A Multi-objective Optimal Placement of Multi-Type DG for Enhancement of Power System Performance by NSGA-II

Kittavit Buayai, I Made Wartana, Sasidharan Sreedharan, and Weerakorn Ongsakul

Abstract—This paper proposes a multi-objective optimal placement of multi-type distributed generator (DG) for enhancement of power system performance. A Pareto-based non-dominated sorting genetic algorithm II (NSGA-II) is proposed to determine locations and sizes of specified number of DG units within the power network. Three objective functions are considered as the indexes of the system performance: maximization the Normal Operation Loadability (NOL) (i.e. the maximum loading which can be supplied by the system while the voltages at all nodes and transmission lines loading are kept within the limits), minimization of the system real power loss and minimization of the annualized investment costs of DG. A fuzzy decision making analysis is used to obtain the final trade off optimal solution. The proposed methodology has been tested on modified IEEE 14-bus system. Test results indicate that NSGA-II is a viable planning tool for practical DG placement in improving the steady state system performance of the power system by the optimal allocation, setting and sizing multi-type DG.

Keywords—Distributed Generation, Multi-objective optimization, NSGA-II, PSAT, System loadability.

1. INTRODUCTION

Distributed generators (DG), based on renewable energy technologies are becoming popular as they address climate change and energy security issues to some extent. Renewable energy based DGs do not contribute to GHG emission and also diversity of sources also increases due to different renewable energy options that address energy security concerns. Apart from climate change and energy security concerns, there are other driving forces for increasing penetration of DG in distribution system [1]. There are a number of technical benefits that the DG can bring such as better voltage profile, loss reduction and reliability improvement.

Several approaches to solve the DG siting and sizing problem in distribution system have been proposed. In [2], they use evolutionary programing approach for optimal placement and size of DG in a radial feeder. The objective is minimize the system real power loss, hybrid distributed generation for a mixed realistic load model is considered. A technique to determine optimal location and sizing of DG units in a MG based on loss sensitivity factor and priority list compare with analytical approach is developed by [3]. A simple methodology for placing a distributed generator with the view of increasing the loadability of the distribution system is presented in [4]. In [5], they use exact loss formula for optimal placement and size of DG in radial distribution system. The objective is minimizing the system real power loss, loadability and voltage stability index. A Genetic Algorithm (GA) combined with power analysis to evaluate DG impacts in system power losses and voltage profile for radial network. The fuzzy power flow is presented in [6]. In [7] DG siting and sizing problem is fulfilled to compromise multi-objective function consisting of energy not-supplied cost, improving cost of network and energy loss cost. In [8, 9], DG siting and sizing problem in distribution network are analyzed to improve only power loss by particle swarm.

From the previous work, we can conclude that the most of the problem of optimal placement and sizing of DG is generally formulated as single-objective optimization problems that optimize a single objective function or transform several objectives to a single objective by aggregating them. Two the most common used of this optimization are the weighted sum method and the ε-constrained method [10]. More study is required to define adequate weights and master objectives, respectively, and the problem is demanding high computational effort. Therefore, multiple objective optimizations are needed in DG placement.

This paper proposes a multi-objective optimal placement of multi-type of DG for enhancement of power system performance. A Pareto-based NSGA-II is proposed to find locations and sizes of a specified number of DG within Power system. Multi-objective functions include maximize NOL within system security margin, minimize system real power loss and annualized investment cost. The final decision will be made by the fuzzy method to find the tradeoff solutions among three different objective functions.

The rest of this paper is organized as follows: Section 2 illustrates the DG placement problem formulation. Section 3 presents a NSGA-II approach for the DG placement. Results and discussions are presented in Section 4. Section 5 summarizes the conclusion and contribution of the paper.

2. DG PLANNING PROBLEM FORMULATION

The normal operation of power system presupposes that
a number of constraint parameters are maintained within predetermined bounds of which the most significant ones are voltage and frequency. The quality of interconnected operation of DG to the grid is specified in terms of operational constraints. DG cluster is assumed to be under the direct supervision and control of the utility operators. The system design assures that there is only unidirectional power flow from DG to the grid and there is safe operation in the event of fault conditions on both sides (DG and grid) by using suitable protection devices. The harmonics analyses are not considered. The model study is conducted in MATLAB - PSAT environment using NSGA-II algorithm. The multi-objective optimization technique to determine the optimal locations and sizes of DG units within power system is as follows:

2.1 Multi-objective

\[
\text{Min } f(x) = [f_1(x), f_2(x), f_3(x)]
\]

(1)

where \(f_1\), \(f_2\), and \(f_3\) represent: normal operation loadability, system real power loss, and annualized investment cost respectively.

2.1.1 Maximize the normal operation loadability

\[
\text{Max } f_1(x,u) = [\lambda]
\]

(2)

Subject to

\[
\text{VL} = \sum_{j=1}^{N_j} BVV_j + \sum_{j=1}^{N_j} OLL_j
\]

(3)

where \(\text{VL}\) is the bus violations and thermal limit factors, \(BVV_j\) and \(OLL_j\) represent the bus voltage violation factor and the overloaded line factor respectively and will be expatiated on later; \(N_j\) and \(N\) are the total load buses and numbers of transmission lines respectively; and \(\lambda\) is a loading parameter of the system, i.e. a scalar variable which multiplies the load direction as follows:

\[
P_d(\lambda) = \lambda P_d\text{lo}\]

(4)

\[
Q_d(\lambda) = \lambda Q_d\text{lo}\]

(5)

If the \(\lambda = 1\), indicates the base load case. The NOL is the maximum loading which can be supplied by the system while the voltages at all nodes and the all branches loading are kept within the limits.

The NOL constrain contains two parts. The first part, \(BVV_j\) in (3) concerns the voltage levels for each bus of the power network. The value of \(BVV_j\) is defined as:

\[
BVV_j = \begin{cases} 
1; & \text{if } 0.9 \leq V_j \leq 1.1 \\
\exp \left( \Gamma_{BVV} \right) & \text{otherwise}
\end{cases}
\]

(6)

where \(BVV_j\) is the bus voltage violation factor at bus \(j\) and \(\Gamma_{BVV}\) represents the coefficient used to adjust the slope of the exponential function in the above equation.

The equation indicates that appropriate voltage magnitudes are close to 1 p.u. The value of \(BVV_j\) equals to 1 if the voltage level falls between the voltage minimal and maximal limits. Outside the range, \(BVV_j\) increases exponentially with the voltage deviation.

The second part, \(OLL_j\), relates to the branch loading and penalizes overloads in the lines. Similar to \(BVV_j\), The value of \(OLL_j\) equals to 1 if the \(j\)th branch loading is less than its rating. \(OLL_j\) increases logarithm (actual logarithm with the overload and it can be calculated from:

\[
OLL_j = \begin{cases} 
1; & \text{if } P_{ij} \leq P_{ij}^{\text{max}} \\
\exp \left( \Gamma_{OLL} \right) \left( 1 - \frac{P_{ij}}{P_{ij}^{\text{max}}} \right) & \text{if } P_{ij} \geq P_{ij}^{\text{max}}
\end{cases}
\]

(7)

where \(P_{ij}\) and \(P_{ij}^{\text{max}}\) are the real power flow between buses \(i\) and \(j\) and the thermal limit for the line between buses \(i\) and \(j\) respectively. \(\Gamma_{OLL}\) is the coefficient which is used to adjust the slope of the exponential function.

2.1.2 Minimize the system real power loss

\[
\text{Min } f_2(x,u) = P_L
\]

(8)

2.1.3 Minimize the annualized Investment Cost

\[
\text{Min } f_3(x,u) = \sum_{i=1}^{N_G} AF_i \times UC_i \times C_{DG_i,\text{max}}
\]

(9)

The annualized investment cost of DG unit \(i\) is assumed to be proportional with the maximum rating of DG, where the unit cost \(UC_i\) is in ($/KVA). The \(UC_i\) is different for different type of generating units. The total of investment cost is transformed to cash value in the beginning of the planning period by using economical expression (i.e. annual cost based on certain interest rate and life span). \(AF_i\) is the annualized factor associated with the installation cost (annual cost based on certain interest rate ‘i’ and life span ‘T’) as shown in (10).

\[
AF_i = \frac{(i/100)(1+i/100)^T}{(1+i/100)^T - 1}
\]

(10)

2.2 Dependent and Control Variables

In the three objective functions, \(x\) is the vector of dependent variables such as slack bus power \(P_{G1}\), load bus voltage \(V_L\), generator reactive power outputs \(Q_G\) and apparent power flow \(S_k\). \(x\) can be expressed as:

\[
x^T = [P_{G1}, V_{k1}, ..., V_{kn}, Q_{G1}, ..., Q_{Gm}, S_1, ..., S_k]
\]

(11)

Furthermore, \(u\) is a set of the control variables such as generator real power outputs \(P_G\) except at the slack bus \(P_{G1}\), generator voltages \(V_G\), the locations of DG units, \(L\).
and their setting parameters. \( \mathbf{u} \) can be expressed as:

\[
\mathbf{u}^T = [P_{G1}, ..., P_{Gn}, P_{DG}, V_{G1}, ..., V_{Gn}, L_1, ..., L_N, Q_{S1}, ..., Q_{Sj}, ..., \lambda]
\]

(12)

where \( N_F \) is the total number of DG devices to be optimally located, and \( N_1 \) to \( N_2 \) are the total numbers of PV and MT respectively. The equality and inequality constraints of the NRPF problem incorporating DG are given bellow.

### 2.3 Equality Constraints

These constraints represent the typical load flow equations as follows:

\[
\begin{align*}
\sum_{i=1}^{N} P_{G_i} &= \sum_{i=1}^{N} P_{in} + P_L \\
\sum_{i=1}^{N} Q_{G_i} &= \sum_{i=1}^{N} Q_{in} + Q_L
\end{align*}
\]

(13)

where \( N \) is the number of buses, \( P_{G_i} \) and \( Q_{G_i} \) are real power reactive power generated by generating unit \( i \) (including slack bus) respectively, in MW.

### 2.4 Inequality Constraints

The inequality constraints are limits of control variables and state variables. Generator active power \( P_{G_i} \), reactive power \( Q_{G_i} \) and voltage \( V_{G_i} \) are restricted by their limits as follows:

\[
\begin{align*}
P_{DG,\text{min}} &\leq P_{DG} \leq P_{DG,\text{max}} \\
Q_{DG,\text{min}} &\leq Q_{DG} \leq Q_{DG,\text{max}} \\
|V|_{\text{min}} &\leq |V| \leq |V|_{\text{max}} \\
P_F &\leq P_{F,\text{max}}
\end{align*}
\]

(14)

The load factor \( \lambda \) is constrained by its limits as:

\[
0 \leq \lambda \leq \lambda_{\text{max}}
\]

(15)

### 2.5 Distributed Generation Model

DG units are modeled as synchronous generators for small hydro power, geothermal power, combined cycles and combustion turbines. They are treated as induction generators for wind and micro hydro power. DG units are considered as power electronic inverter generators such as micro gas turbines, solar power, photovoltaic power and fuel cells [11]. In general, DG can be classified into four types:

- Type 1: DG capable of injecting constant P only (PV)
- Type 2: DG capable of injecting both P and Q (Micro Turbine)
- Type 3: DG capable of injecting constant P but consumes Q (Wind Turbine)
- Type 4: DG capable of delivering Q only (Synchronous condenser).

### 3. NSGA-II FOR DG PLACEMENT

A NSGA-II combined with NRPF based on PSAT [12] is used to solve multi-objective optimization to identify appropriate sizes and locations of a specified number DG unit within power system. The fitness function for the above problem can be written as

\[
f(x) = [f_1(x), f_2(x), f_3(x)] + \sum_{i=1}^{N} (P_{F_i} * U_{SP})
\]

(16)

The final trade off solution is determined by the fuzzy method.

### 3.1 NSGA-II Algorithm

In case of multi-conflicting objectives, there may not exist one solution which is the best compromise for all objectives. Therefore, a “trade-off” solution is needed instead of a single solution in multi-objective optimization. Non-dominated sorting genetic algorithm (NSGA) uses nondominated sorting and sharing has not been widely used mainly because of (i) high computational complexity, (ii) nonelitism approach and (iii) the need for specifying a sharing parameter. NSGA-II is developed to overcome these difficulties [13],[14].

NSGA-II is one of the most efficient algorithms for multi-objective optimization on a number of benchmark problems [14]. In addition, with NSGA-II based approach, the multi-objective of MG planning is retained without the need for any tunable weights or parameters. As a result, the proposed methodology is applicable to solving microgrid planning in a distribution network.

NSGA-II has been developed to determine locations and sizes of DG units within MG area. The NSGA-II procedure can be found in [14] and may be stated as follows:

**Step 1:** Create a random parent population of size \( N \);

**Step 2:** Sort the population based on the nondomination;

**Step 3:** Assign each solution a fitness (or rank) equal to its nondomination level (minimization of fitness is assumed);

**Step 4:** Use the usual binary tournament selection, recombination, and mutation operators to create a new offspring population of size \( N \);

**Step 5:** Combine the offspring and parent population to form extended population of size \( 2N \);

**Step 6:** Sort the extended population based on nondomination;

**Step 7:** Fill new population of size \( N \) with the individuals from the sorting fronts starting from the best;

**Step 8:** Invoke the crowding comparison operator to ensure diversity if a front can only partially fill the next generation (This strategy is called “niching”);

**Step 9:** Repeat the steps 2 to 8 until the stopping criterion is met. The stopping criterion may be a specified number of generations.
It is clear from the above description that NSGA-II uses (i) a fast non-dominated sorting approach, (ii) an elitist strategy, and (iii) no niching parameter [14].

For each iteration \( k \) do:
1) \( R^k = P^k \cup Q^k \) (combine parent and offspring population)
2) \( F = \text{non-dom} \_\text{sort}(R^k) \) (Application the non-dominated sorting on \( k \) \( R^k \))
3) \( P^{k+1} = \Phi \) & \( i = 1 \)
4) until \( \sum_{i=1}^{N} |F_i| \leq N \) (until the parent population is filled)
   a. \( i = i + 1 \)
   b. Calculate the crowding distance for each particle in \( F_i \)
   c. \( P^k = P^{k+1} \cup F^i \)
5) Sort \( (F_i) \) (sort in descending order)
6) \( |P^{k+1}| = |P^{k+1}| \cup F_i \left( N - |P^{k+1}| \right) \) (Choose the first \( N - |P^{k+1}| \) elements of \( F_i \))
7) \( Q^{k+1} \) (use selection, crossover and mutation to create a new population with using \( P^{k+1} \) \( k = k + 1 \))

3.2 Fuzzy Method for Best compromise Solution

Once the Pareto optimal set is obtained, it is practical to select one solution from all solutions that satisfies different goals to some extent. Such a solution is the best compromise solution. In this paper, a simple linear membership function is considered for each of the objective functions. The membership function is defined as follow [15].

\[
\mu_i(z) = \begin{cases} 
1 & f_i(z) \leq f_i^\text{max} \\
\frac{f_i^\text{max} - f_i(z)}{f_i^\text{max} - f_i^\text{min}} & f_i^\text{min} < f_i(z) < f_i^\text{max} \\
0 & f_i(z) \geq f_i^\text{max}
\end{cases}
\]  

(17)

The membership function \( \mu_i(z) \) is varied between 0 and 1, where \( \mu_i(z) = 0 \) indicates incompatibility of the solution with the set, while \( \mu_i(z) = 1 \) means full compatibility. Figure 1 illustrates the graph of this membership function.

The compromised solution can be found by using the normalized membership function [16]. For each non-dominated solution \( k \), the normalized membership function \( \mu^k \) is calculated as:

\[
\mu^k = \frac{\sum_{i=1}^{N_k} \mu_i^k}{\sum_{k=1}^{M} \sum_{i=1}^{N_k} \mu_i^k}
\]  

(18)

In all optimization problems several cases in terms of use of Multi-type DG is considered namely:
1) Base case (without DG).
2) Case 1: PV only.
3) Case 2: GT only.
4) Case 3: coordinated PV and GT.

4. SIMULATIONS

4.1 Analytical Tool and Test System

The load flow analysis used NRPF based on PSAT [12]. Multi-objective optimization problem is solved by NSGA-II.

The power system is the modified IEEE 14-bus test system [12, 17], which consists of two generators, located at bus 1 and 2; three synchronous compensators used only for reactive power support at buses 3, 6 and 8. The system has 11 loads totaling 362.6 MW and 113.96
MVAR, real and reactive load respectively. The IEEE 14-bus test system is depicted in Figure 2.

4.2 Assumptions and Constraints

In this section, it is assumed that:
- Loads are typically represented as constant PQ loads with constant power factor, and increased according to (4) and (5).
- NOL constrains are \( 0 \leq V \leq 1.1 \) and \( S_L \leq 1000 \text{MVA} \)
- The maximum allowable number of DG is two.
- DG placement is not allowed at the same bus.
- All DG resources are evenly distributed within medium voltage area of system.
- Limitation of DG capacity taken into account (as shown on Table A.1 in Appendix), in this paper is not dependent on the category but depend on the total demand of considered power system.

4.3 Evaluation of DG placement within power system

The decision variables considered, are the location and setting of DG units. The DG should be formed at medium voltage side (13.8 kV), consisting of buses 6, 7, 9, 10, 11, 12, 13, and 14. The NSGA-II combined with NR approach is maximized the NOL loadability \( f_1 \), minimizing system real power loss \( f_2 \), and annualized investment cost \( f_3 \). The best parameters for the NSGA-II, selected through ten runs, are given in Table 1. Parameters of all DGs are shown in Table A.1 in Appendix. The number of DG to be installed will be initially specified to two. Simulations have been carried out for optimal placement and size of DG in 3 different DG configurations compared to the base case (without DG) as shown in Table 2.

Table 1. NSGA-II parameters

<table>
<thead>
<tr>
<th>Population</th>
<th>Generation</th>
<th>Pool size</th>
<th>Tour Size</th>
<th>( \eta_c )</th>
<th>( \eta_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>100</td>
<td>25</td>
<td>2</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

4.3.1 Case 1: PV only

The best configuration plan of DG within MG is found at buses 10 and 14 with sizes of 0.7531 p.u. (75.31 MW) and 0.2614 p.u. (26.14 MW), respectively. The process has been repeated for all the three cases and compare to base case as shown in the Table 2. Figure 3 shows the Pareto front, in the objective function space (objective function NOL, system loss and annualized investment cost) for PV only. This set of solutions on the non-dominated frontier is used by the decision maker as the input to select a final compromise solution by using the normalized membership function in (17).

Table 2. Comparison of the results of the 3 cases to the base case

<table>
<thead>
<tr>
<th>Cases</th>
<th>Objectives</th>
<th>Best Compromise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>NOL (pu)</td>
<td>0.85</td>
</tr>
<tr>
<td>Case</td>
<td>RPL (pu)</td>
<td>0.327</td>
</tr>
<tr>
<td>PV only (case 1)</td>
<td>Location (bus)</td>
<td>(10.14, 14)</td>
</tr>
<tr>
<td>PV only (case 1)</td>
<td>Setting (P,Q) in pu</td>
<td>(0.7531, 0.2614)</td>
</tr>
<tr>
<td>MT only (case 2)</td>
<td>Location (bus)</td>
<td>6.9</td>
</tr>
<tr>
<td>MT only (case 2)</td>
<td>Setting (pu)</td>
<td>(0.9612, 0.1207)</td>
</tr>
<tr>
<td>MT only (case 3)</td>
<td>Location (bus)</td>
<td>14.7</td>
</tr>
<tr>
<td>Coordinated PV and MT (case 3)</td>
<td>Setting (pu)</td>
<td>(0.4000, 0.1790, 0.592)</td>
</tr>
</tbody>
</table>

Case 4.3.2: MT only

The best configuration plan of DG within MG is found at buses 6 and 9 with sizes of 0.961 p.u. (96.12 MW) and 1.703 p.u. (170.3 MW), respectively. Their optimal setting of reactive power found to be 0.121 p.u. (12.1 MVAR) and 0.341 p.u. (34.1 MVAR), respectively.

Case 4.3.3: Coordinated PV and MT

The best configuration plan of DG within MG is found at buses 14 and 7 with sizes of 0.400 p.u. (40 MW) and 1.709 p.u. (170.9 MW), respectively. Their optimal settings of reactive power are found to be 0 p.u. (0 MVAR) and 0.592 p.u. (59.2 MVAR), respectively.
Figure 4 shows the comparison of the level of NOL improving, system real power loss and annualized investment cost for the base case, DG type 1, type 2 and type 1&2. Obviously, the DG type 2 is the best plan with respect to system NOL improving of 91.0% and system real power loss reduction of 81.8% compared to the base case. For economic consideration, DG type 1 should be the best plan due to the lowest annualized investment cost. The annualized investment cost for highest NOL level is the lowest at 0.2165 million $/year.

5. CONCLUSION

This paper proposes an efficient multi-objective DG placement methodology. NSGA-II is used to determine locations and sizes of a specified number of distributed generators (DG) within a power system. A fuzzy decision making analysis is used to obtain the final trade off optimal solution. The proposed methodology is tested on IEEE 14-bus system. Using the fuzzy method, DG can improve the system performance by trading off the maximize system NOL, minimize system real power loss and minimize annualized investment cost. Moreover the method does not impose any limitation on the number of objectives. This work will be further extended to address the problem of optimal location of multi-type of DG units to enhance system reliability.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>Loading parameter of the system, in p.u.</td>
</tr>
<tr>
<td>$VL$</td>
<td>Bus violation and thermal limit factors.</td>
</tr>
<tr>
<td>$BVV_i$</td>
<td>Bus voltage violation factor.</td>
</tr>
<tr>
<td>$OLL_i$</td>
<td>Overloaded line factor.</td>
</tr>
<tr>
<td>$N_L$</td>
<td>Total load buses.</td>
</tr>
<tr>
<td>$N_L$</td>
<td>Number of transmission lines.</td>
</tr>
<tr>
<td>$P_{Di}$</td>
<td>Load demand at bus $i$, in MW.</td>
</tr>
<tr>
<td>$Q_{Di}$</td>
<td>Load demand at bus $i$, in MVAR.</td>
</tr>
<tr>
<td>$P_{Di_{base}}$</td>
<td>Load demand at bus $i$ of the base case, in MW.</td>
</tr>
<tr>
<td>$Q_{Di_{base}}$</td>
<td>Load demand at bus $i$ of the base case, in MVAR.</td>
</tr>
<tr>
<td>$V_b$</td>
<td>Actual voltage magnitude at bus $b$, in p.u.</td>
</tr>
<tr>
<td>$P_{ij}$</td>
<td>Real power flow between buses $i$ and $j$, in MW.</td>
</tr>
<tr>
<td>$P_L$</td>
<td>System real power loss, in p.u.</td>
</tr>
<tr>
<td>$Q_L$</td>
<td>System reactive power loss, in p.u.</td>
</tr>
<tr>
<td>$AF_i$</td>
<td>Annualized factor associated with the installation cost of DG unit $i$.</td>
</tr>
<tr>
<td>$UC_i$</td>
<td>Unit cost of DG unit $i$ ($$/kVA).</td>
</tr>
<tr>
<td>$i$</td>
<td>Interest rate (%)</td>
</tr>
<tr>
<td>$T$</td>
<td>Life span in year</td>
</tr>
<tr>
<td>$P_{gi}$</td>
<td>Real power generated by generating unit $i$, in p.u.</td>
</tr>
<tr>
<td>$Q_{gi}$</td>
<td>Reactive power generated by generating unit $i$, in p.u.</td>
</tr>
<tr>
<td>$S_i$</td>
<td>Apparent power flow of transmission line $i$, in p.u.</td>
</tr>
<tr>
<td>$f_{i_{min}}$</td>
<td>Minimum value of the $i^{th}$ objective function among all solutions non-dominated.</td>
</tr>
<tr>
<td>$f_{i_{max}}$</td>
<td>Maximum value of the $i^{th}$ objective function among all solutions non-dominated.</td>
</tr>
<tr>
<td>$\mu_i(z)$</td>
<td>Membership function (varied between 0 and 1).</td>
</tr>
<tr>
<td>$\mu^n$</td>
<td>Normalized membership function.</td>
</tr>
<tr>
<td>NOL</td>
<td>Normal operation loadability, in p.u.</td>
</tr>
<tr>
<td>$P_{Di_{max}}$</td>
<td>Upper real power generating limit of unit $i$, in kW.</td>
</tr>
<tr>
<td>$P_{Di_{min}}$</td>
<td>Lower real power generating limit of unit $i$, in kW.</td>
</tr>
<tr>
<td>$U_{ip}$</td>
<td>the violated constraint</td>
</tr>
<tr>
<td>$N_k$</td>
<td>the total number of violated constraints</td>
</tr>
<tr>
<td>$P_{f_{ip}}$</td>
<td>the penalty factor associated with the violated constraint $U_{ip}$.</td>
</tr>
</tbody>
</table>

REFERENCES


APPENDIX

Table A.1. Parameter for simulation

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Parameter of Simulation</th>
<th>DGs</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DG technology</td>
<td>Photo</td>
<td>Volcanic</td>
<td>Gas Turbine</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>Size (MVA)</td>
<td>0.001 - 200</td>
<td>0.001 - 200</td>
<td>0.001 - 200</td>
</tr>
<tr>
<td>3</td>
<td>Unit cost ($/kVA)</td>
<td>5250</td>
<td>1800</td>
<td>2159</td>
</tr>
<tr>
<td>4</td>
<td>Fuel</td>
<td>Solar energy</td>
<td>Biogas</td>
<td>Wind</td>
</tr>
<tr>
<td>5</td>
<td>Equipment Life (years)</td>
<td>20</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>Interest rate</td>
<td>Economic</td>
<td>0%</td>
<td>75%</td>
</tr>
</tbody>
</table>