Measurement and RTU Placement for State Estimation by Loop Decomposition

Thawatch Kerdchuen* and Weerakorn Ongsakul

Abstract— *This paper proposes a novel loop decomposition technique for optimal measurement and RTU placement for power system state estimation. Loop decomposition is based on the loops of system topology, which does not require the reduced measurement graph. The power injection and power flow measurement are properly installed at buses and branches which make the system observable considering single measurement loss and critical measurement pair free. The optimal number of measurement pairs is equal to minimum number. The minimum number of RTUs is equal to the number of power injection measurement pairs. The observability and critical measurement on the 10-bus, IEEE 14, 30, 57 and 118-bus systems are checked by* $P\delta$ *observability analysis and residual analysis respectively.*

Keywords— **Critical Measurement Pair, Loop Decomposition, Observability, Single Measurement Pair Loss.**

1. INTRODUCTION

Power system state estimation is used to estimate the steady state system conditions using the online measurements. Measurement placement is to provide the sufficient system data for state estimator. The inaccurate measured system conditions may lead to insecure state operation.

Measurement placement should be sufficient to make the system observable for running state estimation [1-4]. Moreover, the number of measurements should be minimized for investment cost savings. The heuristic approach presented the measurement placement in measurement graph [5]. It considers a single measurement pair loss while the system is still observable. This single measurement pair loss requires an additional measurement pair. Bad data or gross error in any single measurement pair of a system without the critical measurement pair can be detected [6-8]. The critical measurement pair is identified by residual analysis. However, the measurement graph is too complicated to construct. In [8], although the measurement placement without critical measurement is considered, the genetic algorithm for optimization is required. Decomposition technique [9] has been used for measurement and phase measurement unit (PMU) placements in multi-area system. This decomposition is used to make subsystem areas observable before data from each area is synchronized by the global positioning system (GPS) satellite trans-mission.

In this paper, loop decomposition technique is used to minimize measurement and RTU placement on the 10-bus, IEEE 14, 30, 57 and 118-bus systems considering single measurement pair loss. The system is decomposed according to system topology. Then, the injection and flow measurement pairs are installed to handle any single measurement pair loss. The absence of critical measurement is verified by the residual analysis.

2. MEASUREMENT PLACEMENT FUNDAMENTAL

The weight least squares (WLS) state estimation equations relating to the measurements and the state vector are

$$\mathbf{J}(\mathbf{x}) = (\mathbf{z}-\mathbf{H}\mathbf{x})^T \mathbf{R}^{-1}(\mathbf{z}-\mathbf{H}\mathbf{x}) \qquad (1)$$

where $\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{e}$ is the measurement vector, $\mathbf{H}$ is the $(m \times N)$ Jacobian matrix $\mathbf{H} = \partial \mathbf{h}(\mathbf{x}) / \partial \mathbf{x}$, $\mathbf{x}$ is the $(N \times 1)$ system state vector, $\mathbf{R}$ is the diagonal covariance matrix, $\mathbf{h}(\bullet)$ is the $(m \times 1)$ nonlinear function vector, $\mathbf{e}$ is the $(m \times 1)$ measurement error vector, m is the number of measurements, and N is the number of buses.

In case measurement pair PQ is used in power system and $\mathbf{H}$ matrix is block diagonal, the observability problem can be decoupled into $P\delta$ observability and QV observability. A power system is defined to be $P\delta$ (QV) observable with respect to a measurement set if the $\mathbf{H}_{P\delta}$ ($\mathbf{H}_{QV}$) matrix is of rank $N-1$ (N) [1, 5]. Therefore, the critical number of real and reactive measurement pairs is $(N-1)$ [5] and additional one of bus voltage magnitude measurement at any bus.

Single Measurement Loss Contingency

For single measurement pair loss contingency, any single injection or flow of real and reactive measurement pair

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can be lost from the power system. This loss is due to either communication failure or measurement failure. When a single measurement pair is lost, the entire row of measurement observability matrix is deleted. Then, the reduced measurement matrix is used to determine observability.

Critical Measurement Identification

If a single measurement pair can be lost from the power system, the system is considered as critical measurement free. In the absence of critical measurement in the power system, the bad data in any single measurement pair can be detected. The residual vector $\mathbf{r}$ is defined as the difference between $\mathbf{z}$ and the corresponding filtered quantities $\hat{\mathbf{z}} = \mathbf{H}\hat{\mathbf{x}}$. The residuals in terms of the elements of matrix $\mathbf{E}$ as follows

$$\begin{aligned}\mathbf{r} &= \mathbf{z} - \hat{\mathbf{z}} = \mathbf{z} - \mathbf{H}\hat{\mathbf{x}} = \mathbf{z} - \mathbf{H}(\mathbf{G}^{-1}\mathbf{H}^{T}\mathbf{z}) \\ &= (\mathbf{I} - \mathbf{H}\mathbf{G}^{-1}\mathbf{H}^{T})\mathbf{z} = \mathbf{E}\mathbf{z}\end{aligned} \tag{2}$$

where $\mathbf{E} = \mathbf{I} - \mathbf{H}\mathbf{G}^{-1}\mathbf{H}^{T}$, $\mathbf{z}$ is a unity vector [6] (This simplification is based on the fact that critical measurement is independently of the measurement values) and $\mathbf{G} = \mathbf{H}^{T}\mathbf{H}$ is a gain matrix. Therefore, the ith component of the residual vector is calculated by

$$r(i) = \sum_{k=1}^{m} E(i,k) \tag{3}$$

For each $z(i)$ of the measurement set, if $r(i)$ and $E(i,i)$ are zero, $z(i)$ is defined as a critical measurement [6-8].

3. LOOP DECOMPOSITION AND MEASUREMENT PAIR PLACEMENT

The rationale behind loop decomposition is that the power flow measurement pair loss can be handled by injection measurement pair. Also the power injection measurement pair loss can also be handled by power flow measurement pair and the adjacent power injection measurement pair. The decomposition loop is a strategic trial and error method used to properly place those measurement pairs. To make the system observable considering single measurement pair loss, the measurement and RTU placement in the loop decomposition procedures are as follows.

Measurement pair placement:

This procedure is to optimally place the measurement pairs for making the number of measurement pairs equal to number of system buses so that the system is observable with single measurement pair loss. The procedure starts with the system loop decomposition.

Step 1: Decompose a power system topology into either a single or multiple loops of subsystems, each loop does not overlap with each other.

Step 2: Power injection measurement pairs are placed at the boundary buses of the loop. The boundary bus is the bus whose branch is incident to other buses outside the loop.

Step 3: Power flow measurement pairs are placed at the internal branches near injection measurement bus which are incident to other buses without power injection measurement pairs. There should be only single power flow measurement pair connected to the same internal bus.

Step 4: At the radial bus outside the loop, power injection measurement pair is placed at all buses next to the end bus and power flow measurement pair is placed on the branch near injection measurement bus incident to the end bus.

Step 5: Place the injection measurement pairs at all buses of the line with intermediate buses inside or outside the loop.

Step 6: Power flow measurement pairs are placed at the branches near injection measurement bus incident to the external buses which are not associated with other loops. There should be only single power flow measurement pair connected to the same external bus.

Step 7: Check observability with single measurement pair loss and critical measurement pair identification. Remove single measurement pair one by one for single measurement loss consideration to check $P\delta$ observability. Also the critical measurement pair is identified by residual analysis. If unobservable or critical measurement pair exists, find the bus without injection measurement pair and replace the incident flow measurement pair with the injection measurement pair. Repeat this step until the system is observable or the system is critical measurement pair free.

RTU placement:

This procedure is to reduce the number of required RTUs.

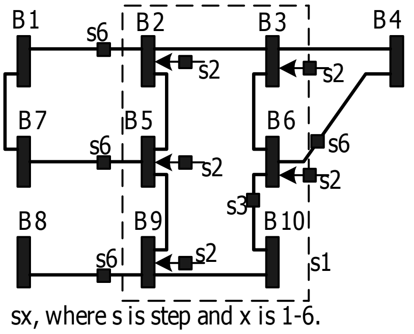
Step 8: For the line with intermediate buses inside each loop, an injection measurement pair is replaced by a flow measurement pair near to the adjacent injection measurement pair bus to minimize the number of power injection measurement pairs inside the observable loop considering single measurement pair loss. Keep repeating this step until every buses with injection measurement pair inside observable loop is considered.

Step 9: For the line with intermediate buses between the loops, an injection measurement pair is replaced by a flow measurement pair near to the adjacent injection measurement pair to minimize the number of power injection measurement pairs considering single measurement pair loss. Keep repeating this step until every buses with injection measurement pair is considered.

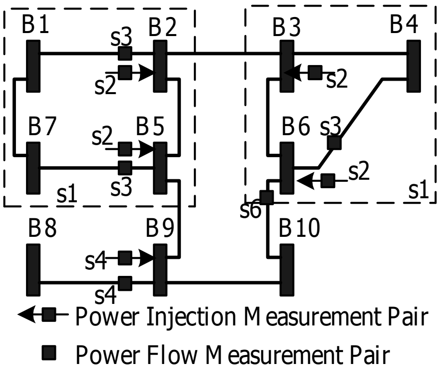
Step 10: For the bus with injection measurement pair and without flow measurement pair on the incident branches, the injection measurement pair is replaced by a flow measurement pair near to the adjacent bus with injection measurement pair to minimize the number of power injection measurement pairs if observable. Keep repeating this step until every buses with injection measurement pair and without flow measurement pair on the incident branches is considered.

4. LOOP DECOMPOSITION RESULTS

The final measurement pair placement for the 10-bus system [5] is illustrated in Fig. 1. The optimal number of measurement pairs and installed RTUs at buses B2, B5, B6 and B9 considering single measurement pair loss is shown in Fig. 2 (a). One loop decomposition on the IEEE 14-bus system with optimal number of measurement pairs and RTUs considering single measurement pair loss is also shown in Fig. 2 (b). The results of loop decomposition method are shown in Table 1. For the IEEE 14, 30, 57 and 118-bus systems, they are observable with minimum number of measurement pairs and reasonable small number of RTUs considering single measurement pair loss.

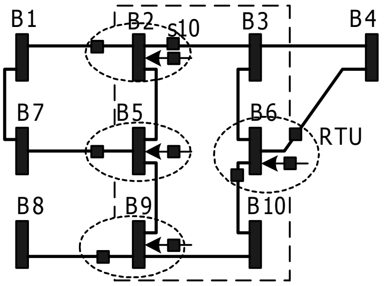


(a)

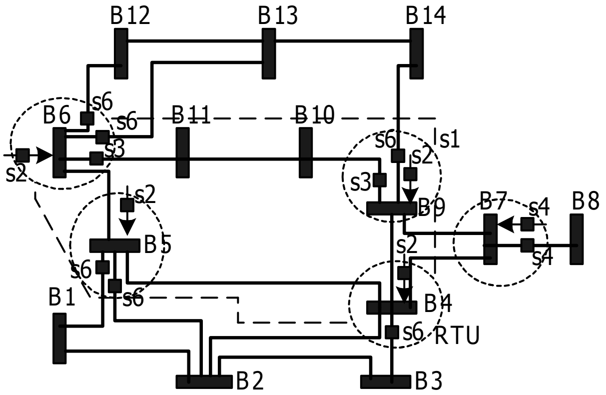


(b)

Fig. 1. Measurement pair placement for 10-bus system (a) one loop decomposition (b) two loops decomposition.



(a)



(b)

Fig. 2. Measurement pair and RTU placement results (a) 10-bus system (b) IEEE 14-bus system

Table 1. Measurement Placement Results of Loop Decomposition Method

System	No. of Loops	No. of Power Injection Meas. Pairs	No. of Power Flow Meas. Pairs	No. of RTUs
10-bus	1	4	6	4
10-bus	2	4	6	4
10-bus [5]	-	4	6	4
IEEE 14-bus	1	5	9	5
IEEE 14-bus [8]	-	5	9	5
IEEE 30-bus	3	11	19	11
IEEE 57-bus	5	30	27	30
IEEE 118-bus	12	45	73	45

In Table 1, even thought the number of measurement pairs and RTUs are the same as [5] and [8], the loop decomposition technique is fast and easy to implement.

5. CONCLUSION

The loop decomposition is fast in placing the measurement pairs and RTUs in the system considering single measurement pair loss. The measurement pair placement contains no critical measurement. Test results on the systems indicate that loop decomposition could result in the minimum number of measurement pairs and reasonable small number of RTUs, leading to investment cost savings.

REFERENCES

[1] G.R. Krumpholz, K.A. Clements and P.W. Davis, (1980). Power System Observability: A Practical Algorithm Using Network Topology. *IEEE Trans. Power App. Syst.*, vol. PAS-99, no. 4, pp. 1534-1542.

[2] A. Monticelli and F. F. Wu, (1985). Network Observability: Theory. *IEEE Trans. Power App. Syst.*, vol. PAS-104, no. 5, pp. 1042-1048.

[3] A. Monticelli and F. F. Wu, (1985) Network Observability: Identification of Observable Islands and Measurement Placement. *IEEE Trans. Power App. Syst.*, vol. PAS-104, no. 5, pp. 1035-1041.

[4] G. N. Korres, P. J. Katsikas, K. A. Clements and P. W. Davis, (2003). Numerical Observability Analysis Based on Network Graph Theory. *IEEE Trans. Power Syst.*, vol. 18, no. 3, pp. 1035-1045.

[5] G. M. Huang, J. Lei and A. Abur, (2003). A Heuristic Approach for Power System Measurement Placement Design. In *Proc. IEEE ISCAS*, vol. 3, pp. 407-410.

[6] M. B. Do Coutto Filho, J. C. S. Souza, F. M. F. Oliveira and M. Th. Schilling, (2001). Identifying Critical Measurements & Sets for Power System State Estimation. Present at *IEEE Porto Power Conference*, Porto, Portugal.

[7] A. S. Costa, T. S. Piazza and A. Mandel, (1990). Qualitative Methods to Solve Qualitative Problems in Power System State Estimation. *IEEE Trans. Power Syst.*, vol. 5, no. 3, pp. 941-949.

[8] J. C. S. Souza, M. B. Do Coutto Filho, M. Th. Schilling and C. Capdeville, (2005). Optimal Metering Systems for Monitoring Power Networks Under Multiple Topological Scenarios. *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 1700-1708.

[9] C. Rakpenthai, S. Premrudeepreechacharn, S. Uatrongjit and N. R. Watson, (2005). Measurement placement for power system state estimation using decomposition technique. *Electric Power Systems Research*, vol. 75, pp. 41-49.