Multi-Objective Optimal Power Flow Considering System Emissions and Fuzzy Constraints

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Abstract—This paper proposes a fuzzy multi-objective optimal real power flow (FMOPF) with transmission line limit and transformer loading constraints. In the proposed FMOPF algorithm, the total system operating cost, total system SO\(_2\), NO\(_x\), and CO\(_2\) emissions fuzzy minimization objectives are solved simultaneously considering fuzzy line flow constraints, using fuzzy linear programming (FLP). The proposed FMOPF is tested with the IEEE 30 bus system. The simulation results shown that the proposed FMOPF can efficiently and effectively trade off among total system operating cost and total system SO\(_2\), NO\(_x\), and CO\(_2\) emissions.

Keywords—Optimal power flow, emission power dispatch, fuzzy linear programming.

1. INTRODUCTION

In power system operation, optimal power flow (OPF) is an extended problem of economic dispatch (ED) to include several parameters such as; generator voltage, transformer tap change, SVC, and include constraints such as; transmission line and transformer loading limits, bus voltage limit, stability margin limit. The objectives may be; minimum generation cost, minimum transmission loss, minimum deviation from target schedule, minimum control shift to alleviate violation, minimum emission. Current interest in the OPF centers around its ability to solve for large scale power system optimal solution that takes account of more objectives, parameter, and constraints of the system.

The main purpose of optimal power dispatch is to minimize the operating cost of the power system satisfying power balance constraints. However, the operating cost minimization objective may not necessarily be the best in term of environment. Several alternatives for achieving emission reductions include adding gas cleaner, switching fuel to low sulfur fuel, and adopting new power dispatch. Among these methods, the power dispatch approach is preferred because it is easily implemented and requires minimum additional costs [1]. Therefore, the unit dispatch considering emission beside cost minimization has received widespread attention for effective short-term option with smaller capital outlay [1-6]. The major environmental concerns in optimal power dispatch include emission of SO\(_2\) [1-2, 4-6], NO\(_x\) [2-6], and CO\(_2\) [5].

In practical power system operation, the single objective such as total operating cost minimization may not be directly applied, since it may lead to unsatisfactory of other aspects such as system security, fuel security, environmental concern. In addition, constraints in conventional OPF are usually given fixed values that have to be met which may lead to over-conservative solutions. When the solution violates the inequality constraints, it is difficult to decide which constraint should be relaxed and the extent of relaxation. Therefore, in the system operator viewpoint, certain trade-off among several objectives and constraints would be desirable rather than a single rigid minimum or maximum solution.

Techniques for treating emission in optimal power dispatch have formed of two major research directions. Many techniques treated the emission as constraints [1, 2, 4]. However, due to the conflicting and non-commensurable nature of operating cost and emission, the earlier techniques formulated the problem to combine emissions minimization into the total operating cost minimization problem [3, 5, 7]. In [5] and [7], the emission minimization objectives were combined into the total cost minimization objective by weighting methods. Nevertheless, in the absence of sufficient information, defining the weighting factors or equivalent cost of emission is incredibly difficult. In [3], the bi-objective power dispatch using fuzzy satisfaction-maximizing decision approach was proposed. Nonetheless, the problem included only minimum NO\(_x\) emission and total operating cost objectives with the linear fuzzy membership function.

This paper proposes the fuzzy multi-objective optimal real power flow (FMOPF) for selecting a final compromise solution for operating cost and emissions minimization problems. In the proposed FMOPF algorithm, the total system operating cost, total system SO\(_2\), NO\(_x\), and CO\(_2\) emissions fuzzy minimization objectives are solved simultaneously using fuzzy linear programming (FLP). The proposed FMOPF is tested with the IEEE 30 bus system. The simulation results shown that the proposed FMOPF can efficiently and effectively trade off among total system operating cost and total system SO\(_2\), NO\(_x\), and CO\(_2\) emissions.

The organization of this paper is as follows. Section 2
addresses the FMOPF problem formulation. The FLP for FMOPF problem is given in Section 3. Numerical results on the IEEE 30 bus test system are illustrated in Section 4. Lastly, the conclusion is given.

2. MOFP PROBLEM FORMULATION

Consider the fact that real power injected at a bus does not change significantly for a small change in the magnitude of bus voltage and the reactive power injected at a bus does not change for a small change in the phase angle of bus voltage. Therefore, the bus voltage magnitudes and transformer tap changes are not included in the total operating cost and emissions multi-objective fuzzy minimization subproblem. Similarly, the real power generations are not included in the total real power loss fuzzy minimization subproblem.

2.1 Total operating cost and emissions multi-objective fuzzy minimization subproblem

The operating cost of the generating unit is expressed as the sum of polynomial function of the real power generation of the unit. Therefore, objective is to minimize total system operating cost,

\[ FC = \sum_{BG} F(P_{Gi}) \]  

subject to the power balance constraints,

\[ P_{Gi} - P_{Di} = \sum_{j=1}^{NR} \left| V_j \right| \left| V_i \right| \cos(\theta_j - \delta_j), i = 1, \ldots, NB, \]  

and fuzzy generator ramp rate constraint,

\[ P_{Gi}^{\text{dec}} - P_{Gi}^{\text{inc}} \cdot \text{Min} \leq P_{Gi} - P_{Gi}^{\text{min}}, \quad i = 1, \ldots, NR. \]  

and the generator minimum and maximum operating limit constraints,

\[ P_{Gi}^{\text{min}} \leq P_{Gi} \leq P_{Gi}^{\text{max}}, \quad i \in BG. \]  

2.2 Real power loss fuzzy minimization subproblem

To minimize the total real power loss, the total real power loss minimization subproblem is solved iteratively with the total fuel cost fuzzy minimization subproblem. The objective is formulated as,

\[ \text{Minimize} \quad \Delta P_{\text{loss}} = \left[ \frac{dP_{\text{loss}}}{d|V|} \right] \left[ \Delta |V| \right] \]  

subject to the fuzzy bus voltage limits constraints,

\[ |V_i| - \Delta \leq |V_i| \leq |V_i| + \Delta, \quad i = 1, \ldots, NV, \]  

where

\[ |V_i| - \Delta = |V_i| - |V_i|^{\text{min}}, \quad \Delta = |V_i| - |V_i|^{\text{max}} \]  

and the transformer tap-change limits constraints,

\[ \Delta T_i^{\text{min}} \leq T_i - T_{\text{ref}}, \quad i = 1, \ldots, NT, \]  

where

\[ \Delta T_i^{\text{min}} = T_i^{\text{min}} - T_{\text{ref}}, \quad \Delta T_i^{\text{max}} = T_i^{\text{max}} - T_{\text{ref}}, \quad i = 1, \ldots, NT. \]  

\[ Q_{Gi}^{\text{min}} \leq Q_{Gi} \leq Q_{Gi}^{\text{max}}, \quad i \in BG. \]  

\[ |V_j|, \quad i \in BG, \quad \text{and} \quad T_i, \quad i = 1, \ldots, NT, \]  

are the unknown control variables obtained from the total real power loss fuzzy minimization subproblem.

\[ R_{Gi}, \quad i \in BG, \quad \text{is the output of the FMOPF algorithm.} \]  

The method is intended to line flow and transformer loading limits constrained economic dispatch in power system. The bus voltages and reactive power optimal controls are not included in the paper.
3. FUZZY LINEAR PROGRAMMING ALGORITHM FOR FMOPF PROBLEM

To solve the FMOPF problem, the goal of decision-maker can be expressed as a fuzzy set and the solution space is defined by constraints that can be modeled by fuzzy set [8]. The multi-objective fuzzy minimization subproblem of the proposed FMOPF can be formulated as,

\[
\text{Maximize } \min \{ \mu_1(x), \mu_2(x), \mu_3(x), \mu_4(x) \}.
\]

subject to \( \mathbf{B} \cdot \mathbf{P}_{\text{gi}} \preceq \mathbf{d} \)

and power balance constraints in (2) and (3), criss inequality line flow limit and transformer loading constraints in (7), and low and high limits of \( P_{\text{gi}} \) in (8).

\( \mathbf{P}_{\text{gi}} \) is the column matrix representing the set of real power generation of the generator connected to bus \( i \). \( \mathbf{d} \) is the vector representing of fuzzy objective functions in Eqs. (1)-(4). Each row of \( \mathbf{B} \) in (10) is represented by a fuzzy set with the membership functions of \( \mu_i(x) \). \( \mu_i(x) \) can be interpreted as the degree to which \( \mathbf{P}_{\text{gi}} \) satisfies the fuzzy objective function. Here, \( \mu_i(x) \) is the degree of satisfaction of \( \mathbf{P}_{\text{gi}} \) for the objective function, whereas \( \mu_j(x) \) to \( \mu_k(x) \) are the degrees of satisfactions of \( \mathbf{P}_{\text{gi}} \) for the total system \( \text{NO}_x, \text{SO}_x, \text{CO}_2 \) emissions, respectively. In this paper, the hyperbolic function is used to represent the nonlinear, S-shaped, membership function [9]. The function can be expressed as,

\[
\mu_i(x) = \frac{1}{2} \tanh \left( \left( \frac{\mathbf{B}_i \cdot \mathbf{P}_{\text{gi}} - \frac{\alpha_i + \beta_i}{2}}{\gamma_i} \right) + \frac{1}{2} \right).
\]

where \( \alpha_i, \beta_i \), and \( \gamma_i \) are the parameters representing the shape of \( \mu_i(x) \) depending on the decision maker and \( \mathbf{B}_i \) is the row \( i \) of \( \mathbf{B} \).

The fuzzy linear programming approach translates the multiple objectives into additional constraints by assigning membership function to each objective. Due to complexity in computation, many literatures set the parameters by heuristics based on operators’ intuition. In this paper, the parameters were set by the worst case principle, which is based on the concept that none of the objective functions can be reduced any further by increasing other objective functions. This can reflect the optimal trade off among the objectives.

\[
\text{Table 1. The generator fuel cost function}
\]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Gen} & \text{Min} & \text{Max} & F(P_{\text{gi}}) = a_i \cdot P_{\text{gi}}^a + b_i \cdot P_{\text{gi}}^b + c_i \cdot P_{\text{gi}}^c + d_i \\
\text{bus} & \text{(MW)} & \text{(MW)} & a_i & b_i & c_i & d_i \\
\hline
1 & 50 & 200 & 0.0010 & 0.092 & 14.5 & -136 \\
2 & 20 & 80 & 0.0004 & 0.025 & 22 & -3.5 \\
5 & 15 & 50 & 0.0006 & 0.075 & 23 & -81 \\
8 & 10 & 50 & 0.0002 & 0.1 & 13.5 & -14.5 \\
11 & 10 & 50 & 0.0013 & 0.12 & 11.5 & 9.75 \\
13 & 12 & 40 & 0.0004 & 0.084 & 12.5 & 75.6 \\
\hline
\end{array}
\]

With the defined membership functions of objective function and fuzzy constraints, the fuzzy optimization problem can be reformulated as,

\[
\text{Maximize } \mu', \quad \text{subject to } \mu' \leq \mu_i(x), \text{ for } i = 1, \ldots, 4.
\]
and 0 ≤ μ′ ≤ 1. 

and power balance constraints in (2) and (3), criss inequality line flow limit and transformer loading constraints in (7), and low and high limits of P\textsubscript{Gj} in (8).

The FLP computational procedure is as follow,

**Step 1:** Solve the linear programming for individual objective function in Eqs. (1)-(4)

**Step 2:** Compute individual objective value of each case.

**Step 3:** Obtain α\textsubscript{i} and β\textsubscript{i} from the minimum and maximum of all objective values computed in step 2.

**Step 4:** Solve the fuzzy linear programming of multi-objective problem using α\textsubscript{i} and β\textsubscript{i} from step 3.

4. SIMULATION RESULTS

The IEEE 30 bus system is used as the test data. It network diagram is shown in Fig.2. The generator fuel cost, and SO2, NOx, and CO2 emissions functions are given in Tables 1 and 2. The generator fuel cost, and SO2, NOx, and CO2 emissions functions are linearized in to 5 piece-wise linear functions.

| **Table 2. The SO2, NOx, and CO2 emissions functions** |
|----------------|----------------|----------------|
| \( E\textsubscript{SO2} (P\textsubscript{Gj}) = a\textsubscript{SO2j} \cdot P\textsuperscript{2}\textsubscript{Gj} + b\textsubscript{SO2j} \cdot P\textsubscript{Gj} + c\textsubscript{SO2j} \cdot P\textsubscript{Gi} + d\textsubscript{SO2j} \) |
| \( E\textsubscript{NOx} (P\textsubscript{Gj}) = a\textsubscript{NOxj} \cdot P\textsuperscript{2}\textsubscript{Gj} + b\textsubscript{NOxj} \cdot P\textsubscript{Gj} + c\textsubscript{NOxj} \cdot P\textsubscript{Gi} + d\textsubscript{NOxj} \) |
| \( E\textsubscript{CO2} (P\textsubscript{Gj}) = a\textsubscript{CO2j} \cdot P\textsuperscript{2}\textsubscript{Gj} + b\textsubscript{CO2j} \cdot P\textsubscript{Gj} + c\textsubscript{CO2j} \cdot P\textsubscript{Gi} + d\textsubscript{CO2j} \) |

<table>
<thead>
<tr>
<th>Gen bus</th>
<th>( a\textsubscript{SO2j} )</th>
<th>( b\textsubscript{SO2j} )</th>
<th>( c\textsubscript{SO2j} )</th>
<th>( d\textsubscript{SO2j} )</th>
<th>( a\textsubscript{NOxj} )</th>
<th>( b\textsubscript{NOxj} )</th>
<th>( c\textsubscript{NOxj} )</th>
<th>( d\textsubscript{NOxj} )</th>
<th>( a\textsubscript{CO2j} )</th>
<th>( b\textsubscript{CO2j} )</th>
<th>( c\textsubscript{CO2j} )</th>
<th>( d\textsubscript{CO2j} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0005</td>
<td>0.150</td>
<td>-17.0</td>
<td>-90.0</td>
<td>0.0012</td>
<td>0.052</td>
<td>18.5</td>
<td>-26.0</td>
<td>-14.0</td>
<td>0.0014</td>
<td>0.092</td>
<td>14.0</td>
</tr>
<tr>
<td>2</td>
<td>0.0014</td>
<td>0.055</td>
<td>12.0</td>
<td>-30.5</td>
<td>0.0004</td>
<td>0.045</td>
<td>12.0</td>
<td>-35.0</td>
<td>12.5</td>
<td>0.0014</td>
<td>0.025</td>
<td>12.5</td>
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<tr>
<td>5</td>
<td>0.0010</td>
<td>0.035</td>
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<td>13.5</td>
<td>0.0016</td>
<td>0.055</td>
<td>13.5</td>
</tr>
<tr>
<td>8</td>
<td>0.0020</td>
<td>0.070</td>
<td>23.5</td>
<td>-34.5</td>
<td>0.0012</td>
<td>0.070</td>
<td>17.5</td>
<td>-74.0</td>
<td>13.5</td>
<td>0.0012</td>
<td>0.010</td>
<td>13.5</td>
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<td>11</td>
<td>0.0013</td>
<td>0.120</td>
<td>21.5</td>
<td>-19.75</td>
<td>0.0003</td>
<td>0.040</td>
<td>8.5</td>
<td>-89.0</td>
<td>21.0</td>
<td>0.0023</td>
<td>0.040</td>
<td>21.0</td>
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<tr>
<td>13</td>
<td>0.0021</td>
<td>0.080</td>
<td>22.5</td>
<td>25.6</td>
<td>0.0014</td>
<td>0.024</td>
<td>15.5</td>
<td>-75.0</td>
<td>22.0</td>
<td>0.0014</td>
<td>0.080</td>
<td>22.0</td>
</tr>
</tbody>
</table>

Step 3 addresses the dispatch results of minimum operating cost, SO2 emission, NOx emission, and CO2 emission, respectively. Table 7 shows the dispatch results of the FMOFP result. The results show that the single objective approaches result in the inferior results in the others objective and less degree of satisfaction.

| **Table 4. Dispatch results of minimum SO2 emission condition** |
|----------------|----------------|----------------|
| **** Generation Cost ** ** |
| BUS | P\textsubscript{GEN} (MW) | Cost ($/h) | Inc-Cost ($/MWh) |
| 1   | 50.00             | 944.00013   | 21.60000         |
| 2   | 68.00             | 1733.87267  | 25.54960         |
| 5   | 50.00             | 1331.49953  | 28.25000         |
| 8   | 36.68             | 625.17418   | 17.43752         |
| 11  | 42.00             | 781.24482   | 18.83320         |
| 13  | 40.00             | 735.59969   | 16.50000         |
| Total Cost = 6151.39102 $/h |
| Total SO2 = 7035.94301 ton/h |
| Total NOx = 5009.04237 ton/h |
| Total CO2 = 5976.29335 ton/h |

| **Table 5. Dispatch results of minimum NOx emission condition** |
|----------------|----------------|----------------|
| **** Generation Cost ** ** |
| BUS | P\textsubscript{GEN} (MW) | Cost ($/h) | Inc-Cost ($/MWh) |
| 1   | 50.00             | 944.00000   | 21.60000         |
| 2   | 69.86             | 1791.70996  | 25.54960         |
| 5   | 50.00             | 1331.49953  | 28.25000         |
| 8   | 36.68             | 625.17418   | 17.43752         |
| 11  | 42.00             | 781.24482   | 18.83320         |
| 13  | 40.00             | 735.59969   | 16.50000         |
| Total Cost = 6161.39996 $/h |
| Total SO2 = 6709.76760 ton/h |
| Total NOx = 5976.29335 ton/h |
| Total CO2 = 5976.29335 ton/h |

Tables 3-6 address the dispatch results of minimum operating cost, SO2 emission, NOx emission, and CO2 emission, respectively. The comparison on the results with total cost minimization, SO2 minimization, NOx Minimization, CO2 minimization, and the proposed FMOFP is shown in Fig.3.

In this test case, the minimum operating cost solution results in the highest SO2 emission of 7035.94 ton/h. Meanwhile, the minimum NOx emissions result in the highest total operating cost and CO2 emissions, of
6161.4 $/h and 6104.2 ton/h, respectively and the minimum CO2 solution results in the highest NOX emission of 5187.67 ton/h.

In contrast, the proposed FMOPF is effectively trades off all objectives in the fuzzy reasoning sense leading to the most compromise solution. Note the FMOPF results in the degree of satisfaction ($\mu'$) of 0.881.

**Table 6. Dispatch results of minimum CO2 emission condition**

<table>
<thead>
<tr>
<th>BUS</th>
<th>P_GEN (MW)</th>
<th>Cost ($/h)</th>
<th>Inc-Cost ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50.00</td>
<td>943.99889</td>
<td>21.60000</td>
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<tr>
<td>2</td>
<td>50.00</td>
<td>1311.30050</td>
<td>28.25000</td>
</tr>
<tr>
<td>5</td>
<td>34.00</td>
<td>571.06491</td>
<td>17.08280</td>
</tr>
<tr>
<td>11</td>
<td>39.56</td>
<td>833.92697</td>
<td>19.25416</td>
</tr>
<tr>
<td>13</td>
<td>34.68</td>
<td>626.79931</td>
<td>15.89414</td>
</tr>
</tbody>
</table>

Total Cost = 6071.4 $/h
Total CO2 = 6071.4 ton/h

**Table 7. Dispatch results of the proposed FMOPF**

<table>
<thead>
<tr>
<th>BUS</th>
<th>P_GEN (MW)</th>
<th>Cost ($/h)</th>
<th>Inc-Cost ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49.98</td>
<td>943.48013</td>
<td>21.59680</td>
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<tr>
<td>2</td>
<td>68.00</td>
<td>1733.87299</td>
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</tr>
<tr>
<td>5</td>
<td>34.00</td>
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<tr>
<td>11</td>
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<tr>
<td>13</td>
<td>34.68</td>
<td>626.79931</td>
<td>15.89414</td>
</tr>
</tbody>
</table>

Total Cost = 6071.4 $/h
Total CO2 = 6071.4 ton/h

**Fig. 3. The comparison on the results with different objective functions.**

**5. CONCLUSIONS**

In this paper, a fuzzy multi-objective optimal real power flow (FMOPF) with transmission line limit and transformer loading constraints is successfully trading off between the total system operating cost, SO2 emission, NOx emission, and CO2 emission, satisfying transmission line limits and transformer loading constraints. The proposed FMOPF results in a compromise solution and can potentially be applied to overcome the difficulties of obtaining the weight or equivalent cost of emission.

**ACKNOWLEDGMENT**

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**NOMENCLATURE**

**Known Variables**

- $EC_{NOx}$, $EC_{SOx}$, and $EC_{CO2}$: the total system NOx, SOx, and CO2 emissions, respectively (ton/h),
- $E_{Nox}$, $E_{Sox}$, and $E_{CO2}$: the NOx, SOx, and CO2 emissions of the generator connected to bus $i$, respectively (ton/h),
- $F(P_{gi})$: the operating cost of the generator connected to bus $i$ ($$/h),
- $BG$: set of buses connected with generators
- $f_l^{max}$: MVA flow limit of line or transformer $l$ (MVA)
- $NT$: total number of on load tap-changing transformers
- $P_{Di}$: total real power demand at bus $i$ (MW)
- $P_{max}^{Gi}$: real power generation of the linearized cost function segment $j$ of generator at bus $i$ (MW)
- $P_{max}^{Gi,j}$: maximum real power generation of the linearized cost function segment $j$ of generator at bus $i$ (MW)
- $P_{max}^{Gi}$: maximum real power generation at bus $i$ (MW)
- $Q_{Di}$: reactive power demand at bus $i$ (MVAR)
- $Q_{max}^{Gi}$: maximum reactive power generation at bus $i$ (MW)
- $Q_{min}^{Gi}$: minimum reactive power generation at bus $i$ (MW)
- $R_{Gi}^{inc}$: ramping rate limit of generator $i$ when increasing real power generation (MW/min)
- $R_{Gi}^{dec}$: ramping rate limit of generator $i$ when decreasing real power generation (MW/min)
- $Min$: time interval in minute (min)
- $S_{ij}$: linearized incremental cost segment $j$ of generator at bus $i$ ($$/MWh)
- $T_{Bi}$: maximum tap setting of transformer $j$ of generator at bus $i$ (MW)
- $T_{Bi}$: minimum tap setting of transformer $i$ (MW)
- $V_{max}^i$: maximum voltage magnitude at bus $i$ (kV)
- $V_{min}^i$: minimum voltage magnitude at bus $i$ (kV)
- $\theta_{ij}$: angle of the $y_{ij}$ element of $Y_{bus}$ (radian)

**Unknown Control Variables**

- $P_{Gi}$: the real power generation of the generator
connected to bus $i$ (MW),

$T_i$ : tap setting of transformer $i$ (MW)

$|V_i|$ : generator voltage magnitude at bus $i$, $i \in BG$ (kV)

State and Output Variables

$FC$ : total system fuel cost ($/h$)

$l_{fl}$ : MVA flow of line or transformer $l$ (MVA)

$NC$ : total number of line flow and transformer loading constraints

$NR$ : total number of generator ramp rate constraints

$NV$ : total number of bus voltage magnitude constraints

$P_i$ : injection real power at bus $i$ (MW)

$P_{loss}$ : total system real power loss (MW)

$Q_Gi$ : reactive power generation at bus $i$ (MVAR)

$|V_j|$ : voltage magnitude at load bus $i$, $i \notin BG$ (kV)

$\delta_{ij}$ : voltage angle difference between bus $i$ and $j$ (radian)

REFERENCES


