SPP Loss Allocation by Cooperative Games

Sopa Heng, Onurai Noohawm and Dulpichet Rerkpreedapong

Abstract— *This paper presents the calculation method of transmission loss allocation including active and reactive losses when an SPP sells electrical power to its customers through 115 kV PEA networks. The concept of circuit theorem and cooperative game theory are together employed in the proposed method. Then, power losses can be allocated to each power producer by using the Shapley value method, and the simulation is performed by the PowerWorld Simulator. From the results, it is found that the SPP location is the key factor of loss allocation. In addition, impartial and reasonable results of loss allocation have been obtained from verifying the effectiveness of the proposed method.*

Keywords— **Loss allocation, cooperative game theory, Shapley, SPP, current injection.**

1. INTRODUCTION

In the past, SPPs (Small Power Producer) could sell electrical power to EGAT (Electricity Generating Authority of Thailand) only. Now SPPs are allowed to sell electricity to their own customers directly. Most of the SPPs are located in PEA (Provincial Electricity Authority) service areas so this can result in a redundant investment in power distribution infrastructure between PEA and SPPs. To mitigate such a problem, PEA has allowed SPPs to sell electrical power to their customers through PEA distribution and transmission networks, and SPPs are charged for PEA infrastructure usages.

Currently, SPP wheeling charges are calculated from the amount of power demand regardless of the SPP location. This way might not be truly reasonable because the SPP location has a significant impact on power losses of the system. Generally, SPPs often increase power losses of networks due to their power flows. On the contrary, some SPPs can reduce the power losses.

Presently, many methodologies have been adopted to compute the transmission charging [1]-[3]. Their advantages and disadvantages are concluded as follows:

- Marginal participation (MP) or incremental cost allocation method is based on marginal participation of generators and loads. This method is uncomplicated to understand and implement because it is found by load flow results, which are MWs of generators injected into the network or loads drawn from the network. Then, the line usage is apportioned among generators and load entities in a proportion to their weighted marginal participation. However, its disadvantage is that the location of slack bus has an effect to marginal loss factor. This is difficult to select the slack bus either individual or dispersed. Also, this method may be cross subsidies on the system.
- Proportional power flow tracing or average participation (AP) method is mostly used to find transmission charges because it is based on proportional sharing principle. Its assumption is that at any network node, the inflows are distributed proportionally between the outf ows and allow tracing in meshed transmission networks [2]. This method is easy to implement but the proportional sharing rule may not give a fair solution.
- Min-max fair tracing method is attractive because it has improved fairness of the model in tracing. However, the model proposed in [3] may not converge to a fair solution, and it also lacks in scalability.

The cooperative game theory is a new method to loss allocation among participants in electricity market [4] – [8]. This method is effective in terms of giving impartial results for loss allocation. In [4], the equivalent current-injected generation and constant-impedance load models were used with the concept of Shapley value from cooperative game theory to allocate power losses to all participants. [5] – [6] proposed the procedures for active and reactive power loss allocation in transmission networks, and at the same time those losses were allocated to individual sources and loads. In [5], the equivalent nodal injection currents of sources and loads were tested with the designed two-step cooperative game together with the Shapley value to allocate power losses. It was a direct analytical method which can be simply understood. In this work, the power losses were decreased by counter-flows generated from capacitive sources. As a result, both sources and loads who contributed in loss reduction should be rewarded a decrease in loss allocation as an economic incentive. In [6], the circuit theory and Aumann-Shapley value for loss allocation was proposed. The network losses were divided into two equal parts, and each of them was allocated to load and generator sides respectively. In this method, the grid system was considered as a black box such that grid parameters were not required for loss

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allocation. However, the Aumann-Shapley method was rather complicated to understand and calculate.

The method of loss allocation proposed in [7] used the cooperative game theory and Shapley value. Both sources and loads were considered as the players of game. This method used Thevenin theorem to transform the sources to equivalent voltage sources, and loads to impedances.

In [8], the method of loss reduction allocation was applied to distributed generation (DG) units in distribution systems. The participation of each unit in reducing the amount of power losses was found, and the cooperative game theory is used to allocate the benefits to each DG. In this paper, the τ method and the Shapley value were compared, and the results were about similar.

This paper presents a method to allocate both active and reactive power losses in PEA's networks to PEA and SPPs, when the grid-connected SPPs are selling electrical power to their customers. The proposed method employs the circuit laws and cooperative game theory to find the losses contributed from each participant in the game. To allocate the losses, the Shapley value method is applied while the players of the game are current injection sources and loads of all nodes. This method has been tested with the 5-bus, 115-kV system. It is found that the SPP's location notably affects the amount of losses allocated to all participants.

2. COOPERATIVE GAME THEORY

In electricity market, the cooperative game theory was widely applied to solve problems regarding cost and loss allocation of transmission networks to their users because this method is convenient to use, and offers impartial and reasonable results. For an n-participant cooperative game, the number of coalitions of n players is equal to 2^n. For an example of three players ($n=\{1,2,3\}$), it results in eight coalitions as follows. $S=\{\},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}$ when S is subsets of set n.

In this paper, the Shapley value is used to allocate the amount of current and transmission loss to each player. The player k will receive payoff allocation as shown in (1).

$$\phi_k(v)=\sum_{\substack{S\subseteq n\\ k\in n}}\frac{(n-|S|)!(|S|-1)!}{n!}\left[v(S)-v(S-\{k\})\right] \quad (1)$$

where: ϕ_k = payoff allocation to player k

S = group of players as subsets of n including player k

$v(S)$ = characteristic function of coalition S

3. METHODOLOGY

In this paper, the methodology presented in [5] is used to allocate losses because of its practicality. It could simultaneously allocate both active and reactive power losses which satisfy all circuit laws. In this method, the players are nodal injection currents which play two cooperative games. In the first game, current allocation for branch l is performed. In the second game, the losses are allocated to all players. The procedure is illustrated as follows:

1. Let a player of the game be injected current at node k ($\bar{I}_k^n$) including injections from sources and loads, when n is the number of players or number of nodes excluding the ground node. It can be calculated from the complex power equation in (2).

$$S_k=\bar{V}_k\hat{I}_k \quad (2)$$

where: S_k = complex power injected at node k

$\bar{V}_k$ = phasor voltage at node k

$\hat{I}_k$ = complex conjugate of injected phasor current at node k

Thus, the equation of injected current at node k is given by (3) – (5).

$$\bar{I}_{k_S}^n=\frac{\left(P_{Sk}-jQ_{Sk}\right)}{\hat{V}_k}\ ,\ k=1,2,\cdots,n \quad (3)$$

$$\bar{I}_{k_D}^n=\frac{\left(-P_{Dk}+jQ_{Dk}\right)}{\hat{V}_k}\ ,\ k=1,2,\cdots,n \quad (4)$$

$$\bar{I}_k^n=\bar{I}_{k_S}^n+\bar{I}_{k_D}^n\ ,\ k=1,2,\cdots,n \quad (5)$$

where: $\bar{I}_{k_S}^n,\bar{I}_{k_D}^n$ = injected phasor current from source and load respectively at node k

P_{Sk},Q_{Sk} = active and reactive power injected at node k respectively

$\bar{I}_k^n$ = injected phasor current at node k

2. Calculate the current of branch l using Ohm's law as expressed in equation (6)

$$\bar{I}_l^b=\frac{\left(\bar{V}_i^n-\bar{V}_j^n\right)}{\bar{Z}_l} \quad (6)$$

where: $\bar{V}_i^n,\bar{V}_j^n$ = phasor voltages at node i and j

$\bar{Z}_l$ = impedance of branch l

$\bar{I}_l^b$ = phasor current of branch l

3. Calculate the responding current of branch l from the current injection into node k both from sources and loads by using Ohm's and Kirchhoff's laws as given by (7) – (10).

$$\bar{V}^n=\bar{Z}^n\bar{I}^n \quad (7)$$

where: $\bar{V}^n$ = column vectors of the bus voltages

$\bar{I}^n$ = column vectors of the injection currents entering the buses

$\bar{Z}^n$ = bus impedance matrix

The matrix in Eq. (7) can be formulated in terms of equations as given by (8).

$$\bar{V}_i^n = \sum_{k=1}^{n} \bar{Z}_{ik}^n \bar{I}_k^n \quad , \quad i = 1,2,\cdots,n \tag{8}$$

where: $\bar{Z}_{ik}^n$ = element in the *i*th row and *j*th column of the bus impedance matrix

Thus, the the responding current of branch l from the current injection into node k both sources and loads is given by (9)

$$\left.\begin{aligned} \bar{I}_l^b &= \frac{\left(\bar{V}_i^n - \bar{V}_j^n\right)}{\bar{Z}_l} = \frac{\sum_{k=1}^{n} \bar{Z}_{ik}^n \bar{I}_k^n - \sum_{k=1}^{n} \bar{Z}_{jk}^n \bar{I}_k^n}{\bar{Z}_l} \\ &= \frac{\sum_{k=1}^{n}\left(\bar{Z}_{ik}^n - \bar{Z}_{jk}^n\right)\bar{I}_k^n}{\bar{Z}_l} \end{aligned}\right\} \tag{9}$$

Let $\bar{I}_{kl}^b$ be the total responding current of branch l from injection current ($\bar{I}_k^n$) at node k. It is given by (10).

$$\bar{I}_{kl}^b = \frac{\left(\bar{Z}_{ik}^n - \bar{Z}_{jk}^n\right)}{\bar{Z}_l} \bar{I}_k^n \quad , \quad k = 1,2,\cdots,n \tag{10}$$

Substituting Eq. (5) into Eq. (10) results in (11).

$$\bar{I}_{kl}^b = \frac{\left(\bar{Z}_{ik}^n - \bar{Z}_{jk}^n\right)}{\bar{Z}_l}\left[\bar{I}_{k_S}^n + \bar{I}_{k_D}^n\right] \quad , \quad k = 1,2,\cdots,n \tag{11}$$

The total responding current of branch l ($\bar{I}_{kl}^b$) is decomposed into two parts including those from source and load as given by (12)-(15).

$$\bar{I}_{k_S l}^b = \frac{\left(\bar{Z}_{ik}^n - \bar{Z}_{jk}^n\right)}{\bar{Z}_l} \bar{I}_{k_S}^n \tag{12}$$

$$\bar{I}_{k_D l}^b = \frac{\left(\bar{Z}_{ik}^n - \bar{Z}_{jk}^n\right)}{\bar{Z}_l} \bar{I}_{k_D}^n \tag{13}$$

where:
$$\bar{I}_{kl}^b = \bar{I}_{k_S l}^b + \bar{I}_{k_D l}^b \quad , \quad 1,2,\cdots,n \tag{14}$$

and
$$\bar{I}_l^b = \sum_{k=1}^{n} \bar{I}_{kl}^b \tag{15}$$

4. Start the first cooperative game such that the player of game is the injected current at node k ($\bar{I}_k^n$), and the number of players is equal to the number of nodes (n). The characteristic function of game is $\bar{I}_l^b(S)$, when S is the group of players of the game.

Next, the current of branch l is allocated to each player using the Shapley value according to Eq. (1). The current fraction of player k can be calculated by Eq. (16).

$$\bar{I}_{kl}^n\left(\bar{I}_l^b\right) = \sum_{\substack{S \subseteq n \\ k \in n}} \frac{(n-|S|)!(|S|-1)!}{n!}\left[\bar{I}_l^b(S) - \bar{I}_l^b(S - \{k\})\right] \tag{16}$$

From Eq. (16), the player k will receive current fraction of branch l equal to $\bar{I}_{kl}^b$ according to Eq. (11).

5. Next, begin the second cooperative game. In this game, the characteristic function of game is active and reactive power losses on branch l. To find the loss fraction of each player ($\bar{I}_{kl}^b$), it can be calculated from step 4 as follows.

Rewrite $\bar{I}_{kl}^b$ as (17)

$$\bar{I}_{kl}^b = a_{kl} + jb_{kl} \tag{17}$$

From the power loss equations, the characteristic function of game is given by (18)-(19).

$$P_l(S) = \left|\sum_{k \in S} \bar{I}_{kl}^b\right|^2 r_l = \left|\sum_{k \in S} a_{kl}\right|^2 r_l + \left|\sum_{k \in S} b_{kl}\right|^2 r_l \tag{18}$$

$$Q_l(S) = \left|\sum_{k \in S} \bar{I}_{kl}^b\right|^2 x_l = \left|\sum_{k \in S} a_{kl}\right|^2 r_l + \left|\sum_{k \in S} b_{kl}\right|^2 x_l \tag{19}$$

where: P_l, Q_l = active and reactive power losses on branch l

r_l, x_l = resistance and reactance of branch l

Then, the losses on branch l are allocated to each player using the Shapley value. The loss fraction of player k can be calculated by Eq. (20).

$$L_{kl}^P\left(P_l\right) = \sum_{\substack{S \subseteq n \\ k \in n}} \frac{(n-|S|)!(|S|-1)!}{n!}\left[P_l(S) - P_l(S - \{k\})\right] \tag{20}$$

From Eq. (20), the loss fraction of total active power on branch l of player k (L_{kl}^P) is expressed by (21).

$$L_{kl}^P = r_l a_{kl} \sum_{h=1}^{n} a_{hl} + r_l b_{kl} \sum_{h=1}^{n} b_{hl} \quad , \quad k = 1,2,\cdots,n \tag{21}$$

Eq. (21) can be expressed in dot product form as (22).

$$L_{kl}^P = \left(\bar{I}_{kl}^b \bullet \bar{I}_l^b\right) r_l \quad , \quad k = 1,2,\cdots,n \tag{22}$$

where: $\bar{I}_{kl}^b = \left[a_{kl} \; b_{kl}\right]^T$

$$\bar{I}_l^b = \sum_{k=1}^{n} \bar{I}_{kl}^b$$

Substituting Eq. (14) into Eq. (22) leads to (23)-(24).

$$L_{k_S l}^{P} = \left(\bar{I}_{k_S l}^{b} \bullet \bar{I}_{l}^{b}\right) r_l \quad (23)$$

$$L_{k_D l}^{P} = \left(\bar{I}_{k_D l}^{b} \bullet \bar{I}_{l}^{b}\right) r_l \quad (24)$$

Eq. (23) and (24) express the loss fraction of total active power on branch l of player k from source and load.

6. From the above equations, find the loss fraction of total active power of player k at all branches ($L_{k_S}^{P}, L_{k_D}^{P}$) in the network using (25)-(26).

$$L_{k_S}^{P} = \sum_{l=1}^{b} L_{k_S l}^{b} \quad (25)$$

$$L_{k_D}^{P} = \sum_{l=1}^{b} L_{k_D l}^{b} \quad (26)$$

7. Similarly, the loss fraction of total reactive power on branch l of player k from source and load is given by (27)-(28).

$$L_{k_S l}^{Q} = \left(\bar{I}_{k_S l}^{b} \bullet \bar{I}_{l}^{b}\right) x_l \quad (27)$$

$$L_{k_D l}^{Q} = \left(\bar{I}_{k_D l}^{b} \bullet \bar{I}_{l}^{b}\right) r_l \quad (28)$$

And the loss fraction of total reactive power of player k at all branches ($L_{k_S}^{Q}, L_{k_D}^{Q}$) in the network is given by (29)-(30).

$$L_{k_S}^{Q} = \sum_{l=1}^{b} L_{k_S l}^{b} \quad (29)$$

$$L_{k_D}^{Q} = \sum_{l=1}^{b} L_{k_D l}^{b} \quad (30)$$

8. Finally, the total losses allocated to SPPs and their customers are added up to find the total active and reactive power losses. The total losses for PEA can be calculated in the same way.

4. CASE STUDY

The test system is a looped network because PEA's 115-kV transmission lines are typically looped for increasing the system security. There are 5-bus, 115-kV system including 7 transmission lines and 5 high-voltage customers. The customers at bus 2 and 3 are purchasing power from the SPP, and the others are PEA's customers as shown in Figure 1. This paper presents 2 case studies to analyze the effects of SPP's location. Source, load and line data are given in Table 1 to 3 respectively.

Case I: The SPP located at bus 4 results in active and reactive power loss allocation to sources and loads as shown in Table 4. The results of total active and reactive power loss fractions of SPP and PEA are given in Table 5.

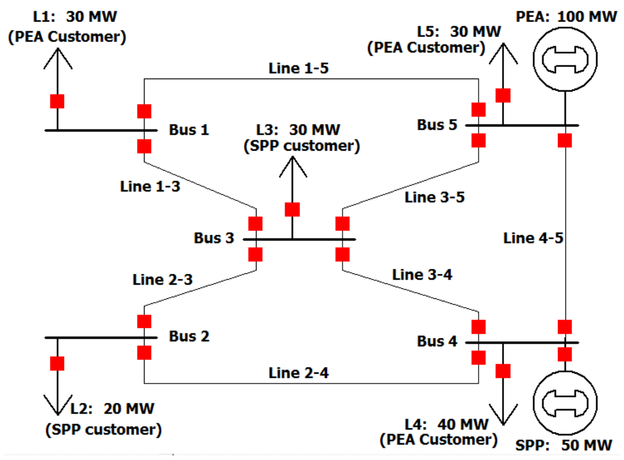

Fig.1. A 5-bus, 115-kV transmission system (Base MVA = 100 and Base kV = 115 kV)

Table 1. Source Data

Case	Bus	Generator (MVA)	Power providers
I	4	50.00 + j0.00	SPP
	5	100.77 + j97.94	PEA
II	1	50.00 + j0.00	SPP
	5	101.03 + j99.41	PEA

Table 2. Load Data

Bus	Load (MW)	Power providers
1	30.00 + j18.59	PEA
2	20.00 + j12.39	SPP
3	30.00 + j18.00	SPP
4	40.00 + j24.78	PEA
5	30.00 + j18.35	PEA

Table 3. Line parameters

l	Line	R (Ohm)	X (Ohm)
1	1-3	0.01719	0.12200
2	1-5	0.02149	0.15250
3	2-3	0.01719	0.12200
4	2-4	0.02149	0.15250
5	3-4	0.01504	0.10675
6	3-5	0.01504	0.10675
7	4-5	0.02149	0.15250

Table 4. Case I: Source and Load Loss allocation Results

Bus		Bus Power (MVA)	Loss Fractions (MVA)
Source	4	50.00 + j0.00	-0.1613 - j1.1443
	5	100.77 + j97.94	0.9185 + j6.5176
Total		**150.77 + j97.94**	**0.7572 + j5.3733**
Load	1	30.00 + j18.59	0.0730 + j0.5181
	2	20.00 + j12.39	0.0823 + j0.5838
	3	30.00 + j18.00	0.0566 + j0.4018
	4	40.00 + j24.78	0.0236 + j0.1672
	5	30.00 + j18.35	-0.1703 - j1.2082
Total		**150.00 + j92.11**	**0.0652 + j0.4627**
Total System Loss			**0.8224 + j5.8360**

Table 5. Case I: SPP and PEA Loss allocation Results

Power Providers	Bus		Bus Power (MVA)	Loss Fractions (MVA)
SPP	Source	4	50.00 + j0.00	-0.1613 - j1.1443
	Load	2	20.00 + j12.39	0.0823 + j0.5838
		3	30.00 + j18.00	0.0566 + j0.4018
Total Loss				**-0.0224 - j0.1587**
PEA	Source	5	100.77 + j97.94	0.9185 + j6.5176
	Load	1	30.00 + j18.59	0.0730 + j0.5181
		4	40.00 + j24.78	0.0236 + j0.1672
		5	30.00 + j18.35	-0.1703 - j1.2082
Total Loss				**0.8448 + j5.9947**
Total System Loss				**0.8224 + j5.8360**

Table 6. Case II: Source and Load Loss allocation Results

Bus		Bus Power (MVA)	Loss Fractions (MVA)
Source	1	50.00 + j0.00	-0.0348 - j0.2469
	5	101.03 + j99.41	1.0255 + j7.2764
Total		**151.03 + j99.41**	**0.9907 + j7.0295**
Load	1	30.00 + j18.59	-0.0854 - j0.6059
	2	20.00 + j12.39	0.1090 + j0.7737
	3	30.00 + j18.00	0.0497 + j0.3528
	4	40.00 + j24.78	0.1559 + j1.1066
	5	30.00 + j18.35	-0.1916 - j1.3595
Total		**150.00 + j92.11**	**0.0376 + j0.2677**
Total System Loss			**1.0283 + j7.2972**

Table 7. Case II: SPP and PEA Loss allocation Results

Power providers	Bus		Bus Power (MVA)	Loss Fractions (MVA)
SPP	Source	1	50.00 + j0.00	-0.0348 - j0.2469
	Load	2	20.00 + j12.39	0.1090 + j0.7737
		3	30.00 + j18.00	0.0497 + j0.3528
Total Loss				**0.1239 + j0.8796**
PEA	Source	5	101.03 + j99.41	1.0255 + j7.2764
	Load	1	30.00 + j18.59	-0.0854 - j0.6059
		4	40.00 + j24.78	0.1559 + j1.1066
		5	30.00 + j18.35	-0.1916 - j1.3595
Total Loss				**0.9044 + j6.4176**
Total System Loss				**1.0283 + j7.2972**

Case II: The SPP is relocated to bus 1, and results in active and reactive power loss allocation to sources and loads as shown in Table 6. The results of total active and reactive power loss fractions of SPP and PEA are shown in Table 7.

The results given in Table 5 and 6 show that the SPP location at bus 4 results in a negative loss fraction (-0.0224-j0.1587 MVA) because of this location generates counter flows. While the SPP located at bus 1 results in a positive loss fraction (0.1239 + j0.8796 MVA). Therefore, the SPP location has significant effects to power losses in the system. The appropriate SPP location can reduce losses as shown in Case I.

5. CONCLUSION

The application of the cooperative game and Shapley value to complex loss allocation is performed. The results show the losses are allocated reasonably and impartially to all players. The loss fraction is positive if it flows in the same direction of the branch's dominant current. Otherwise, it is negative. The allocation method using the Shapley value reveals the benefits in terms of loss reduction caused by the player of the game as shown in Case I. In other words, the player or SPP at bus 4 reduced the complex losses of the system. As a result, the SPP should be rewarded a decrease in costs resulted from loss allocation. The results of both case studies also show that the network loss allocation depends on the SPP location and network topology. At last, this method is appealing to an SPP to choose a suitable location for minimizing costs regarding loss allocation, and transmission losses in the network.

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