cost in an 85-bus system obtained from Case A.

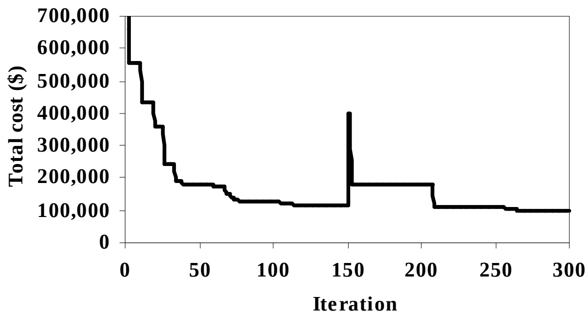

Fig. 2. Convergence characteristic of total cost in an 85-bus system obtained from Case B.

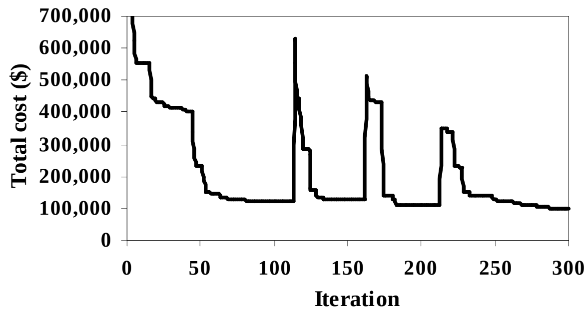

Fig. 3. Convergence characteristic of total cost in an 85-bus system obtained from Case C.

It can be seen from Table 1 that the proposed adaptive PSO have lower total capacitor, peak power loss cost, energy cost, capacitor cost, and total cost. In Figure 1, the solution of case A converges at iteration 160 with a total cost of $104,635. Case B, whose solution is activated just only one time at iteration 150 as shown in Figure 2, gives a total cost of $98,322. In case C, the solution is activated three times at iterations 113, 162, and 212 as shown in Figure 3, gives a total cost of $96,265. The total cost of cases B and C are about 6% and 8% lower than that of case A respectively.

The tests results of an 85-bus distribution system demonstrate that with activation, better solutions are found, indicating the effectiveness of the proposed adaptive PSO. However, because the appropriate number of activations for both methods cannot be determined in advance, experimental runs are still required but it is worth doing so as there is a high possibility to obtain improved solutions. Note that this proposed technique, as in many heuristic techniques, does not guarantee the globally optimal solution, but at least this adaptive strategy could be useful.

5. SOLUTION TECHNIQUE BY ADAPTIVE PSO

The adaptive PSO based approach takes the following steps:

Step 1: Input the line and bus data of a distribution system, all operational constraints, the values of all the variables associated with Equation (1), and PSO parameters.

Step 2: Generate an initial population of particles with random positions and velocities. The dimension of each particle is $m \times nl$, where m is the number of buses in the system and nl is the number of load levels.

Step 3: Set iteration index $t = 0$

Step 4: For each particle, perform an AC three-phase distribution power flow and harmonic power flow to obtain a power losses, bus voltages, and THD values for all the load levels.

Step 5: Calculate the total cost of each particle using Equation (1) and check all the constraints. If any of the constraints is violated, a penalty term is added to the total cost. The calculated total cost is served as a fitness value of particle.

Step 6: Compare the fitness value of each particle with the personal best, $Pbest$. If the fitness value is lower than $Pbest$, set this value as the current $Pbest$, and record the particle corresponding to this $Pbest$ value.

Step 7: Select the minimum value of $Pbest$ from all particles to be the current global best, $Gbest$, and record the particle corresponding to this $Gbest$ value.

Step 8: Check whether $t = t_a$, where t_a is the activation point. If yes, all particles should be activated, or else the velocity and position of particles are updated by Equations (8) and (9).

Step 9: If the maximum number of iteration is reached, the particle associated with the current $Gbest$ is the optimal solution, and then go to Step 10. Otherwise, set $t = t + 1$ and return to Step 4.

Step 10: From the optimal solution obtained in Step 9, the types and sizes of capacitors to be placed at each bus are identified. If the sizes of capacitor for all load levels are different, then the capacitor at this bus can be considered as switched type. On the other hand, if the sizes of capacitor are identical for all load levels, the capacitor at this bus is fixed type. The capacitor size for each bus is defined as the maximum value of capacitor found in any load level at that bus.

Step 11: Calculate energy loss cost, peak power loss cost, capacitor placement cost, the total cost, voltages and THD levels for all buses using the obtained optimal solution.

Step 12: Print out the results.

6. THREE-PHASE POWER FLOW AND HARMONIC POWER FLOW

For an unbalanced distribution system, a power flow algorithm with complete three-phase model is required to determine power losses and bus voltages at fundamental frequency. The technique in [9], based on the backward-forward sweep technique and tailored for radial distribution systems, is employed. This three-phase power flow algorithm develops a "bus-injection to branch-current matrix (BIBC)" based on the topological structure of distribution system to indicate the relationship between load currents and branch currents. All bus voltages are then computed by iterative process using BIBC matrix. The three-phase bus voltage in the k^{th} iteration of power flow calculation is stated by:

$$[\mathbf{V}]^{(k)} = [\mathbf{V_0}] - [\mathbf{BIBC}]^T[\mathbf{Z}][\mathbf{BIBC}][\mathbf{I}]^{(k)} \tag{10}$$

where
$[\mathbf{V}]^{(k)}$ = vector of three-phase bus voltages
$[\mathbf{V_0}]$ = vector of three-phase voltage at the slack bus
$[\mathbf{BIBC}]$ = three-phase bus-injection to branch-current matrix
$[\mathbf{Z}]$ = three-phase primitive impedance matrix
$[\mathbf{I}]^{(k)}$ = three-phase vector of load currents

$[\mathbf{V}]^{(k)}$ and demand load at every bus are used to calculate $[\mathbf{I}]^{(k+1)}$, so that $[\mathbf{V}]^{(k+1)}$ in the next iteration can be evaluated. This process is repeated until the difference of bus voltages between a current iteration and the previous one is smaller than the prespecified tolerance. Note that all vectors and matrices in Equation (10) are in three-phase format; therefore, elements in $[\mathbf{V}]^{(k)}$, $[\mathbf{V_0}]$, and $[\mathbf{I}]^{(k)}$ are 3×1 subvectors and elements in $[\mathbf{Z}]$ and $[\mathbf{BIBC}]$ are 3×3 submatrices.

The harmonic power flow is performed under different harmonic orders to find harmonic voltages and harmonic losses. In this paper, harmonic power flow is based on the technique proposed in [10], where harmonic voltages are computed by:

$$[\mathbf{V}_i^h] = [\mathbf{Y}_i^h]^{-1}[\mathbf{I}_i^h] \tag{11}$$

where
$[\mathbf{V}_i^h]$ = vector of three-phase bus voltages at harmonic order h for load level i
$[\mathbf{Y}_i^h]$ = three-phase bus admittance matrix at harmonic order h for load level i
$[\mathbf{I}_i^h]$ = vector of three-phase harmonic current at harmonic order h for load level i

All the variables in Equation (11) are expanded to three-phase format, so that elements in $[\mathbf{V}_i^h]$ and $[\mathbf{I}_i^h]$ are 3×1 subvectors whereas elements in $[\mathbf{Y}_i^h]$ are 3×3 submatrices.

$[\mathbf{Y}_i^h]$ is formulated from the feeder admittances of the network, all load admittances, and all shunt capacitor admittances. The values of all the admittances are varied with harmonic order as defined in [10]. Nonlinear loads are treated as harmonic sources which inject harmonic currents into the system. Elements in $[\mathbf{I}_i^h]$ are, therefore, calculated by:

$$I_{j,i}^{p,h} = \rho_j^p \frac{P_{L(j,i)}^p - jQ_{L(j,i)}^p}{h(V_{j,i}^{p,1})^*} \tag{12}$$

where $I_{j,i}^{p,h}$ = injection current in phase p of bus j for load level i at harmonic order h

ρ_j^p = nonlinear portion of load in phase p of bus j

$P_{L(j,i)}^p$ = real power demand in phase p of bus j for load level i

$Q_{L(j,i)}^p$ = reactive power demand in phase p of bus j for load level i

$(V_{j,i}^{p,1})*$ = conjugate of voltage in phase p of bus j for load level i at fundamental frequency

p = $\{A,B,C\}$

h = order of harmonic being considered

7. CASE STUDY

The distribution system of PEA, designated as Kalasin Feeder 5, is used as the test system in this work. The system has 28 buses and 19 load points. Its configuration, load data including feeder data are provided in Figure 4 [11].

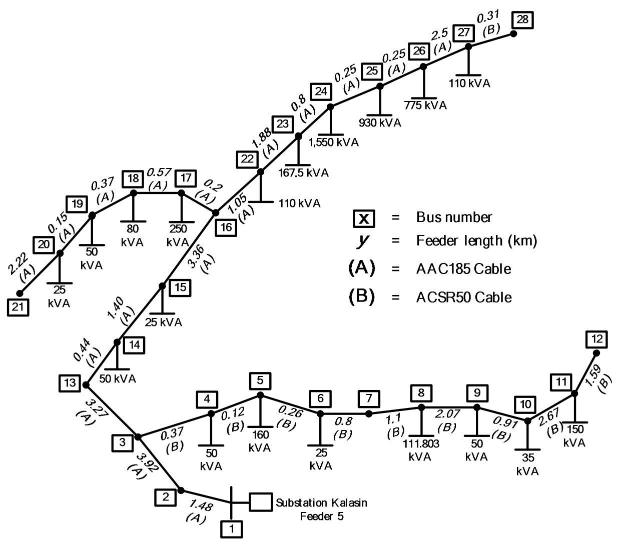

Fig. 4. Kalasin feeder 5 distribution system.

The base values for voltage and power are 22 kV and 100 MVA. The power factor of all load points is assumed as 0.85. The harmonic orders of interest are 5, 7, 11, 13, 17, 19, 23, and 25. It is assumed that capacitors are placed or replaced to each bus by discrete size of 300 kVAr three-phase fixed capacitor or switched capacitor bank. The reactive power from capacitors at each bus is, therefore, equally divided to install at each phase by the same amount.

A two-step load pattern, peak and off-peak period, is given in Table 2. The off-peak duration is 11 hours a day and the peak duration is 13 hours a day. The operational constraints and cost data are listed in Table 3. Note that cost of peak power loss (k_p) is zero because from the view point of PEA, who purchases electricity from Electricity Generating Authority of Thailand (EGAT), PEA is not obliged to pay the cost of peak demand.

For PSO parameters, the number of particles in swarm and maximum number of iteration are 100 and 300. The values of PSO acceleration constants are given as 2.0, the PSO inertia weigh is linearly decreased from 0.9 to 0.4 in each iteration and the activation point for the adaptive PSO technique is set at iteration 150.

In order to examine the effects of unbalanced loading and nonlinearity of load on the optimal solution of capacitor placement problem, 15 cases are investigated with different values of these two factors as shown in Table 4. Case 1 represents the base case in which all three phase loads are balanced and entirely linear. For other case, for example in case 5, 5% unbalanced loading means that the load of phase A is 5% higher that of phase B but lower than that in phase C by the same amount, while 15% nonlinearity of load indicates 15% of the loads are assumed to be nonlinear.

Table 2. Load duration data

Load level	Load (p.u.)	Duration (hr)
Off-peak	1.0	4,015
Peak	1.2	4,745

Table 3. Operational constraints and cost data

Minimum voltage limit	0.95 p.u.
Maximum voltage limit	1.05 p.u.
Maximum THD limit	5%
Maximum capacitor for each bus	1,500 kVAr
Cost of energy loss (off-peak period)	1.1154 Baht/kWh
Cost of energy loss (peak period)	2.9278 Baht/kWh
Cost of peak power loss	-
Cost of fixed type capacitor	32,000 Baht/bank
Cost of switched type capacitor	43,200 Baht/bank

Table 4. Fifteen cases

Case	%UB	%NL	Case	%UB	%NL
1	0	0	9	10	30
2	0	15	10	15	0
3	0	30	11	15	15
4	5	0	12	15	30
5	5	15	13	20	0
6	5	30	14	20	15
7	10	0	15	20	30
8	10	15			

%UB = %Unbalanced loading %NL = %Nonlinearity of load

8. RESULTS AND DISCUSSION

8.1 Results before capacitor placement

At first, three-phase power flow and harmonic power flow algorithms are performed to obtain the bus voltages, THD levels, and total losses of all cases before capacitor placement. Numerical results from case 15 (extreme case) during peak period, as shown in Figures 5 and 6, indicate the effect of unbalanced loading and nonlinearity of load on voltage profile and THD values of phases A, B, and C. As expected, the voltage and THD of each phase vary with its loading.

Before capacitor placement, the minimum voltage including maximum THD level found in any phase of all buses at any load levels, and total loss are graphically summarized in Figures 7 to 9. It is found that there are some bus voltages which are lower than the minimum voltage (0.95 p.u.) as shown in Figure 7 for all cases. It is very interesting to note that with the same unbalanced loading, a greater nonlinearity of load results in a higher minimum voltage. The maximum voltages for all cases are always at slack bus, so they are not shown in the figure.

Voltage distortions are presented in the system in the cases with nonlinear loads as seen in Figure 8. From this figure, the highest values of THD found in the system are violated the maximum permissible THD level (5%) only in the cases with 30% nonlinearity of load (i.e. case 3, 6, 9, 12, and 15). The total losses of all 15 cases are shown in the bar graph of Figure 9. This figure indicates that more total losses are introduced due to an increase in unbalanced loading, while a change of load nonlinearity in the same value of unbalanced loading results in slight difference in the total loss.

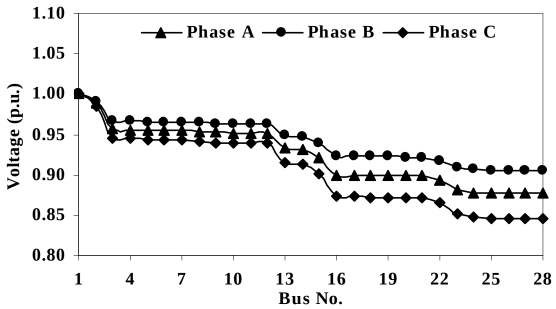

Fig. 5. Bus voltage profile at peak period for Case 15 before capacitor placement.

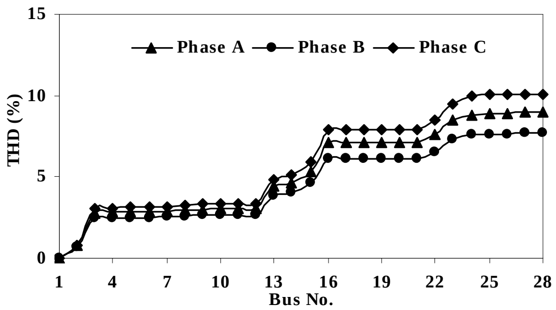

Fig. 6. THD values along the feeders at peak period for Case 15 before capacitor placement.

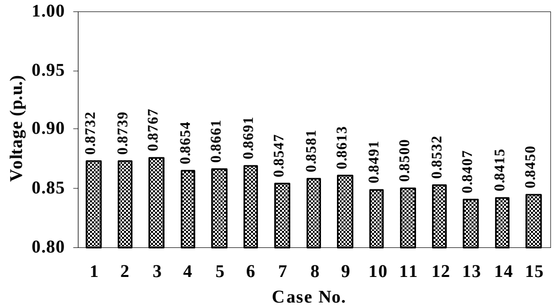

Fig. 7. Minimum voltages found in all cases before capacitor placement.

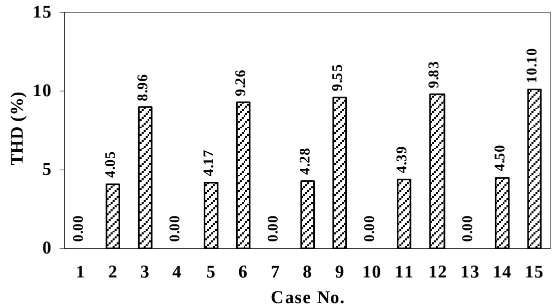

Fig. 8. Maximum THD values found in all cases before capacitor placement.

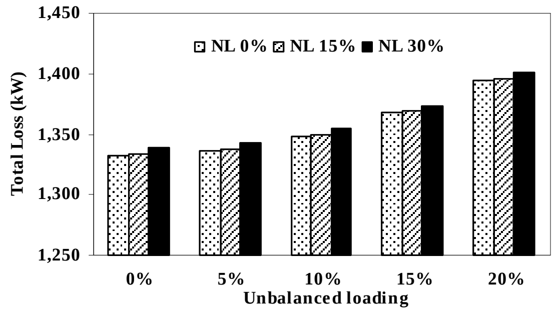

Fig. 9. Total loss of all cases before capacitor placement.

8.2 Performance of the adaptive PSO

Based on the fifteen cases shown in Table 4, their total costs can be calculated by the conventional PSO and the proposed adaptive PSO, as illustrated in Figure 10. It is obviously seen that the solutions obtained from the proposed adaptive PSO can offer lower total costs compared with those derived by the conventional PSO for every scenario appearing in the fifteen cases. These results confirm the advantages of the adaptive PSO technique in searching for better solutions.

8.3 Effects of unbalanced loading and nonlinearity of load on the optimal solutions

The effects of unbalanced loading and nonlinearity of load are investigated using the optimal solutions determined by the proposed adaptive PSO approach. The summary of constraint satisfaction, fixed and switched type capacitor kVAr to be placed, and the comparison of total loss and total cost between before and after capacitor placement are presented in Tables 5 to 7. Figures 11 to 13 graphically show the total capacitor kVAr required for reactive power compensation, as well as reduction in the total loss and total cost respectively.

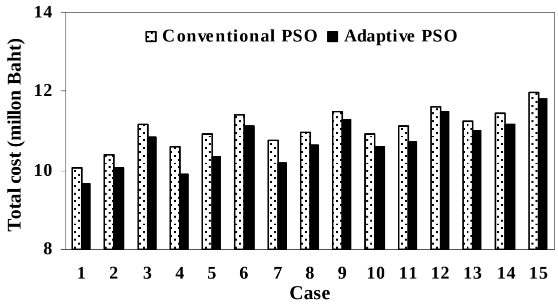

Fig. 10. Comparison of total cost between conventional PSO and adaptive PSO

It can be seen from Table 5 that the obtained optimal solutions of 15 cases satisfy all the constraint given in Section 2, namely, bus voltages, THD levels, and maximum capacitor kVAr for each bus. Table 6 indicates that the total capacitor kVAr associated with the obtained optimal solution vary with the change of unbalanced loading and nonlinearity of load. It is also found in Figure 11 that the increasing of these two factors gives the more total capacitor kVAr required for compensation.

The optimal capacitor placements introduce the reduction in total loss and total cost compared with that before capacitor placement as listed in Table 7. The percentage in reduction of total loss is slightly different regard to the variation of unbalanced loading and nonlinearity of load as shown in Figure 12. It is around 33-37% for all cases. On the other hand, the percentage in reduction of total cost is decreased with increased unbalanced loading and nonlinearity of load as seen in Figure 13. This is expected as an increase of these two factors needs more investment on capacitor. The maximum reduction in total cost of about 28% is taken place in case 1 with entirely linear and balanced load connected to the network, while the minimum reduction of total cost of around 17%, is given in case 15.

The above observations are evidences for the effects of unbalanced loading and load nonlinearity on the optimal solution of capacitor placement problem. For this reason, both unbalanced loading and the presence of nonlinear loads should be considered when the optimal location and size of capacitors are determined. Otherwise, a solution without concerning these two factors is a non-optimal solution.

Table 5. Summary of constraints satisfaction for optimal solution in all cases

Case	Min. voltage (p.u.)	Max. voltage (p.u.)	Max. THD (%)	Max. capacitor placed at any bus (kVAr)
1	0.9546	1.000	0.00	1,200
2	0.9541	1.0012	3.43	1,500
3	0.9750	1.0060	4.81	1,500
4	0.9504	1.0000	0.00	1,500
5	0.9546	1.0009	4.11	1,200
6	0.9695	1.0125	4.99	1,500
7	0.9505	1.0009	0.00	1,500
8	0.9519	1.0055	3.96	1,200
9	0.9648	1.0178	4.99	1,500
10	0.9503	1.0061	0.00	1,500
11	0.9521	1.0123	3.18	1,500
12	0.9633	1.0238	4.88	1,500
13	0.9501	1.0099	0.00	1,500
14	0.9502	1.0121	3.24	1,500
15	0.9676	1.0231	4.75	1,500

Table 6. Fixed and switched type capacitors required for optimal solution in all cases

Case	Fixed type capacitor (kVAr)	Switched type capacitor (kVAr)	Total kVAr (kVAr)
1	9,600	2,100	11,700
2	8,700	4,800	13,500
3	5,100	12,000	17,100
4	5,700	6,300	12,000
5	7,800	6,300	14,100
6	10,200	7,500	17,700
7	8,100	5,700	13,800
8	6,300	9,000	15,300
9	7,800	10,500	18,300
10	7,800	7,800	15,600
11	10,200	6,000	16,200
12	7,500	11,700	19,200
13	4,200	12,300	16,500
14	6,900	10,500	17,400
15	8,700	11,100	19,800

Table 7. Total loss and total cost before and after capacitor placement

Case	Total loss (kW)		Total cost (Baht)	
	Before	After	Before	After
1	1,322.76	829.02	13,530,285	9,675,243
2	1,334.10	841.01	13,543,364	10,072,660
3	1,338.83	857.28	13,589,995	10,821,851
4	1,336.61	837.99	13,570,495	9,896,125
5	1,337.94	857.76	13,583,569	10,337,989
6	1,342.67	896.14	13,630,200	11,107,645
7	1,348.15	845.06	13,691,338	10,179,085
8	1,349.49	870.24	13,704,392	10,631,717
9	1,354.22	905.06	13,751,025	11,271,614
10	1,367.47	855.68	13,893,610	10,579,519
11	1,368.81	882.86	13,906,634	10,709,031
12	1,373.54	925.72	13,953,268	11,506,100
13	1,394.68	881.21	14,178,653	10,996,677
14	1,396.01	891.79	14,191,634	11,154,244
15	1,400.75	940.96	14,238,271	11,801,461

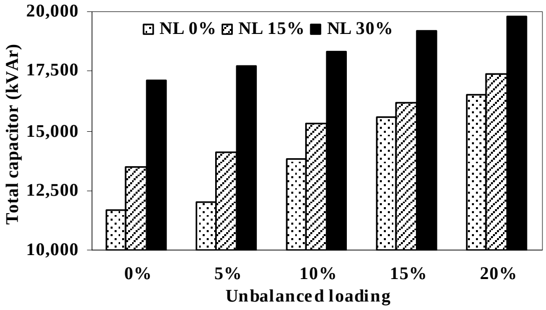

Fig. 11. Total capacitor kVAr required for optimal solution for all cases.

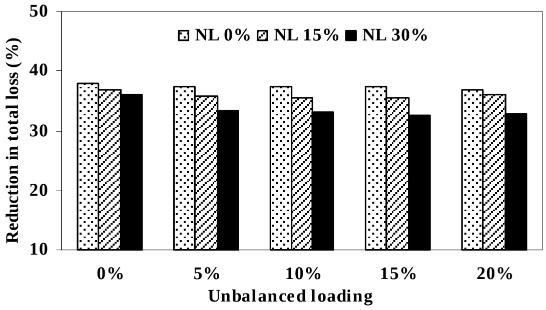

Fig. 12. Reduction in total loss for all cases after optimal capacitor placement.

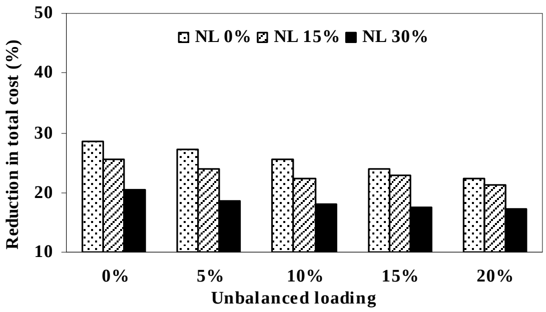

Fig. 13. Reduction in total cost for all cases after optimal capacitor placement.

9. CONCLUSION

This paper has presented an adaptive PSO-based optimization technique to determine optimal capacitor placement in an unbalanced loading distribution system connected to nonlinear loads. A distribution system of PEA, which consists of 28 buses and 19 load points with a two-step load pattern, is used to illustrate the performance of this proposed technique. Numerical results from different values of unbalanced loading and nonlinearity of load demonstrate that the obtained optimal solutions introduce a saving in total cost while satisfying all the specified constraints. The effects of unbalanced loading and load nonlinearity on the optimal solution are also examined. It is found that total loss in the network and total capacitor kVAr required for reactive power compensation are increased when unbalanced loading and load nonlinearity are increased whereas the saving in total cost is decreased owing to the more investment cost for capacitors.

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