# On the Coordinated Control of Multiple HVDC Links: Modal Analysis Approach

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***Abstract***— *There are several possibilities to improve the small-signal stability in a power system. One adequate option is to make use of available power system components that possess high controllability properties such as, for instance, high voltage direct current (HVDC) systems. This paper presents results from a study aimed at the investigation of small-signal stability enhancement achieved by proper coordinated control of multiple HVDC links. Modal analysis was used as the main tool for the theoretical investigation. The obtained results indicate that the coordinated control of several HVDC links in a power system may assist achieving in an essential increase of damping in the power system. Another important conclusion from the paper is that the possibilities of the coordination of the HVDCs to a certain extent depend on the structure of the grid, which can be investigated by examining the controllable subspace of the linearzied model of the power system.*

***Keywords***— **Coordinated control, HVDC, Modal analysis, Power System Stability.**

## 1. INTRODUCTION

Modern interconnected electric power systems are characterized by large dimensions and high complexity of the structure and the dynamic phenomena associated with the power system operation and control. Power system deregulation that took place in many countries worldwide was one of the driving forces that stimulated a fuller utilization of power systems, which in some cases lead to a reduced stability margin, as the power systems became more stressed. Under these circumstances it becomes quite important to seek new possibilities of enhancement of both transient and small-signal stability of the power systems.

There are several obvious ways of improving power system stability, namely, (*1*) building new transmission lines, (*2*) installing new generation capacities, (*3*) better utilization of the existing equipment in the power system, or (*4*) a combination of the above. This paper is primarily concerned with third option, since compared to the other options it is less costly and can be easily implemented in a real power system. The central idea of the study presented in this paper is the utilization of several HVDC links for small-signal stability enhancement.

The central purpose of conventional HVDC transmission is to transfer a certain amount of electrical power from one node to another and to provide the fast controllability of real power transfer. If the HVDC link is operated in parallel with a critical ac line the load-flow of the ac line can be controlled directly. The presence of an HVDC can assist in improving the stability margin in the power system [1]. In case there are several HVDC links in the system, there is also a possibility of coordinating the HVDC links to enhance the operation of the system [by, for example, altering the load-flow patterns] and to improve the system stability evermore.

The power systems are known to be operated most of the time in the so-called `quasi-steady state'. That is to say, the power systems are always subject to various—often small—disturbances [2]. A change in the loading level or capacitor switching is typical examples of such small disturbances that sometimes give rise to oscillations in the power system. The oscillations are often positively damped and their magnitudes decrease after a while thus the system remains stable.

In case of negative damping of the oscillations the situation is opposite and may result in loss of synchronism unless preventive measures are taken.

## 2. CASE STUDY

The aim of this paper is to perform modal analysis using coordinated control of two conventional HVDC links in a benchmark power system. This paper also explores the possibilities brought by the controllability and coordination of the HVDC links to enhance the rotor angle stability upset by a disturbance.

In the benchmark power system, as is in all realistic cases, the turbine action is very slow compared to the fast controllability of the HVDC. The theory in this paper is based on small-signal analysis by linearizing the system around the stable or unstable equilibrium point. The modal analysis provides valuable information about the inherent dynamic characteristics of the system. By controlling the current through the HVDCs and using state feedback it is possible to move the eigenvalues to pre-specified locations in the complex plane and thereby increase the damping.

## 3. TEST POWER SYSTEM

The system in this case study consists of three generators connected to nodes A, B, and C. The HVDC links are

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connected between nodes A and C and between B and C. The power production is large in node A and the load center is assumed to be in node C, i.e., there is a significant power flow from nodes A to C. The power can go either through the ac lines or through the HVDC links. An overview of the test power systems is shown in Fig. 1.

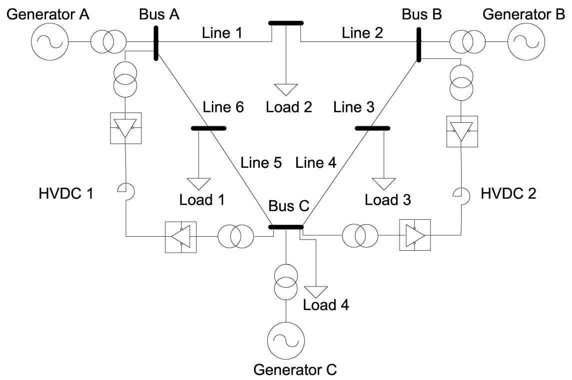


**Fig. 1. Test Power system.**

## 4. SYSTEM MODEL

The HVDC link is of a classical type, which implies that it consumes reactive power and that the active power can be controlled. The HVDC link consists of an ideal rectifier, inverter, and series reactor which models the dc-line and creates a smooth dc current. The power through the HVDC is controlled by controlling the firing angle, α. The firing angle is controlled by a basic PI-controller and the input is the set point current. The generators are modelled by the one axis model, described by equations (1)-(3) [3], thus three states per generator are used. The dynamics of the HVDC [4] are described in equations (4)-(16), where the subscripts "*r*" and "*i*" refer to rectifier and inverter, respectively.

$$\dot{\delta} = \omega \tag{1}$$

$$\dot{\omega} = \frac{1}{M}\left(P_m - P_e - D\omega\right) \tag{2}$$

$$\dot{E}'_q = \left(E_f - \frac{x_d}{x'_d}E'_q + \frac{x_d - x'_d}{x'_d}V\cos(\delta - \theta)\right) \tag{3}$$

where

- $\delta$ is the rotor angle
- $\omega$ is the rotor synchronous speed
- $E'_q\angle\delta$, $V\angle\theta$ are the voltage phasors at the internal and terminal buses
- $T_{do}'$ is the d-axis transient open-circuit time constant
- $P_m$ is the mechanical power applied to the generator shaft
- $D$ is the generator's shaft damping constant
- $M$ is the machine inertia
- $x_d$ is the d-axis synchronous reactance
- $x_d'$ is the d-axis transient reactance
- $E_f$ is the constant generator field voltage

$$\dot{I}_d = \frac{1}{L_d}(V_{d_r} - V_{d_i}) - \frac{R_d}{L_d}I_d \tag{4}$$

$$\dot{x}_r = K_I(I_{d_{setp}} - I_d) \tag{5}$$

$$\dot{x}_i = K_I(I_d - I_{d_{setp}}) \tag{6}$$

$$\cos(\alpha_r) = x_r + K_P(I_{d_{setp}} - I_d) \tag{7}$$

$$V_{d_r} = \frac{3\sqrt{2}}{\pi}V_{r_{pu}}\cos(\alpha_r) - \frac{3}{\pi}X_{C_r}I_d \tag{8}$$

$$S_r = -\frac{3\sqrt{2}}{\pi}\frac{V_{b_{dc}}I_{b_{dc}}}{S_{b_{dc}}}V_{r_{pu}}I_d \tag{9}$$

$$P_r = -\frac{V_{b_{dc}}I_{b_{dc}}}{S_{b_{dc}}}V_{d_r}I_d \tag{10}$$

$$Q_r = -\sqrt{S_r^2 - P_r^2} \tag{11}$$

$$\cos(\gamma_i) = x_i + K_P(I_d - I_{d_{setp}}) \tag{12}$$

$$V_{d_i} = \frac{3\sqrt{2}}{\pi}V_{i_{pu}}\cos(\gamma_i) - \frac{3}{\pi}X_{C_i}I_d \tag{13}$$

$$S_i = \frac{3\sqrt{2}}{\pi}\frac{V_{b_{dc}}I_{b_{dc}}}{S_{b_{dc}}}V_{i_{pu}}I_d \tag{14}$$

$$P_i = \frac{V_{b_{dc}}I_{b_{dc}}}{S_{b_{dc}}}V_{d_i}I_d \tag{15}$$

$$Q_i = -\sqrt{S_i^2 - P_i^2} \tag{16}$$

where

- $V_{d_r}$, $V_{d_i}$ are the per unit dc terminal voltages at the rectifier and inverter
- $V_{b_{dc}}$, $I_{b_{dc}}$, $S_{b_{dc}}$ are the base quantities at the dc side for the voltage, current and power, respectively
- $X_{c_r}$, $X_{c_i}$ are the unit commutation reactances
- $R_d$, $L_d$ are the per unit dc line parameters
- $V_{r_{pu}}$, $V_{i_{pu}}$ are the per unit ac bus voltages
- $I_{d_{setp}}$ is the current set point through the HVDC

## 5. MODAL ANALYSIS

To study the power system small-signal stability problem, an appropriate model for the machines, loads and HVDC dynamics is required. The behavior of a power can be described by a set of first order nonlinear ordinary differential equations and a set of nonlinear algebraic equations [3].

$$\dot{x} = f(x, y, u) \quad (17)$$

$$0 = g(x, y, u) \quad (18)$$

where

$x = (\delta^T \omega^T E_q^{'T} I_d^T x_r^T x_i^T)^T$ is the vector containing the state variables, $y = (\theta^T V^T)^T$ is the vector designating algebraic variables, and finally $u = I_{d_{setp}}$ the defined as the vector of control variables.

In the small signal stability analysis, the equations (17) and (18) are linearized at an equilibrium point and the higher order terms are neglected. The linearization gives the structure as follows:

$$\begin{cases} \Delta\dot{x} = f_x\Delta x + f_y\Delta y + f_u\Delta u \\ 0 = g_x\Delta x + g_y\Delta y + g_u\Delta u \end{cases} \Rightarrow \quad (19)$$

$$\begin{aligned} \Delta\dot{x} &= (f_x - f_y g_y^{-1} g_x)\Delta x + (f_u - f_y g_y^{-1} g_u)\Delta u \\ &= J_x\Delta x + J_u\Delta u. \end{aligned} \quad (20)$$

The prefix Δ means a small increment in corresponding variables. The matrices in equation (19) and (20) can be found in appendix.

The Lyapunov's first stability method is the fundamental analytical basis for power system small-signal stability assessment. It is based on eigenvalue analysis and provides valuable information of the behavior of the system, i.e. the time domain characteristics of a system mode. It is usual to associate each eigenvalue $\lambda_i$ with a mode of the system. Real eigenvalues represent non-oscillatory modes, where a negative one corresponds to decaying mode, while a positive one relates to aperiodic instability. Complex eigenvalues are associated with system oscillatory modes, the pair of complex eigenvalues with negative real parts indicate a decreasing oscillatory behavior, and those with positive real parts result in an increasing oscillatory behavior. The damping of the i:th mode is defined as follows:

$$\xi_i = \frac{-\sigma_i}{\sqrt{\sigma_i^2 + \omega_i^2}} \quad (21)$$

where

$\sigma_i = \Re e(\lambda_i)$

$\omega_i = \Im m(\lambda_i)$

By controlling the HVDCs and thereby controlling the modes it is thereby possible to change the behavior of the system. The controllable subspace provides information how the eigenvalues can be moved i.e. how the system can be controlled. The Kalman decomposition transforms the state space model into controllable, uncontrollable, observable and unobservable subspaces [5],[6]. Where the controllable and uncontrollable subspaces are of interest, since they provide information about how the eigenvalues can be moved. The matrices $J_x$ and $J_y$ span the controllability matrix and the image of the corresponding map is the controllable space. The proposed method can thereby be used for practical and large scale power systems. By following the above described procedure the controllability gramian provides the information how the poles can be moved, i.e., how the damping in the system can be increased.

The eigenvalues can nor be moved arbitrary in the controllable subspace due to limitations in the current through the HVDCs.

## 6. SIMULATION STUDY

In the test power system the eigenvalues of $J_x$ in equation (20) are determined. It results in two oscillatory modes which are referring to the system model. The eigenvalues can be found in Table 1.

The damping in the system is about 5%. By using state feedback it is possible to increase the damping. The control is performed as showed in Fig. 2. This is full state feedback which means all states are used in the feedback, if not all the states are available, it possible to perform state estimation. The unavailable signals can be estimated by an observer, but this situation is left outside of the scope of this paper.

**Table 1. Eigenvalues of the linearized system**

| $\lambda_{\text{no feedback}}$ | $\xi_{\text{no feedback}}$ | $\lambda_{\text{feedback}}$ | $\xi_{\text{feedback}}$ |
|---|---|---|---|
| -0.552 + 9.780i | 0.0564 | -2.887 | 1 |
| -0.552 - 9.780i | 0.0564 | -2.887 | 1 |
| -1.179 + 13.258i | 0.0886 | -3.464 | 1 |
| -1.179 - 13.258i | 0.0886 | -3.464 | 1 |
| -0.102 | 1 | -2 | 1 |
| -0.496 | 1 | -2.25 | 1 |
| -0.716 | 1 | -2.5 | 1 |
| -0.849 | 1 | -2.75 | 1 |
| -1.138 | 1 | -3 | 1 |
| -1.138 | 1 | -3.25 | 1 |
| -2.373 | 1 | -3.5 | 1 |
| -2.373 | 1 | -3.75 | 1 |
| 0 | - | -4 | 1 |
| 0 | - | 0 | - |
| 0 | - | 0 | - |

The output may be chosen arbitrarily from the states or a linear combination of the states. However, in this paper generator A is taken as the reference and the outputs are the deviation in speed of generator B and generator C, i.e. *output one* = $\omega_C$-$\omega_A$ and *output two* = $\omega_B$-$\omega_A$. The output is related to the states by a matrix which is linearly combining the state to the output. This is shown in Fig. 2 and the relating matrix is denoted as $J_y$.

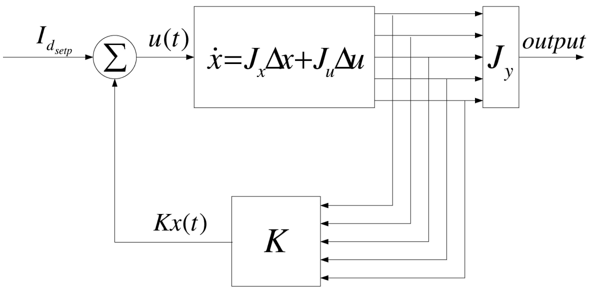


**Fig. 2. State feedback.**

In the case study in this paper all the eigenvalues can be moved arbitrarily. One possibility is to increase the damping or remove the oscillatory modes completely. In the test power system the closed loop system has the eigenvalues given in Table 1.

Two disturbances are applied to the system, the disturbances are due to load changes. Case one is due to a decrease of load 1 and increase of load 3. Case two is due to decrease of load 2 and increase of load 3. In both the cases the increase and the decrease are equal, which is done to preserve the equilibrium point. Fig. 3 and Fig. 4 show the output signals when disturbance one is applied. It is obvious that inspection of the figures reveals that the system behaves more nicely when using the state feedback and the oscillatory behavior should be totally eliminated, since all $\omega_i$.=0 Fig. 5 and **Fig. 6** show the system behavior under disturbance two, and also in this case, the response is better in the controlled system compared to the uncontrolled.

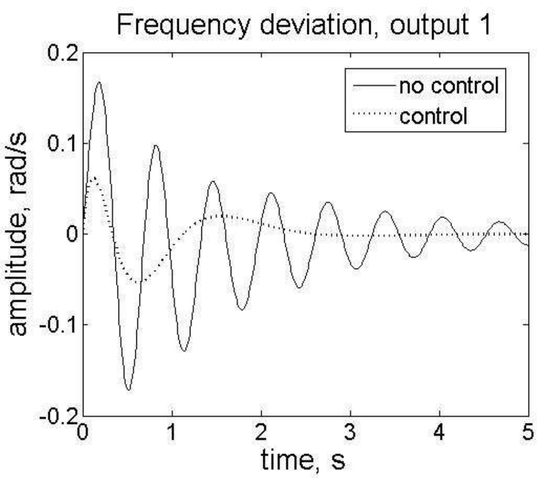


**Fig. 3. Disturbance one i.e. increase of load 3 and decrease of load 1. Output one i.e. speed difference of generator C and A.**

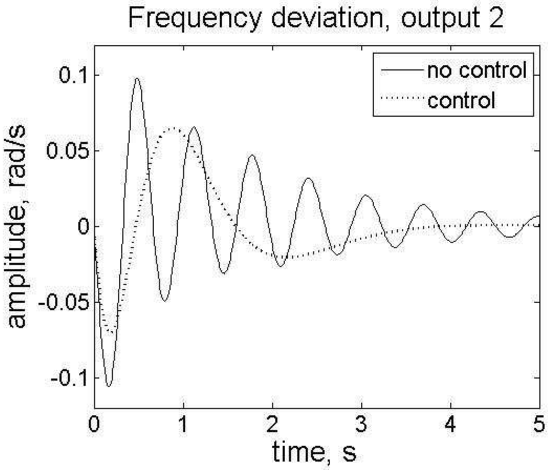


**Fig. 4. Disturbance one i.e. increase of load 3 and decrease of load 1. Output two i.e. speed difference of generator B and A.**

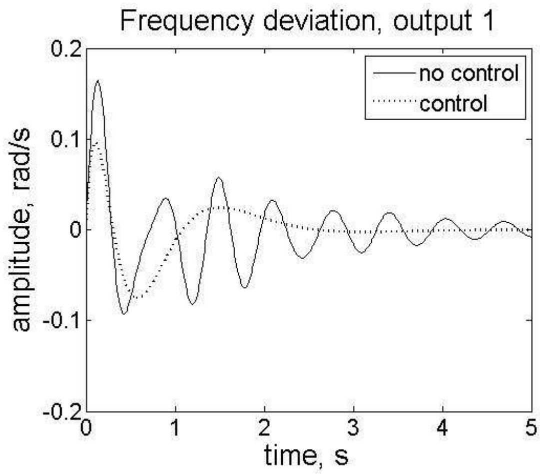


**Fig. 5. Disturbance two i.e. increase of load 3 and decrease of load 2. Output one i.e. speed difference of generator C and A.**

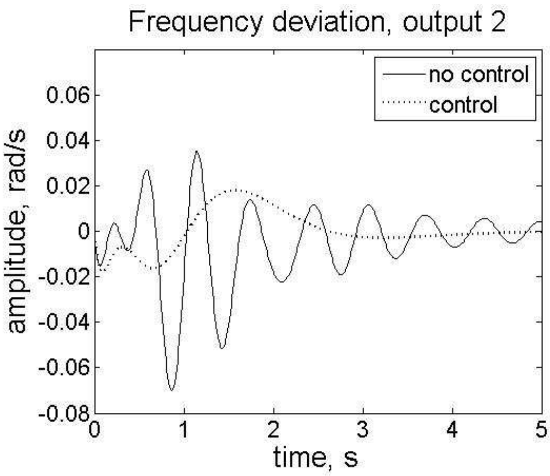


**Fig. 6. Disturbance two i.e. increase of load 3 and decrease of load 2. Output two i.e. speed difference of generator B and A.**

## 7. CONCLUSIONS

This paper presents modal analysis of a power system with several HVDC links and investigates the possibility to increase the damping in the system by changing the modes in the system. The obtained results strongly indicate that essential enhancement of small-signal stability can be achieved by proper coordination of the control of the HVDC links.

The paper also describes the ideas about the controllability of the modes, which are depending of the structure of the grid and may be determined as the controllable subspace.

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## APPENDIX

In the following section the Jacobian matrices of the linearized system are given.

The following is given: Let $N$ be the number of network buses, and $n_g$ be the number of generators. Also let $n_x = 3(n_g + n_{HVDC})$ be the number all state variables, where $n_{HVDC}$ is the number of HVDC-links. And finally $n_y = 2N$ be the number of all system algebraic variables. Then the Jacobian matrices $f_x$, $f_y$, $f_u$, $g_x$, $g_y$ and $g_u$ have the structure as follows:

$$f_x = \frac{\partial f}{\partial x} = \begin{pmatrix} \mathbf{0} & \mathbf{f}^x_{1,2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{f}^x_{2,1} & \mathbf{f}^x_{2,2} & \mathbf{f}^x_{2,3} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{f}^x_{3,1} & \mathbf{0} & \mathbf{f}^x_{3,3} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{f}^x_{4,4} & \mathbf{f}^x_{4,5} & \mathbf{f}^x_{4,6} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{f}^x_{5,4} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{f}^x_{6,4} & \mathbf{0} & \mathbf{0} \end{pmatrix}$$

which is of order $n_x \times n_x$

for $k = 1 \dots n_g$ and for $m = 1 \dots n_{HVDC}$

$$f^x_{1,2}(k,k) = 1$$

$$f^x_{2,1}(k,k) = -\frac{1}{M_k}\frac{E'_{qk}V_k}{x'_{dk}}\cos(\delta_k - \theta_k)$$

$$f^x_{2,2}(k,k) = -\frac{D}{M_k}$$

$$f^x_{2,3}(k,k) = -\frac{1}{M_k}\frac{V_k}{x'_{dk}}\sin(\delta_k - \theta_k)$$

$$f^x_{3,1}(k,k) = -\frac{x_{dk} - x'_{dk}}{T'_{dk}x'_{dk}}V_k\sin(\delta_k - \theta_k)$$

$$f^x_{3,3}(k,k) = -\frac{x_{dk}}{T'_{dk}x'_{dk}}$$

$$f^x_{4,4}(m,m) = -\frac{3\sqrt{2}K_p}{L_d\pi}(V_{i_{pu}} + V_{r_{pu}}) + \frac{3}{\pi}(X_{c_i} - X_{c_r}) - \frac{R_d}{L_d}$$

$$f^x_{4,4}(m,m) = -\frac{3\sqrt{2}K_p}{L_d\pi}(V_{i_{pu}} + V_{r_{pu}}) + \frac{3}{\pi}(X_{c_i} - X_{c_r}) - \frac{R_d}{L_d}$$

$$f^x_{4,5}(m,m) = \frac{3\sqrt{2}V_{r_{pu}}}{\pi L_d}$$

$$f^x_{4,6}(m,m) = -\frac{3\sqrt{2}V_{i_{pu}}}{\pi L_d}$$

$$f^x_{5,4}(m,m) = -K_I$$

$$f^x_{6,4}(m,m) = K_I$$

$$f_y = \frac{\partial f}{\partial y} = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{f}^y_{2,1} & \mathbf{f}^y_{2,2} \\ \mathbf{f}^y_{3,1} & \mathbf{f}^y_{3,2} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$$

which is of order $n_x \times n_y$

for $k = 1 \dots n_g$

$$f^y_{2,1}(k,k) = \frac{E'_{qk}V_k}{M_k x'_{dk}}\cos(\delta_k - \theta_k)$$

$$f^y_{2,2}(k,k) = -\frac{E'_{qk}}{M_k x'_{dk}}\sin(\delta_k - \theta_k)$$

$$f^y_{3,1}(k,k) = \frac{x_{dk} - x'_{dk}}{x'_{dk}T_{dk}}V_k\sin(\delta_k - \theta_k)$$

$$f^y_{3,2}(k,k) = \frac{x_{dk} - x'_{dk}}{x'_{dk}T_{dk}}\cos(\delta_k - \theta_k)$$

$$f_u = \frac{\partial f}{\partial u} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{f}^u_{4,1} & \mathbf{f}^u_{5,1} & \mathbf{f}^u_{6,1} \end{pmatrix}$$

which is of order $n_{HVDC} \times n_x$

for $m = 1 \dots n_{HVDC}$

$$f^u_{4,1}(m,m) = \frac{3\sqrt{2}}{\pi L_d}K_P(V_{r_{pu}} + V_{i_{pu}})$$

$$f^u_{5,1}(m,m) = K_I$$

$$f^u_{6,1}(m,m) = -K_I$$

$$g_x = \frac{\partial g}{\partial x} = \begin{pmatrix} \mathbf{g}^x_{1,1} & \mathbf{0} & \mathbf{g}^x_{1,3} & \mathbf{g}^x_{1,4} & \mathbf{g}^x_{1,5} & \mathbf{g}^x_{1,6} \\ \mathbf{g}^x_{2,1} & \mathbf{0} & \mathbf{g}^x_{2,3} & \mathbf{g}^x_{2,4} & \mathbf{g}^x_{2,5} & \mathbf{g}^x_{2,6} \end{pmatrix}$$

which is of order $n_y \times n_x$

for $k = 1 \dots n_g$ and $m = 1 \dots n_{HVDC}$

$$g^x_{1,1}(k,k) = -\frac{E_{qk}V_k}{x'_{dk}}\cos(\theta_k - \delta_k)$$

$$g^x_{1,3}(k,k) = \frac{V_k}{x'_{dk}}\sin(\theta_k - \delta_k)$$

$$g^x_{1,4}(k,m) = \begin{cases} -\frac{V_{bcd}I_{bdc}}{S_b}\left(\frac{3\sqrt{2}}{\pi}V_k(x_r - K_P I_{d_m}) + \frac{6}{\pi}X_{cr}I_{d_m}\right) & * \\ \frac{V_{bcd}I_{bdc}}{S_b}\left(\frac{3\sqrt{2}}{\pi}V_k(x_i + K_P I_{d_m}) + \frac{6}{\pi}X_{ci}I_{d_m}\right) & ** \\ 0 & *** \end{cases}$$

$$g^x_{1,5}(k,m) = \begin{cases} -\frac{V_{bcd}I_{bdc}}{S_b}\frac{3\sqrt{2}}{\pi}V_k I_{d_m} & * \\ 0 & *** \end{cases}$$

$$g^x_{1,6}(k,m) = \begin{cases} \frac{V_{bcd}I_{bdc}}{S_b}\frac{3\sqrt{2}}{\pi}V_k I_{d_m} & ** \\ 0 & *** \end{cases}$$

$$g_{2,1}^{x}(k,k) = -\frac{E_{qk}V_k}{x'_{dk}}\sin(\theta_k - \delta_k)$$

$$g_{2,3}^{x}(k,k) = -\frac{V_k}{x'_{dk}}\cos(\theta_k - \delta_k)$$

$$g_{2,4}^{x}(k,m) = \begin{cases} -C\frac{V_{bcd}I_{bdc}}{S_b}\left(\frac{3\sqrt{2}}{\pi}V_k(x_r - K_P I_{d_m} + \frac{6}{\pi}X_{cr}I_{d_m}\right) & * \\ -C\frac{V_{bcd}I_{bdc}}{S_b}\left(\frac{3\sqrt{2}}{\pi}V_k(x_i + K_P I_{d_m} + \frac{6}{\pi}X_{ci}I_{d_m}\right) & ** \\ 0 & *** \end{cases}$$

$$g_{2,5}^{x}(k,m) = Cg_{1,5}^{x}(k,m)$$

$$g_{2,6}^{x}(k,m) = -Cg_{1,6}^{x}(k,m)$$