Optimal PMU Placement by Stochastic Simulated Annealing for Power System State Estimation

Thawatch Kerdchuen and Weerakorn Ongsakul

Abstract— *This paper proposes stochastic simulated annealing (SSA) for solving optimal phasor measurement unit (PMU) placement in the power system for state estimation. The placement of PMU is used to detect bad data. The critical measurement free system can detect any single measurement bad data. Critical measurement identification is included as a penalty function. The topologically observable concept is used to check observability. Total cost of SSA is less than hybrid genetic algorithm and simulated annealing (HGS) especially in the large systems.*

Keywords— **Power system state estimation, Stochastic simulated annealing, Observability, PMU placement.**

1. INTRODUCTION

The rapid growth of computer and communication technology is challenging to power system monitoring and control. All phasor measurement units (PMUs) in power system might be synchronized either by satellite or fiber optic systems. PMU can measure bus voltage magnitude, bus voltage phase angle and real and reactive current flow in the incident lines [1]. Conventional power system state estimation uses power flow and injection measurements connected via remote terminal unit (RTU) to control centre. Then, nonlinear state estimator in energy management system (EMS) is processed. If PMU is used, linear power system state estimation can be used [2, 3].

So far, a few PMU is placed to enable bad data detection [1]. In power system with conventional measurement, bad data is detected by additional PMU. Power system state estimation with bad data detection is satisfied for the measurement system without critical measurement. Critical measurement is identified by Peters-Wilkinson method [1]. However, several methods are introduced for critical measurement identification [4, 5]. In [4], critical measurement is easily identified by residual analysis. In [6], the entire measurement system for state estimation is connected via several PMUs but bad data detection is not considered. In [4], bad data detection is considered for optimal measurement placement. Remote terminal unit (RTU) with conventional measurement is placed by genetic algorithm (GA). Residual analysis is used to identify the critical measurement.

In this paper, optimal PMU placement is proposed for state estimation. Critical measurement identification by residual analysis is included in the cost function of SSA and hybrid GA and SA (HGS) [9]. The “0” and “1” at system bus are coding for PMU placement. The topologically observable concept is used to check observability. This observability concept is easily observed that all buses are connected by a single connected graph. Results are shown both only system observable and observable considering critical measurement free.

2. FUNDAMENTAL OF PMU PLACEMENT

PMU placement is generally required to make the system observable. Moreover, the reliable measurement system is required such as bad data. Critical measurement free is necessary for bad data detection in any measurement.

Measurement Jacobian with PMU for Observability Analysis

The linear model for real power and bus phase angles of conventional state estimation are expressed in following form

$$\mathbf{z}_P = \mathbf{H}_{P\delta}\boldsymbol{\delta} + e_P \tag{1}$$

where

$\mathbf{z}_P$	real power measurement vector of real power flow and injection measurements
$\boldsymbol{\delta}$	bus phase angle vector
$\mathbf{H}_{P\delta}$	measurement Jacobian matrix for real power measurements versus all bus voltage angles
$\mathbf{e}_P$	real power measurement error vector.

PMU can measure both voltage phasor of its own bus and current phasors on incident branches. This typical measuring configuration is shown in Figure 1.

In Figure 1, a PMU is installed at bus B, thus a bus voltage phasor and three current phasors are measured. Each incident branch, the current phasor measurement between buses i and j can be written in rectangular coordinates as shown in Figure 2, where y and y_{sh} are defined as series admittance and shunt admittance respectively.

Thawatch Kerdchuen (corresponding auhtor) is with Rajamangala University of Technology Isan, Nakhonratchasima, Thailand. E-mail: thawatch.ke@gmail.com.

Weerakorn Ongsakul is with Energy Field of Study, Asian Institute of Technology, Thailand. E-mail: ongsakul@ait.ac.th.

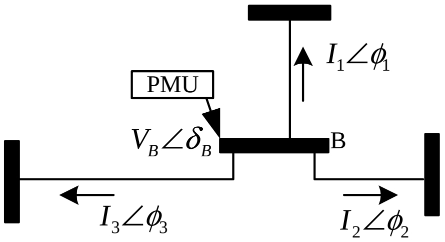


Fig. 1. Phasor measurements by a PMU

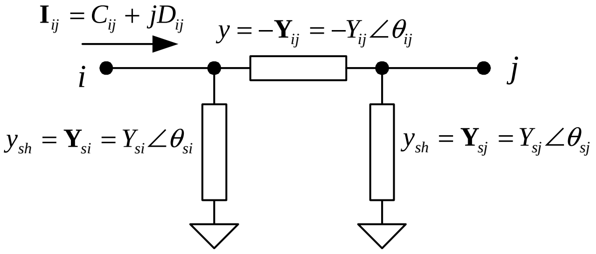


Fig. 2. Transmission line model

The expressions C_{ij} and D_{ij} are

$$C_{ij}=V_iY_{si}\cos(\delta_i+\theta_{si})+V_jY_{ij}\cos(\delta_j+\theta_{ij})-V_iY_{ij}\cos(\delta_i+\theta_{ij}) \quad (2)$$

$$D_{ij}=V_iY_{si}\sin(\delta_i+\theta_{si})+V_jY_{ij}\sin(\delta_j+\theta_{ij})-V_iY_{ij}\sin(\delta_i+\theta_{ij}) \quad (3)$$

The power system state vector is given as $\mathbf{x}=\left[V_1\ V_2...V_n\ \delta_1\ \delta_2...\delta_n\right]^T$. Thus, the entries of measurement Jacobian **H** corresponding to the real and reactive parts of current phasors are:

$$\frac{\partial C_{ij}}{\partial V_i}=Y_{si}\cos(\delta_i+\theta_{si})-Y_{ij}\cos(\delta_i+\theta_{ij})$$

$$\frac{\partial C_{ij}}{\partial V_j}=Y_{ij}\cos(\delta_i+\theta_{ij})$$

$$\frac{\partial C_{ij}}{\partial \delta_i}=-V_iY_{si}\sin(\delta_i+\theta_{si})+V_iY_{ij}\sin(\delta_i+\theta_{ij}) \quad (4)$$

$$\frac{\partial C_{ij}}{\partial \delta_j}=-V_jY_{ij}\sin(\delta_j+\theta_{ij}) \quad (5)$$

$$\frac{\partial D_{ij}}{\partial V_i}=Y_{si}\sin(\delta_i+\theta_{si})-Y_{ij}\sin(\delta_i+\theta_{ij})$$

$$\frac{\partial D_{ij}}{\partial V_j}=Y_{ij}\sin(\delta_i+\theta_{ij})$$

$$\frac{\partial D_{ij}}{\partial \delta_i}=V_iY_{si}\cos(\delta_i+\theta_{si})+V_iY_{ij}\cos(\delta_i+\theta_{ij}) \quad (6)$$

$$\frac{\partial D_{ij}}{\partial \delta_j}=V_jY_{ij}\cos(\delta_j+\theta_{ij}) \quad (7)$$

The system states are estimated if the measurement system is observable. Since the observability is independent to the branch parameter, all branch impedances are assumed as j1.0 p.u., and all bus voltages are assumed as 1.0 p.u. Based on (4) to (7) and the assumption of impedances and voltages, thus the real part of current phasor can be written as

$$real(\mathbf{I}_{ij})=\delta_i-\delta_j$$

Therefore, the linear model measurement Jacobian $\mathbf{H}_{P\delta}$ in (1) when PMU installed at bus i can be written as $\mathbf{H}_{I\delta}$

$$\mathbf{H}_{I\delta}=\begin{array}{c|ccccc|} & .. & \delta_i & \delta_j & \delta_k & .. \\ \hline .. & .. & .. & .. & .. & .. \\ \delta_i & .. & 1 & 0 & 0 & .. \\ \mathbf{I}_{ij} & .. & 1 & -1 & 0 & .. \\ \mathbf{I}_{ik} & .. & 1 & 0 & -1 & .. \\ .. & .. & .. & .. & .. & .. \end{array}$$

The above measurement Jacobian $\mathbf{H}_{I\delta}$ assumes the installed PMU at bus i and two incident branches. The topological observability is easily introduced to analyze. If all buses are connected by current flow measurement, the system is observable. Similarly, the system is said to be topologically observable if rank of $\mathbf{H}_{I\delta}$ is equal to $N-1$, where N is the number of system buses. In this topologically observable consideration, the row δ_i of $\mathbf{H}_{I\delta}$ should be deleted, since all connected buses are emerged only via current flow measurement.

Critical Measurement Identification

The WLS estimator will minimize the index $J(\mathbf{x})$, defined as follows.

$$J(\mathbf{x})=(\mathbf{z}-\mathbf{Hx})^T\mathbf{W}(\mathbf{z}-\mathbf{Hx}) \quad (8)$$

Matrix **W** is a diagonal matrix whose elements are measurement weight factors. If bad data or gross error occurs in a measurement and makes unable to estimate the system state, measurement is defined as a critical measurement (*cm*). Thus, in case of single measurement can be lost from the power system that means power system is absence of critical measurement. Therefore the absence of critical measurement in power system, bad data in any single measurement pair is detected. In filtering process, the state estimate $\hat{x}$ which minimizes $J(x)$ in (8) can be obtained from:

$$\frac{\partial J}{\partial x}=\mathbf{H}^T\mathbf{W}(\mathbf{z}-\mathbf{H}\hat{\mathbf{x}})=0$$

$$\hat{\mathbf{x}}=\mathbf{G}^{-1}\mathbf{H}^T\mathbf{z} \quad (9)$$

where $\mathbf{G}=\mathbf{H}^T\mathbf{H}$ is gain matrix. The residual vector **r**, defined as the difference between **z** and the corresponding filtered quantities $\hat{\mathbf{z}}=\mathbf{H}\hat{\mathbf{x}}$. In a dataset received for processing, the i^{th} measurement is declared critical if:

$$r(i)=z(i)-\hat{z}(i)=0 \tag{10}$$

$$\sigma_E(i)=\sqrt{E(i,i)}=0 \tag{11}$$

Using of (9) and (10), the residuals in terms of elements of matrix **E** as follows:

$$\begin{aligned}\mathbf{r} &= \mathbf{z}-\hat{\mathbf{z}}=\mathbf{z}-\mathbf{H}\hat{\mathbf{x}}=\mathbf{z}-\mathbf{H}(\mathbf{G}^{-1}\mathbf{H}^T\mathbf{z})\\ &=(\mathbf{I}-\mathbf{H}\mathbf{G}^{-1}\mathbf{H}^T)\mathbf{z}=\mathbf{E}\mathbf{z}\end{aligned} \tag{12}$$

where $\mathbf{E}=\mathbf{I}-\mathbf{H}\mathbf{G}^{-1}\mathbf{H}^T$ and **z** is unity vector (This simplification is based on the fact that *cm* property established from equation (10) to (12) is independently of measurement values). Therefore, the i^{th} component of residual vector is calculated by:

$$r(i)=\sum_{k=1}^{m}E(i,k) \tag{13}$$

For each $z(i)$ of measurement set, if $r(i)$ and $E(i,i)$ are zero, then declare $z(i)$ as critical measurement [4].

3. PMU PLACEMENT PROBLEM FORMULATION

The objective function of optimal PMU placement is to minimize the cost of those PMUs placement in the power system. The number of PMUs is directly dependent on the costs of PMU. Thus, the objective is to minimize the total number of PMUs as follows

$$\text{Min} \quad Cost(N_{PMU})=\sum_{i=1}^{N_{PMU}}PMU_i \tag{14}$$

subjects to the observability constraints

$$zero\_pivot=1 \tag{15}$$

or

$$rank(\mathbf{G}_{I\delta})=N-1 \tag{16}$$

or

$$rank(\mathbf{H}_{I\delta})=N-1 \tag{17}$$

where N_{PMU} is the total number of PMUs, and PMU_i is the i^{th} PMU of entire system. Matrix $\mathbf{G}_{I\delta}$ and $\mathbf{H}_{I\delta}$ in (16) and (17) are related with the terms of current flow measurement of PMU installation. Constraint (15) is used when triangular factorization or numerical method is used for observability analysis. In (13), zero pivot encounters during factorization. Constraints (16) and (17) are used when $P\delta$ observability concept used. Similarly, the system is topologically observable if constraint (17) is satisfied.

Cost evaluation of solution is following to (14) with penalties. Penalties include observability, and critical measurement. However, the minimal penalty part requirement is observability.

$$\text{Min} \quad Cost(N_{PMU})=N_{PMU}+Penalties \tag{18}$$

$$\begin{aligned}&Penalties=Penalty1+Penalty2\\ &Penalty1=\left[N-1-rank(\mathbf{H}_{I\delta})\right](N)\\ &Penalty2=(\text{No. of } cm)(N)\end{aligned} \tag{19}$$

First penalty is appeared if system is unobservable. The *penalty*2 is occurred if the system is with critical measurement.

4. SSA IMPLEMENTATION

This SSA is derived from adaptive SA with very fast annealing [8]. The important components for optimal PMU placement solving are solution coding and new solution generating.

Solution Coding

Random solution bits of solution coding represent position of PMUs in a power system. For example, the 10-bus system with 12 branches is typical shown in Figure 3.

0 1 0 0 1 1 0 0 1 0

Fig. 3. Typical random bits solution of 10-bus system for SSA initialization.

In Figure 3, PMUs are installed at buses 2, 5, 6 and 9. These solution bits are used to form measurement Jacobian $\mathbf{H}_{I\delta}$. Then, cost function in (18) is evaluated.

New Solution Generating

Initial solution is perturbed to generate new solution. Perturbing method of SSA uses bit flipping and bit exchanging. Fifty percent probability is applied between bit flipping and bit exchanging. Position for bit flipping and positions for bit exchanging are randomly generated. Perturbing method is shown in Figure 4.

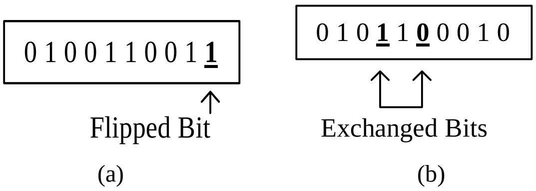


Fig. 4. Typical new solution creating (a) bit flipping (b) bit exchanging.

SSA Process

SSA process for solving optimal PMU placement is shown as follows:

```
Step 1: The solution is randomly initialized ($XInt$), also initial temperature $T_0 = 50$ and temperature length $T_L = N$.
Step 2: Solution evaluation of initial solution ($CostInt$) using (18), set old solution $Xold = XInt$, set best solution $Xb = XInt$ and set old best cost $Bold = CostInt$.
Step 3: Set iteration $k = 1$, set maximum evaluation step $maxstep = 400$ and the same result counter $S = 0$
Step 4: If $k \le maxstep$ and $S \le 100$, set sub-iteration $k_1 = 1$. Otherwise go to Step 5.
    Step 4.1: If $k_1 \le T_L$. Otherwise go to Step 4.2
        Step 4.1.1: the new solution ($Xnew$) is created by the $Xold$ perturbing
        Step 4.1.2: $Xnew$ cost evaluation
        Step 4.1.3: if $Xnew$ cost $\le$ $Xold$ cost, $Xold = Xnew$ and $Xb = Xnew$. Else if $e^{(Xold\ \mathrm{cost} - Xnew\ \mathrm{cost})/T} > rand$, $Xold = Xnew$. Otherwise $Xold = Xold$.
        Step 4.1.4: Set $k_1 = k_1 + 1$ and return to Step 4.1
    Step 4.2: update temperature $T = T_0 e^{(-ck^q)}$ [8], where $c = 2e^{(-\log(maxstep)/bits)}$, $q$ is quenching factor, 0.5, and $bits$ is number of solution bits
    Step 4.3: If $Bold = Xb$, $S = S + 1$. Otherwise $S = 0$.
    Step 4.4: Set $Bold = Xb$.
    Step 4.5: Set $k = k + 1$, return to Step 4.
Step 5: The best solution is $Xb$
```

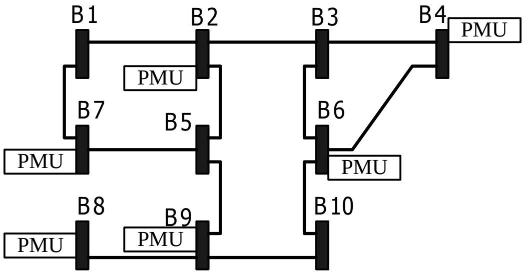
This solution updating Step 4.1.3 makes the diversity of solution, and the new direction of search shall be addressed by new solution generating. Temperature length is defined by the number of solution bits. However if we need to reduce the computing time, temperature length can be decreased.

5. NUMERICAL RESULTS

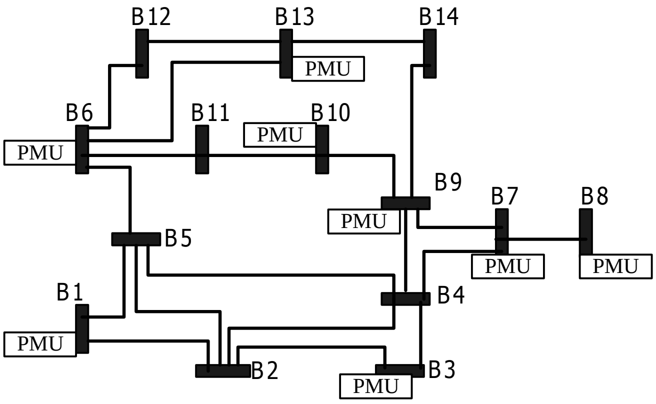
The total number of PMUs and their locations whether with observable or observable with critical measurement free are given in Table 1. To compare, HGS [9] is also used to solve optimal PMU placement. HGS is based on GA that uses SA acceptance criterion for chromosome selection. Population size, crossover and mutation probabilities are determined by experiments. Numerical results by SSA and HGS are shown in Table 1. Also the typical PMU placements are shown in Figure 5.

Table 1. Numerical PMU placement in several systems

System	Number of PMUs			
	Observable		Observable without *cm*	
	HGS	SA	HGS	SA
10-bus	4	4	6	6
IEEE 14-bus	4	4	8	8
IEEE 30-bus	10	10	18	18
IEEE 57-bus	20	19	29	28
IEEE 118-bus	36	34	65	63



(a)



(b)

Fig. 5. Typical optimal PMU placement with *cm* free for (a) 10-bus system (b) IEEE 14-bus system

In Table 1, the number of PMU for making the observable system is less than that for making the observable system with critical measurement free. For critical measurement free, any single flow current measurement of PMU can be lost while the system is still observable. Therefore, the number of PMUs is higher considering only observable system condition.

6. CONCLUSION

Optimal PMU is placed in power system for power system state estimation. Critical measurement free is included for bad data in any single measurement detection ability. SA with stochastic new solution generating is introduced as SSA. SSA result has indicated that the number of PMUs and placement sites are lower than HGS, leading to investment cost savings.

REFERENCES

[1] Chen, J. and Abur, A. 2006. Placement of PMUs to Enable Bad Data Detection in State Estimation. *IEEE Trans. Power Syst.*, vol. 21, no. 4, pp. 1608-1615.

[2] Thorp, J.S., Phadke, A.G. and Karimi, K.J. 1985 Real Time Voltage-Phasor Measurements for Static State Estimation. *IEEE Trans. Power Apparatus and Systems*, vol. PAS-104, no. 11, pp. 3098-3104.

[3] Phadke, A.G., Thorp, J.S. and Karimi, K.J. 1986. State Estimation with Phasor Measurement. *IEEE Trans. Power Syst.*, vol. PWRS-1, no. 1, pp. 233-238.

[4] Filho, M.B.D.C.J., de Souza, C.S. and Oliveira, F. M.F. 2001. Identifying Critical Measurements & Sets for Power System State Estimation. Present at IEEE Porto Power Conference, Porto, Portugul.

[5] Simoes-Costa, A., Piazza, T.S. and Mandel, A. 1990. Qualitative methods to solve qualitative problems in power system state estimation. *IEEE Trans. Power Syst.*, vol. 5, no. 3, pp. 941-949.

[6] Peng, J., Sun, Y. and Wang, H.F. 2006. Optimal PMU Placement for Full Network Observability Using Tabu Search Algorithm. *Electrical Power & Energy System*, vol. 28, pp.223-231.

[7] Baldwin, T.L., Mili, L., Boisen, M.B., and Adapa, R. 1993. Power System Observability with Minimal Phasor Measurement Placement. *IEEE Trans. Power Syst.*, vol. 8, no. 2, pp. 707-715.

[8] Ingber, L. 1993. Simulated Annealing: Practice versus Theory. *Mathematical and Computer Modelling*, 18, 29-57.

[9] Kerdchuen, T. and Ongsakul, W. 2006. Optimal Measurement placement for Power System State Estimation Using Hybrid Genetic Algorithm and Simulated Annealing. Presented at Powercon, Chongqing, Chaina.