Augmented Lagrange Hopfield Network for Combined Economic and Emission Dispatch with Fuel Constraint

Vo Ngoc Dieu, Nguyen Phuc Khai, and Le Quoc Uy

Abstract— *This paper proposes an augmented Lagrange Hopfield network (ALHN) for solving combined economic and emission dispatch (CEED) problem with fuel constraint. In the proposed ALHN method, the augmented Lagrange function is directly used as the energy function of continuous Hopfield neural network (HNN), thus this method can properly handle constraints by both augmented Lagrange function and sigmoid function of continuous neurons in the HNN. For dealing with the bi-objective economic dispatch problem, the slope of sigmoid function in HNN is adjusted to find the Pareto-optimal front and then the best compromise solution for the problem will be determined by fuzzy-based mechanism. The proposed method has been tested on many cases and the obtained results are compared to those from other methods available the literature. The test results have shown that the proposed method can find good solutions compared to the others for the tested cases. Therefore, the proposed ALHN could be a favourable implementation for solving the CEED problem with fuel constraint.*

Keywords— **Augmented Lagrange Hopfield network, combined economic and emission dispatch.**

NOMENCLATURE

a_i, b_i, c_i	emission coefficients for thermal unit i
d_i, e_f, f_i	fuel cost coefficients for thermal unit i
B_{ij}, B_{0i}, B_{00}	transmission loss formula coefficients
F_{ik}	fuel delivery for thermal unit i during subinterval k, in tons
F_i^{min}, F_i^{max}	lower and upper fuel delivery limits for thermal unit i, in tons
M	number of subintervals of scheduled period
N	total number of thermal units;
P_{Dk}	load demand of the system during subinterval k, in MW
P_{Lk}	transmission loss of the system during subinterval k, in MW
P_{ik}	output power of thermal unit i during subinterval k, in MW
P_i^{min}, P_i^{max}	lower and upper generation limits of thermal unit i, in MW
Q_{ik}	fuel consumption function of thermal unit i in subinterval k, in tons/h
t_k	duration of subinterval k, in hours
X_{i0}	initial fuel storage for unit i, in tons
X_{ik}	fuel storage for unit i during subinterval k, in tons
X_i^{min}, X_i^{max}	lower and upper fuel storage limits for thermal unit i, in tons
$V_{p,ik}$	output of continuous neuron representing for output power P_{ik}
$V_{f,ik}$	output of continuous neuron representing for fuel delivery F_{ik}
$V_{x,ik}$	output of continuous neuron representing for fuel storage X_{ik}
$V_{\lambda,k}$	output of multiplier neuron associated with power balance constraint
$V_{\gamma,k}$	output of multiplier neuron associated with fuel delivery constraint
$V_{\eta,ik}$	output of multiplier neuron associated with fuel storage constraint
$U_{p,ik}$, $U_{f,ik}$, $U_{x,ik}$	inputs of continuous neurons corresponding to the outputs $V_{p,ik}$, $V_{f,ik}$ and $V_{x,ik}$, respectively
$U_{\lambda,k}$, $U_{\gamma,k}$, $U_{\eta,ik}$	inputs of multiplier neurons corresponding to the outputs $V_{\lambda,k}$, $V_{\gamma,k}$ and $V_{\eta,ik}$, respectively
ΔP_k	power balance constraint error in subinterval k, in MW
ΔF_k	fuel delivery constraint error in subinterval k, in tons
ΔX_{ik}	fuel storage constraint error for unit i during subinterval k, in tons
$\Delta V_{p,ik}$, $\Delta V_{f,ik}$, $\Delta V_{x,ik}$	iterative errors of continuous neurons
σ	slope of sigmoid function of continuous neurons
α_p, α_f, α_x	updating step sizes for continuous neurons
α_λ, α_γ, α_η	updating step sizes for multiplier neurons
φ_{0i}, φ_{1i}, φ_{2i}	fuel consumption coefficients for thermal unit i
λ_k, γ_k, η_{ik}	Lagrangian multipliers associated with power balance, fuel delivery and fuel storage constraints, respectively
$\beta_{\lambda,k}$, $\beta_{\gamma,k}$, $\beta_{\eta,sk}$	penalty factors associated with power balance, fuel delivery and fuel storage constraints, respectively

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1. INTRODUCTION

Economic dispatch with fuel constraint is an important part of utility for operation and planning since it is a complex problem with a long range of time periods and a large set of constraints and variables. The fuel used by a thermal generating unit may be obtained from different contracts at different prices. The fuel contracts are generally under a take-or-pay agreement including both maximum and minimum limits on delivery of fuel to generating units over life of the contract. The fuel storage for units is usually within a specified limit to allow for inaccurate load forecasts and the inability to deliver on time of suppliers [1]. On the other hand, thermal generating units generate toxic gases during power production due to fossil fuels and this is also considered as a source of environment pollution [2]. With recently increasing concern on environment impact of power generation, the power generation dispatch is required to reduce the emission level while meeting load demand [3]. However, both the fuel cost and emission level conflict together since the pollution minimization will lead to maximizing the fuel cost and vice versa. Therefore, they both must be simultaneously considered to attain a practical compromise operation and this is termed combined economic and emission dispatch (CEED) problem. For solving this problem, one usually finds a set of compromise solutions by simultaneously optimizing all objectives to form a Pareto-optimal front which represents the tradeoffs among conflicting objective functions.

The objective of the CEED problem with fuel constraint is to minimize both total fuel cost and emission from thermal units while satisfying power balance, fuel delivery, fuel storage constraints together with fuel delivery, fuel storage, and generator operating limits. The schedule time horizon for this problem can be decomposed into long-term (weeks to year) [4], short-term (days to week) [5], daily (hours to day) [6], and real-time (minutes to hour) [7] problems. In the long-term schedule problem, the schedule time is divided into sub-periods (months or weeks) to obtain optimal fuel use strategy.

The economic dispatch with fuel constraint for thermal units have been investigated in [7] using linear programming (LP) and network flow programming (NFP). In the LP method [8], the total time period is divided into discrete time increments and the objective function is made up a sum of linear functions where each of function is a function of one or more variables from one step time. In the NFP method [9], the input/output characteristics of generating units can be linear or non-linear which will form linear or non-linear network. For the non-linear network, the problem is solved as a sequence of linear networks with artificial limits calculated from the current solution of the linear network and used for calculating the next solution of the linear network. Nonetheless, these methods suffer difficulties in solving optimization problems. The computation efforts in the NFP will drastically increase when there exist some convex branches in the flow network whereas LP requires linearization of objectives and constraints. On the other hand, the CEED problem has been attracted several researchers. Conventional methods have been applied for solving the problem such as Newton-Raphson (NR) method [10], linear programming (LP) [11], Lagrangian relaxation (LR) method [12], etc. The advantage of these methods is that they can quickly find optimal solution for a problem. However, they suffer some difficulties when dealing with complex and large-scale nonlinear problems such as matrix inversion in NR method, linearization in LP method and duality gap in LR method. For dealing with more complicated problems, several artificial intelligent based methods have been used such as genetic algorithm (GA) [13], evolutionary programming (EP) [14], differential evolution (DE) **Error! Reference source not found.**, and particle swarm optimization (PSO) [16][17]. These population based search methods are suitable for finding near optimal solution for non-convex complicated problems. However, for large-scale problems, these methods become very slow in finding solution and the near optimal solution is not always obtained. Neural networks are also popular for solving the CEED problem [18], [19]. Hopfield neural network (HNN) is the most popular neural network applied to optimization problems and has been successfully applied to the CEED dispatch problem with fuel constraint [19]. Though the HNN can easily handle maximum and minimum constraints for continuous variables based on a sigmoid function, its formulation still suffers some difficulties such as constraint linearization, parameter selection associated with energy function that may lead to local optima if they are not precisely chosen, and map from the problem to the HNN.

In this paper, an augmented Lagrange Hopfield network (ALHN) is proposed for solving combined economic dispatch with fuel constraint. In the proposed ALHN method, the augmented Lagrange function is directly used as the energy function of continuous Hopfield neural network (HNN), thus this method can properly handle constraints by both augmented Lagrange function and sigmoid function of continuous neurons in HNN. For dealing with the bi-objective economic dispatch problem, the slope of sigmoid function in HNN is adjusted to find the Pareto-optimal front and then the best compromise solution for the problem will be determined by fuzzy-based mechanism. The proposed method has been tested on three systems with many cases considered and the obtained results are compared to those from other methods available the literature including recursive approach (RA) [24], simplified recursive approach (SRA) [25], Newton Raphson (NR) [10], fuzzy logic controlled genetic algorithm (FCGA) [13], analytical strategy (AS) [26], multi-objective chaotic particle swarm optimization (MOCPSO) [16], multi-objective chaotic ant swarm optimization (MOCASO) [17].

The remaining organization of this paper is follows. Section 2 addresses the formulation of the combined economic dispatch problem with fuel constraint. Augmented Lagrange Hopfield neural network implement for the problem is described in Section 3. Numerical results are presented in Section 4. Finally, the conclusion is given.

2. PROBLEM FORMULATION

Assuming that the entire schedule time horizon is divided into M subintervals each having a constant load demand P_{Dk} and that all generating units are available and remain on-line for M subintervals. The objective is to simultaneously minimize generation cost F_1 and emission level F_2 of generating units over the M subintervals such that the constraints for power balance, fuel delivery and fuel storage for any given subinterval as well as maximum-minimum fuel delivery, fuel storage, and generator operating constraints for each generating unit are satisfied.

We propose an h-factor to combine generation F_1 and emission level F_2 of generating units.

Mathematically, the problem formulation for a system having N thermal generating units scheduled in M subintervals is as follows [19]:

$$\text{Min } \{F_1 + h.F_2\} \tag{1}$$

where F_1 and F_2 respectively representing the total fuel cost and emission functions are defined based on the quadratic function as follows:

$$F_1 = \sum_{k=1}^{M} \sum_{i=1}^{N} t_k \left(a_i + b_i P_{ik} + c_i P_{ik}^2 \right) \tag{2}$$

$$F_2 = \sum_{k=1}^{M} \sum_{i=1}^{N} t_k \left(d_i + e_i P_{ik} + f_i P_{ik}^2 \right) \tag{3}$$

The h-factor is calculated as follows:

$$h_i = \frac{a_i + b_i P_i^{\max} + c_i \left(P_i^{\max} \right)^2}{d_i + e_i P_i^{\max} + f_i \left(P_i^{\max} \right)^2} \tag{4}$$

subject to

(a) Power balance constraints

The total power supply from the generating units must be sufficient supplying to forecasted load demand of the system and power transmission loss for the whole schedule time horizon:

$$\sum_{i=1}^{N} P_{ik} - P_{Lk} - P_{Dk} = 0 \text{ ; } k = 1,\ldots, M \tag{5}$$

where system power loss is determined by the Kron's formula [8] as follows:

$$P_{Lk} = \sum_{i=1}^{N} \sum_{j=1}^{N} P_{ik} B_{ij} P_{jk} + \sum_{i=1}^{N} B_{0i} P_{ik} + B_{00} \tag{6}$$

(b) Fuel delivery constraint

The total fuel delivery to the generating units must satisfy their demand during the considered schedule time horizon:

$$\sum_{i=1}^{N} F_{ik} - F_{Dk} = 0 \text{ ; } k = 1, \ldots, M \tag{7}$$

(c) Fuel storage constraint

The fuel storage for the generating units must be sufficient for their consumption during the considered schedule time horizon:

$$X_{ik} = X_{ik-1} + F_{ik} - t_k Q_{ik} \text{ ; } i = 1, \ldots, N;\ k = 1, \ldots, M \tag{8}$$

where the fuel consumption of generating units are expressed as a function of power generation:

$$Q_{ik} = \varphi_{0i} + \varphi_{1i} P_{ik} + \varphi_{2i} P_{ik}^2 \tag{9}$$

(d) Generator operating limits

The power outputs from the generators are limited by their capacity of generation:

$$P_i^{\min} \le P_{ik} \le P_i^{\max} \text{ ; } i = 1, \ldots, N;\ k = 1, \ldots, M \tag{10}$$

(e) Fuel delivery limits

The fuel delivery to generating units is limited by the capacity of suppliers:

$$F_i^{\min} \le F_{ik} \le F_i^{\max} \text{ ; } i = 1, \ldots, N;\ k = 1, \ldots, M \tag{11}$$

(f) Fuel storage limits

The fuel storage for generating units is limited by the capacity of the storages:

$$X_i^{\min} \le X_{ik} \le X_i^{\max} \text{ ; } i = 1, \ldots, N;\ k = 1, \ldots, M \tag{12}$$

The fuel storage at subinterval k in (7) can be rewritten in terms of initial fuel storage as follows:

$$X_{ik} = X_{i0} + \sum_{l=1}^{k} \left(F_{il} - t_l Q_{il} \right) \tag{13}$$

in which, the initial fuel storage X_{i0} is given.

3. AUGMENTED LAGRANGE HOPFIELD NETWORK IMPLEMENTATION

For implementation of the bi-objective problem in ALHN, the two objectives are combined in the Lagrangian function which is used as energy function in HNN. By adjusting the sigmoid slope of continuous neurons the obtained corresponding solutions will form a Pareto-optimal front and then the best compromise solution will be determined by fuzzy-based mechanism [20]. The principle of multi-objective optimization and the fuzzy-based mechanism are given in Appendix.

The augmented Lagrange function L of the problem is formulated as follows

$$L = \sum_{k=1}^{M}\sum_{i=1}^{N} t_k \left[ah_i + bh_i P_{ik} + ch_i P_{ik}^2 \right]$$
$$+\sum_{k=1}^{M}\left[\lambda_k \left(P_{Lk} + P_{Dk} - \sum_{i=1}^{N} P_{ik} \right) + \frac{1}{2}\beta_{\lambda,k}\left(P_{Lk} + P_{Dk} - \sum_{i=1}^{N} P_{ik} \right)^2 \right]$$
$$+\sum_{k=1}^{M}\left[\gamma_k \left(\sum_{i=1}^{N} F_{ik} - F_{Dk} \right) + \frac{1}{2}\beta_{\gamma,k}\left(\sum_{i=1}^{N} F_{ik} - F_{Dk} \right)^2 \right]$$
$$+\sum_{k=1}^{M}\sum_{i=1}^{N}\left[\begin{array}{l} \eta_{ik}\left(X_{ik} - X_{i0} - \sum_{l=1}^{k}\left(F_{il} + t_l Q_{il} \right) \right) + \\ +\frac{1}{2}\beta_{\eta,ik}\left(X_{ik} - X_{i0} - \sum_{l=1}^{k}\left(F_{il} + t_l Q_{il} \right) \right)^2 \end{array} \right] \tag{14}$$

where: ah_i, bh_i, ch_i are defined from the h-factor: $ah_i = a_i + h_i.d_i$; $bh_i = b_i + h_i.e_i$; $ch_i = c_i + h_i.f_i$;

To represent in augmented Lagrange Hopfield neural network, $3N$xM continuous neurons and $(N+2)$xM multiplier neurons are required. The energy function E of the problem is formulated based on the augmented Lagrangian function in terms of neurons as follows.

$$E = \sum_{k=1}^{M}\sum_{i=1}^{N} t_k \left[ah_i + bh_i V_{p.ik} + ch_i V_{p,ik}^2 \right]$$
$$+\sum_{k=1}^{M}\left[V_{\lambda k} \left(P_{Lk} + P_{Dk} - \sum_{i=1}^{N} V_{p,ik} \right) + \frac{1}{2}\beta_{\lambda k}\left(P_{Lk} + P_{Dk} - \sum_{s=1}^{N} V_{p,ik} \right)^2 \right]$$
$$+\sum_{k=1}^{M}\left[V_{\gamma k} \left(\sum_{i=1}^{N} V_{f,ik} - F_{Dk} \right) + \frac{1}{2}\beta_{\gamma,k}\left(\sum_{s=1}^{N} V_{f,ik} - F_{Dk} \right)^2 \right]$$
$$+\sum_{k=1}^{M}\sum_{i=1}^{N}\left[\begin{array}{l} V_{\eta,ik}\left(V_{x,ik} - X_{i0} - \sum_{l=1}^{k}\left(V_{f,il} + t_l Q_{il} \right) \right) + \\ +\frac{1}{2}\beta_{\eta,ik}\left(V_{x,ik} - X_{i0} - \sum_{l=1}^{k}\left(V_{f,il} + t_l Q_{il} \right) \right)^2 \end{array} \right]$$
$$+\sum_{k=1}^{M}\sum_{i=1}^{N}\left(\int_0^{V_{p,ik}} g^{-1}(V)dV + \int_0^{V_{f,ik}} g^{-1}(V)dV + \int_0^{V_{x,ik}} g^{-1}(V)dV \right) \tag{15}$$

In (14), the sums of integral terms are Hopfield terms where their global effect is a displacement of solutions toward the interior of the state space [21].

The dynamics of augmented Lagrange Hopfield network for updating neuron inputs are defined as follows.

$$\frac{dU_{p,ik}}{dt} = -\frac{\partial E}{\partial V_{p,ik}} \tag{16}$$

$$\frac{dU_{f,ik}}{dt} = -\frac{\partial E}{\partial V_{f,ik}} \tag{17}$$

$$\frac{dU_{x,ik}}{dt} = -\frac{\partial E}{\partial V_{x,ik}} \tag{18}$$

$$\frac{dU_{\lambda,k}}{dt} = +\frac{\partial E}{\partial V_{\lambda,k}} \tag{19}$$

$$\frac{dU_{\gamma,k}}{dt} = +\frac{\partial E}{\partial V_{\gamma,k}} \tag{20}$$

$$\frac{dU_{\eta,ik}}{dt} = +\frac{\partial E}{\partial V_{\eta,ik}} \tag{21}$$

The inputs of neurons are updated based on their dynamics as follows:

$$U_{p,ik}^{(n)} = U_{p,ik}^{(n-1)} - \alpha_p \frac{\partial E}{\partial V_{p,ik}} \tag{22}$$

$$U_{f,ik}^{(n)} = U_{f,ik}^{(n-1)} - \alpha_f \frac{\partial E}{\partial V_{f,ik}} \tag{23}$$

$$U_{x,ik}^{(n)} = U_{x,ik}^{(n-1)} - \alpha_x \frac{\partial E}{\partial V_{x,ik}} \tag{24}$$

$$U_{\lambda,k}^{(n)} = U_{\lambda,k}^{(n-1)} + \alpha_\lambda \frac{\partial E}{\partial V_{\lambda,k}} \tag{25}$$

$$U_{\gamma,k}^{(n)} = U_{\gamma,k}^{(n-1)} + \alpha_\gamma \frac{\partial E}{\partial V_{\gamma,k}} \tag{26}$$

$$U_{\eta,ik}^{(n)} = U_{\eta,ik}^{(n-1)} + \alpha_\eta \frac{\partial E}{\partial V_{\eta,ik}} \tag{27}$$

The outputs of continuous neurons representing for output power, fuel delivery and fuel storage of power plants are calculated from on their inputs by a sigmoid function [22]:

$$V_{p,ik} = g(U_{p,ik}) = \left(\frac{P_i^{\max} - P_i^{\min}}{2} \right)\left[1 + \tanh\left(\sigma U_{p,ik} \right) \right] + P_i^{\min} \tag{28}$$

$$V_{f,ik} = g(U_{f,ik}) = \left(F_i^{\max} - F_i^{\min} \right)\left(\frac{1 + \tanh\left(\sigma U_{f,ik} \right)}{2} \right) + F_i^{\min} \tag{29}$$

$$V_{f,ik} = g(U_{f,ik}) = \left(F_i^{\max} - F_i^{\min} \right)\left(\frac{1 + \tanh\left(\sigma U_{f,ik} \right)}{2} \right) + F_i^{\min} \tag{30}$$

where σ determines the shape of the sigmoid function. The shape of the sigmoid function is shown in Fig. 1.

The outputs of multiplier neurons representing Lagrangian multipliers are determined by a transfer function:

$$\mathrm{V}_{\lambda,k} = g(U_{\lambda,k}) = U_{\lambda,k} \tag{31}$$

$$\mathrm{V}_{\gamma,k} = g(U_{\gamma,k}) = U_{\gamma,k} \tag{32}$$

$$\mathrm{V}_{\eta,ik} = g(U_{\eta,ik}) = U_{\eta,ik} \tag{33}$$

The proof of convergence of the ALHN method is given Appendix.

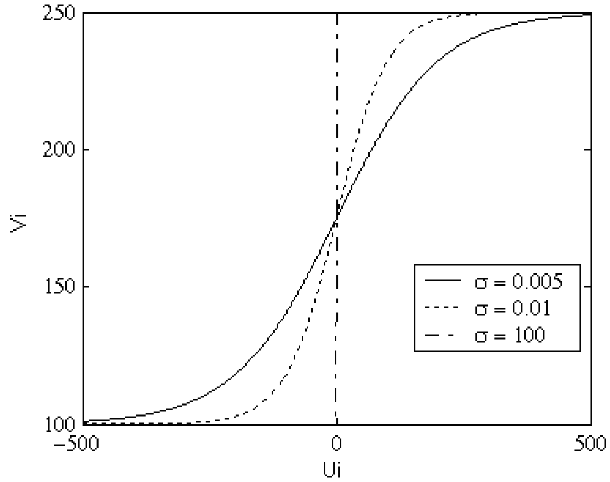

Fig. 1. Sigmoid function with different slopes

3.1 Selection of parameters

All positive parameters in the ALHN model have to be selected in advance including slope of sigmoid function and updating step sizes for neurons and penalty factors associated with constraints. These parameters are selected by experiments and the proper parameters will lead to fast convergence of the network. Among the parameters, the value of σ has directly effect on the priority of objectives in the problem. If σ is greater than 1, the fuel objective is more important than the emission one. In contrast, if σ is smaller than 1, the emission objective is more important. Therefore, the non-dominated solutions for the problem will be obtained by adjusting the value of σ from smaller than 1 to very large values. The penalty factors associated with all constraints are equally chosen and usually fixed to a small value. The other parameters including α_f, α_x, α_γ and α_η will be tuned depending on the considered problem. It is observed that the larger the values of these parameters, the closer the system act to being a discrete system, producing values at the upper and lower limits of each neuron. On the contrary, the smaller the values of them, the slower the convergence of the network. For simplicity, the values of α_f and α_x can be equally chosen. It is similar manner for α_γ and α_η.

3.2 Initialization

The algorithm requires initial conditions for inputs and outputs of all neurons. In this paper, the initial outputs of neurons are selected as follows.

For the continuous neurons representing for power output and fuel delivery of units, their outputs are initiated by "mean distribution" [23]:

$$V_{p,ik}^{(0)} = P_{Dk} \frac{P_i^{\max}}{\sum_{i=1}^{N} P_i^{\max}} \tag{34}$$

$$V_{f,ik}^{(0)} = F_{Dk} \frac{F_i^{\max}}{\sum_{i=1}^{N} F_i^{\max}} \tag{35}$$

For the neurons representing for fuel storage of power plants, their outputs are initiated at the medium value between the maximum and minimum values of fuel storage:

$$V_{x,ik}^{(0)} = \left(X_i^{\max} + X_i^{\min}\right)/2 \tag{36}$$

For the multiplier neurons associated with power balance constraint, their outputs are initialized by mean values as follows:

$$V_{\lambda k}^{(0)} = \frac{1}{N}\sum_{i=1}^{N} \frac{t_k\left(b_i + 2c_i V_{p,ik}^{(0)}\right) + t_k\left(e_i + 2 f_i V_{p,ik}^{(0)}\right)}{1 - \dfrac{\partial P_{Lk}}{\partial V_{p,ik}}} \tag{37}$$

The outputs of other multiplier neurons are initiated from zeros. The inputs of all neurons are calculated corresponding to their outputs via the inversion of corresponding sigmoid and transfer functions.

3.3 Stopping criteria

The algorithm of the ALHN will be terminated when either the maximum error Err_{max} including both constraint and iterative errors is lower than a predefined tolerance ε or maximum number of iterations N_{max} is reached.

The constraint and iterative errors at iteration *n* are calculated as follows.

$$\Delta P_k^{(n)} = \left| P_{Dk} + P_{Lk} - \sum_{i=1}^{N} V_{p,ik}^{(n)} \right| \tag{38}$$

$$\Delta F_k^{(n)} = \left| \sum_{i=1}^{N} V_{f,ik}^{(n)} - F_{Dk} \right| \tag{39}$$

$$\Delta X_{ik}^{(n)} = \left| V_{x,ik}^{(n)} - X_{i0} - \sum_{l=1}^{k}\left(V_{f,il}^{(n)} + t_l Q_{il}\right) \right| \tag{40}$$

$$\Delta V_{p,ik}^{(n)} = \left| V_{p,ik}^{(n)} - V_{p,ik}^{(n-1)} \right| \tag{41}$$

$$\Delta V_{f,ik}^{(n)} = \left| V_{f,ik}^{(n)} - V_{f,ik}^{(n-1)} \right| \tag{42}$$

$$\Delta V_{x,ik}^{(n)} = \left| V_{x,ik}^{(n)} - V_{x,ik}^{(n-1)} \right| \tag{43}$$

The maximum error of the model is determined:

$$Err_{\max}^{(n)} = \max\left\{\Delta P_k^{(n)}, \Delta F_k^{(n)}, \Delta X_{ik}^{(n)}, \Delta V_{p,ik}^{(n)}, \Delta V_{f,ik}^{(n)}, \Delta V_{x,ik}^{(n)}\right\} \tag{44}$$

3.3 Overall procedure

Overall procedure of ALHN for solving the CEED problem with fuel constraint is as follows:

Step 1: Select parameters for the network as in Section 3.1 and choose stopping criteria as in Section

3.3.

Step 2: Initialize inputs and outputs for all neurons as in Section 3.2.

Step 3: Set number of iteration $n = 1$.

Step 4: Calculate dynamics of neurons from equations (16) - (21).

Step 5: Update inputs of neurons from equations (22) - (27).

Step 6: Calculate outputs of neurons from equations (28) - (33).

Step 7: Calculate maximum error as in Section 3.3.

Step 8: If $Err_{max} > \varepsilon$ and $n < N_{max}$, $n = n + 1$ and return to Step 4.

Step 9: Calculate total cost and emission, and stop.

4. NUMERICAL RESULTS

The proposed ALHN is tested on six-unit and three-plant systems without fuel constraint and a five-unit system with fuel constraint. The algorithm of ALHN is coded in Matlab platform and run on a 2.1 GHz with 2 GB RAM PC. For stopping criteria, the maximum tolerance ε is set to 10^{-4}.

4.1 Case 1: Six-unit system neglecting fuel constraint

The test system from [24] includes six units supplying to a load demand of 900 MW.

4.1.1 Case 1a: Neglecting power loss

When power loss is neglected, the total power generation is balanced to load demand. Three cases are considered in this cases including best economic dispatch, best emission dispatch and combined economic and emission dispatch.

Table 1 shows a comparison of best compromise solution from ALHN to that from RA method and simplified RA (SRA) method [25]. To find the best compromise solution by ALHN, a Pareto-optimal front is obtained first as shown in Fig. 2 and then the fuzzy-based mechanism is used. For comparison of the best compromise solution among the methods, a price penalty factor (PPF) method [18] is used as in Table 1. The explanation and calculation of PPF are given Appendix.

For the cases with single objective optimization, the total cost and emission amount by the proposed method are closed to those from RA method. However, for the case of best compromise solution, the total equivalent cost from the proposed ALHN is less than that from RA method.

Table 1: Comparison of best compromise solution for Case 1a

	RA [24]	SRA [25]	ALHN
Fuel cost ($/h)	46,131.80	46,131.80	46,191.32
Emission (kg/h)	679.241	679.241	676.4765
Emission PPF ($/kg)	44.7880	44.7880	44.7880
Equiv. cost of emission ($/h)	30,421.85	30,421.85	30,298.03
Total equiv. cost ($/h)	76,554.65	76,554.65	76,489.35

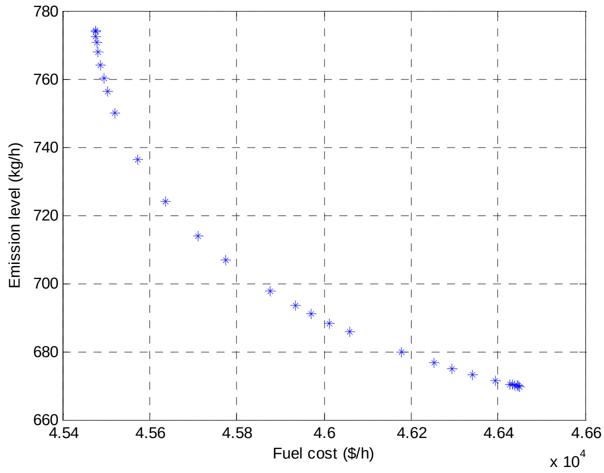

Fig. 2. Pareto-optimal front for fuel cost and emission from Case 1a

4.1.2 Case 1b: Considering power loss

When power loss is included, the total power generation is balanced to load demand plus power loss. In this case, three load demands are considered including 500 MW, 700 MW, and 900 MW. Table 2 shows the comparison of best compromise solutions from the proposed method to those from NR [10] and FCGA methods. The Pareto-optimal fronts for fuel cost and emission corresponding to the load demands are given in Figs. 3. For all the compared cases in Tables 2, the proposed method can find better solutions in terms of less total costs and emission levels for single objective problems and less total equivalent costs for CEED than the others.

4.2 Case 2: Three-plant system neglecting fuel constraint

The test system has three plants with six generating units supplying to a total load demand of 900 MW as shown in **Error! Reference source not found.**. The unit data is from [26] and also given in **Error! Reference source not found.**.

In calculation, the system power loss in terms of load demand and B-matrix coefficients is derived out in [26]. The total costs, emission amount, and equivalent costs obtained from the ALHN method for best CEED are compared to those from AS method [26], MOCPSO [16], and MOCASO [17] in Tables 3, respectively. The Pareto-optimal front obtained by the ALHN for this case is shown in Fig.4. In all cases, the total cost in best economic dispatch, total emission in best emission dispatch, and total equivalent cost in CEED from the proposed method are less than those from the others. For the computational time, the proposed method also obtains solution faster than the others for best economic dispatch and best emission dispatch. Note the computational times obtained in AS, and both MOCPSO and MOCASO are from a Intel Pentium III processor, 996 MHz, 416 MB of RAM PC and a Intel (R) Core (TM) 2 Duo CPU PC with 2.2 GB of RAM, respectively.

Table 2: Comparison of best compromise solution for Case 1b

Load		NR [10]	FCGA [13]	ALHN
500 MW	Fuel cost ($/h)	28,550.15	28,231.06	28,423.7037
	Emission (kg/h)	312.513	304.90	280.3083
	Emission PPF ($/kg)	43.8983	43.8983	43.8983
	Equiv. cost of emission ($/h)	13,718.7894	13,384.5917	12,305.06
	Total equiv. cost ($/h)	42,268.9394	41,615.6517	41,206.8448
700 MW	Fuel cost ($/h)	39,070.74	38,408.82	38,816.1969
	Emission (kg/h)	528.447	527.46	479.8875
	Emission PPF ($/kg)	44.7880	44.7880	44.7880
	Equiv. cost of emission ($/h)	23,668.0842	23,623.8785	21,493.2
	Total equiv. cost ($/h)	62,738.8242	62,032.6985	60,897.5768
900 MW	Fuel cost ($/h)	50,807.24	49,674.28	50,340.0820
	Emission (kg/h)	864.060	850.29	776.2410
	Emission PPF ($/kg)	47.8222	47.8222	47.8222
	Equiv. cost of emission ($/h)	41,321.2501	40,662.7384	37,121.55
	Total equiv. cost ($/h)	92,128.4901	90,337.0184	87,461.63

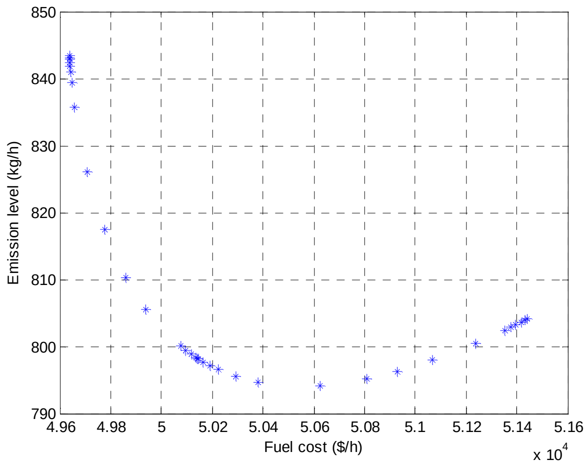

Fig. 3. Pareto-optimal front for fuel cost and emission in Case 1b with load demand of 900 MW

Table 3: Comparison of best compromise solution for Case 2

Method	AS [26]	MOCPSO [16]	MOCASO [17]	ALHN
Fuel cost ($/h)	47,804.55	47,549.87	47,804.59	47,949.78
Emission (kg/h)	843.42	823.36	843.41	735.1226
Emission PPF ($/kg)	47.8222	47.8222	47.8222	47.8222
Equiv. cost of emission ($/h)	40,334.20	39,374.89	40,333.72	35,155.18
Total cost ($/h)	88,138.75	86,924.76	88,138.31	83,104.98

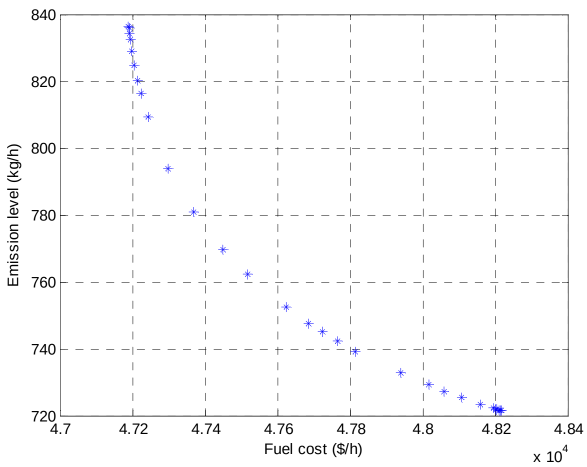

Fig.4. Pareto-optimal front for fuel cost and emission in Case 2

4.3 Case 3: Five-unit system with fuel constraint

The system consists of five thermal units in [19] with fuel constraints supplying to load demand for a 3-week period.

Three sub-cases with differently initial fuel storage are considered for this system. The initial values of fuel storage are given in Table 5.

Table 5: Initial storage for the five-unit system in Case 3

Sub-case	Unit	1	2	3	4	5
1	$X_i^{(0)}$ (tons)	2000	5000	5000	8000	8000
2	$X_i^{(0)}$ (tons)	2000	5000	5000	500	8000
3	$X_i^{(0)}$ (tons)	2000	2500	2500	8000	500

When the system power loss is considered, the obtained results from the ALHN method are given in Table 6 and Pareto-optimal fronts for the sub-cases are shown in Figs.5-7.

Table 6: Results for Case 3 by ALHN

Sub-case		Best compromise
1	Fuel cost ($)	1,034,680.0121
	Emission (kg)	655,494.8034
2	Fuel cost ($)	1,034,673.6641
	Emission (kg)	655,511.7391
3	Fuel cost ($)	1,034,684.4679
	Emission (kg)	655,467.2724

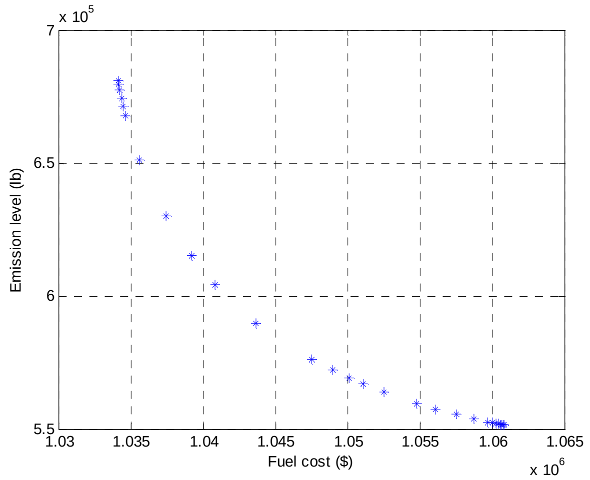

Fig. 5. Pareto-optimal front for fuel cost and emission in Sub-case 1 of Case 3

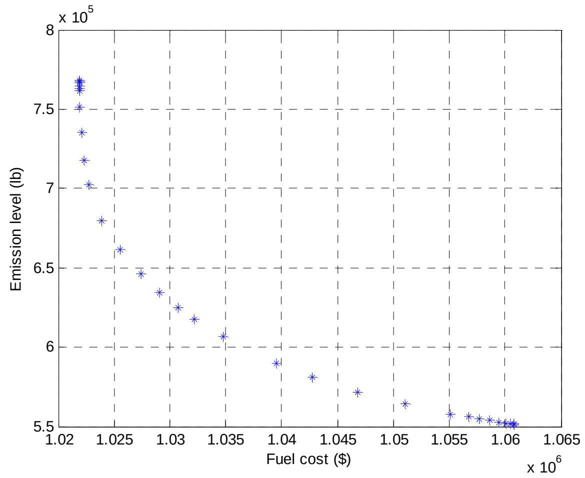

Fig. 6. Pareto-optimal front for fuel cost and emission in Sub-case 2 of Case 3

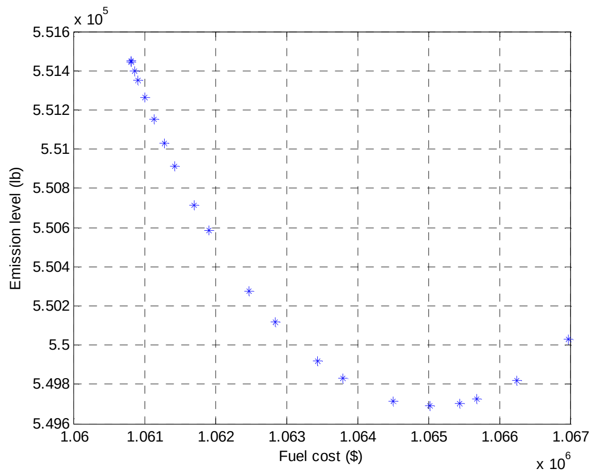

Fig. 7. Pareto-optimal front for fuel cost and emission in Sub-case 3 of Case 3

5. CONCLUSION

In this paper, the ALHN method has been efficiently implemented for solving the CEED problem with fuel constraint. By directly using the augmented Lagrange function as the energy function for Hopfield network together with sigmoid function of continuous neurons, the problem constraints are properly handled. Moreover, ALHN is a recurrent neural network with parallel processing which leads to quick convergence to the optimal solution for the optimization problems. For obtaining different non-dominated solutions of a multi-objective optimization problem, the slope of sigmoid function of continuous neurons is adjusted from very small to very large values. The result comparisons from the many tested cases have shown that the proposed ALHN can obtain better optimal solutions than many other methods. Therefore, ALHN could be a favourable implementation for solving the CEED with complicated constraints.

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