Cuckoo Search Algorithm for Maximum Loadability of Power Systems

Dieu Ngoc Vo and Kien Trung Vo

Abstract— *This paper proposes a cuckoo search algorithm (CSA) for solving the maximum loadability of power systems. The objective of the maximum loadability in power systems is to find the maximum value of power factor so as all unit and system constraints are satisfied. The proposed CSA is a new developed method inspired from the brood behavior of the cuckoo species for solving optimization problems. The main advantage of the CSA method is its effectiveness for optimization problems with few control parameters. The main operations used in CSA are the Lévy flights and the probability of alien eggs discovered in the host bird's nest to generate new solutions. The proposed CSA method has been tested on the IEEE 30 bus and 118 bus systems and the result comparisons with other methods in the literature has indicated that the proposed CSA is more efficient than other methods. Therefore, the proposed CSA can be a very favorable method for solving the maximum loadability of power systems.*

Keywords— **Cuckoo search algorithm, maximum loadability, load factor, Lévy flights.**

1. INTRODUCTION

In the past decades, the power systems in the world have become larger and more complex to meet the social and economic development. Therefore, the transmission system has played a very important role for power systems to be stalely operated. The maximum loadability has attracted special attention in power system [1].

There have been many methods proposed for solving the maximum loadability problem. For the conventional methods, many methods have been proposed for solving the problem such as continuous power flow (CPF) [2-4], interior point (IP) method [5], direct method [6-7], fast voltage stability index (FVSI) [8], P-e curve [9], and trust region (TR) [10]. Among the methods, the CPF method has been early proposed and become popular for determining the maximum loadability point in power systems. However, this method may lead to inexact result if the chosen step length is large. Moreover, as the power system operates near the maximum loadability point, the CPF method may suffer the problem of convergence and a proper step size selection depends on the experience of the researcher. The IP method is very effective for determine the maximum loadability of power systems. However, if the step size in the IP method is not properly selected, it may lead to difficulty for this method to deal with nonlinear and non-smooth problems. Moreover, this method is also sensitive to the initial condition and stopping criteria. In the P-e curve method, the load voltage is considered as a function of real power demand or total power demand. There are two solutions for this method including high current - low voltage and low current - high voltage. In fact, the system usually operates at upper branch of the curve which is corresponding to low current - high voltage solution. The intersection of the upper branch and low branch of the curve is the voltage collapse point. In the TR method, the power balance equation at the maximum loadability is considered as the optimal function. The solution for this method is the approximate quadratic function. If the function is exact, the dimension of the region is increased; otherwise the dimension of the region is reduced. In general, the conventional methods can be efficiently implemented for solving the problem. However, these methods may suffer difficulty when dealing with complex systems. Recently, many other artificial intelligence based methods have been also implemented for solving the maximum loadability problem such as genetic algorithm (GA) [11], evolutionary programming (EP) [12], ant colony optimization (ACO) [13], particle swarm optimization (PSO) [14-16], and hybrid methods [17-19]. Among these methods, GA is the early method used for solving the problem. However, this method can suffer many disadvantages such as local optima and long computational time. The PSO method has been widely used for solving optimization problem and the advantages of this method is simple and fast computation. However, this method My suffer local optima for complex problems. The hybrid methods usually utilize the advantages of the component methods. Although these methods are effective for complex problems, they are usually complex for handling may control parameters and slow computation due to combination of different methods.

In this paper, a cuckoo search algorithm (CSA) is proposed for solving the maximum loadability of power systems. The objective of the maximum loadability in power systems is to find the maximum value of power factor so as all unit and system constraints are satisfied. The proposed CSA is a new developed method inspired from the brood behavior of the cuckoo species for solving optimization problems. The main advantage of the CSA method is its effectiveness for optimization

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problems with few control parameters. The main operations used in CSA are the Lévy flights and the probability of alien eggs discovered in the host bird's nest to generate new solutions. The proposed CSA method has been tested on the IEEE 30 bus and 118 bus systems and the obtained results have been compared to those from other methods in the literature.

2. PROBLEM FORMULATION

The objective of the maximum loadability problem is to maximize the load factor of the power system satisfying system constraints such as power generation limits, bus voltage limits, tap changer limits and switchable capacitor limits [3]. Mathematically, the maximum loadability problem is formulated as follows:

$$\text{Max } f = \lambda \quad (1)$$

where λ is load factor representing the load incensement of system from the current operating point to the maximum point as

$$0 \le \lambda \le \lambda_{critical} \quad (2)$$

in which, λ = 0 for the base case and $\lambda = \lambda_{critical}$ for the voltage collapse case.

The load demand at load buses is simultaneously increased as follows [14]:

$$P_{di} = P_{di0} + \lambda P_d \quad (3)$$

$$Q_{di} = Q_{di0} + \lambda Q_d \quad (4)$$

where P_{di} is the real power at load bus i; P_{di0} is the initial real power at load bus i; Q_{di} is the reactive power at load bus i; Q_{di0} is the initial real power at load bus i.

As the real power P_{di} and reactive power Q_{di} at load bus i increase with respect to P_{di0} and Q_{di0}, (3) and (4) can be rewritten:

$$P_{di} = P_{di0}(1+\lambda) \quad (5)$$

$$Q_{di} = Q_{di0}(1+\lambda) \quad (6)$$

subject to

- Real and reactive power balance

$$P_{gi} - P_{di} = |V_i| \sum_{i=1}^{N_b} |Y_{ij}||V_j| \cos(\delta_i - \delta_j - \theta_{ij}) \quad (7)$$

$$Q_{gi} - Q_{di} = |V_i| \sum_{i=1}^{N_b} |Y_{ij}||V_j| \sin(\delta_i - \delta_j - \theta_{ij}) \quad (8)$$

where N_b is number of buses; $|V_i|$ and δ_i are voltage magnitude and angle at bus i, respectively; $|Y_{ij}|\angle\theta_{ij}$ is the i^{th} element of the bus admittance matrix.

As load increased, the value of F_{pi} and F_{qi} is defined as follows: [10]

$$F_{pi} = P_{gi} - P_{di} - |V_i| \sum_{i=1}^{N_b} |Y_{ij}||V_j| \cos(\delta_i - \delta_j - \theta_{ij}) \quad (9)$$

$$F_{qi} = Q_{gi} - Q_{di} - V_i| \sum_{i=1}^{N_b} |Y_{ij}||V_j| \sin(\delta_i - \delta_j - \theta_{ij}) \quad (10)$$

where P_{di} and Q_{di} are from (5) and (6), respectively.

- Real power upper and lower limits

$$P_{gi,max} \le P_{gi} \le P_{gi,min},\ i = 1,\ldots,N_g \quad (11)$$

where N_g is number of generation buses.

- Reactive power upper and lower limits

$$Q_{gi,max} \le Q_{gi} \le Q_{gi,min},\ i = 1,\ldots,N_g \quad (12)$$

- Upper and lower limits of bus voltage magnitude and angle limits

$$|V_i|_{min} \le |V_i| \le |V_i|_{max},\ i = 1,\ldots,N_b \quad (13)$$

$$\delta_{i,min} \le \delta_i \le \delta_{i,max} \quad (14)$$

- Transformer tap changer limits

$$T_{k,min} \le T_k \le T_{k,max},\ k = 1,\ldots,N_t \quad (15)$$

where N_t is number of transformers.

- Switchable shunt capacitor bank limits

$$Q_{ci,min} \le Q_{ci} \le Q_{ci,max},\ i = 1,\ldots,N_c \quad (16)$$

where N_c is number of switchable capacitor bank.

3. IMPLEMENTATION OF CSA FOR THE MAXIMUM LOADABILITY PROBLEM

The proposed CSA was developed by Yang and Deb in 2009 [20] inspired from the behaviors of the cuckoo species in the nature combined with Le1vy flights. The behavior of the cuckoo species is laying their eggs in the nest of other species which have similar egg with the cuckoo species. For surviving, the cuckoo species usually hatch before the host bird and young cuckoo birds also look similar the young bird of the host bird. If the host bird discovers a strange egg or young bird in its nest, it whether destroys the nest or abandon the nest to construct a new one. On the other hand, many animals or insects have their behavior similar to Lévy flights which is a random walk with the heavy-tailed step-lengths of probability distribution [20].

The CSA is developed based on the three rules as follows:

- Each cuckoo bird lays an egg at a certain time to any preselected nest among the host nests.

- The best egg with best quality will be transferred to the next generation.

- The number of host nests is fixed, and the alien by the cuckoo species can be discovered with a probability of p_a in the range [0,1]. For this case, the host bird whether throws alien egg away or abandons the nest to construct a new one.

For implementation of CSA to the problem, the map from the problem to the proposed method is as follows:

- Each egg in a nest represents a solution for the problem and the egg which is not discovered by the host bird will be considered the best solution for the next step.

- The quality of the obtained solutions will be evaluated by a fitness function defined as a combination of the objective function and penalized constraints associated a penalty factor.

- At each step, the new solutions (eggs) are generated by Lévy flights and the probability of an alien egg discovered by the host bird will replace the worse ones.

The steps for implementation of CSA to the problem are as follows.

3.1 Initialization

Each nest X_d, d = 1,…, N_p which is the number of nests representing the power factor λ is initialized as follows:

$$X_d = X_{d,min} + rand_1*(X_{d,max} - X_{d,min}) \quad (17)$$

where $X_{d,max} = \lambda_{critical}$, $X_{d,min} = 0$, and $rand_1$ is the random number in [0,1].

After the nests initialized, the power flow problem will be solved for the fixed power factors. In this paper, the power flow problem is solved by Newton-Raphson using Matpower toolbox [21]. The fitness function for evaluation of the obtained solutions defended as:

$$Fit_d = K_1*(1/\lambda) + K_2*\sum_{i=1}^{N_b}(F_{pi}^2 + F_{qi}^2) + K_3\sum_{i=1}^{N_g}FP_i^2 + K_4*\sum_{i=1}^{N_g}FQ_i^2 + K_5*\sum_{i=1}^{N_d}FV_i^2 \quad (18)$$

where K_1, K_2, K_3 and K_4 are penalty factors; N_b, N_g and N_d are number of buses, generation buses and load buses, respectively; FP_i is the real power mismatch at generation bus i determined by:

$$FP_i = \begin{cases} P_{gi} - P_{gi,\max} & \text{If } P_{gi} > P_{gi,\max} \\ P_{gi,\min} - P_{gi} & \text{If } P_{gi} < P_{gi,\min} \\ 0 & \text{Otherwise} \end{cases} \quad (19)$$

FQ_i is the reactive power mismatch at generation bus i calculated by:

$$FQ_i = \begin{cases} Q_{gi} - Q_{gi,\max} & \text{If } Q_{gi} > Q_{gi,\max} \\ Q_{gi,\min} - Q_{gi} & \text{If } Q_{gi} < Q_{gi,\min} \\ 0 & \text{Otherwise} \end{cases} \quad (20)$$

and FV_i is the voltage mismatch as bus i:

$$FV_i = \begin{cases} V_i - V_{i,\max} & \text{If } V_i > V_{i,\max} \\ V_{i,\min} - V_i & \text{If } V_i < V_{i,\min} \\ 0 & \text{Otherwise} \end{cases} \quad (21)$$

Each initial nest is set to $X_{best,d}$ representing the best case for each nest and the best nest is set to G_{best} representing the best nest among the population.

3.2 New Solution Generation via Lévy Flights

The new solution generation via Lévy flights using Mantegna algorithm [22] is calculated as follows:

$$X_{d,new} = X_{best} + \alpha \times rand_2 \times \Delta X_{d,new} \quad (22)$$

where the positive α is the step length of the Lévy flights, $rand_2$ is a random number in [-1,1], and the incremental $\Delta X_{d,new}$ is calculated by:

$$\Delta X_{d,new} = \nu \frac{\sigma_x(\beta)}{\sigma_y(\beta)}\left(X_{best,d} - G_{best}\right) \quad (23)$$

in which,

$$\nu = \frac{rand_x}{\left|rand_y\right|^{1/\beta}} \quad (24)$$

$$\sigma_x(\beta) = \left[\frac{\Gamma(1+\beta)\times\sin\left(\frac{\pi\beta}{2}\right)}{\Gamma\left(\frac{1+\beta}{2}\right)\times\beta\times 2^{\left(\frac{\beta-1}{2}\right)}}\right]^{1/\beta} \quad (25)$$

$$\sigma_y(\beta) = 1 \quad (26)$$

where $rand_x$ and $rand_y$ are random distributions with standard deviation of $\sigma_x(\beta)$ and $\sigma_y(\beta)$, β is the distribution factor (0.3<β<1.99), and Γ is a gamma distribution function.

The new obtained solution should also satisfy the limits of the power factor.

$$X_{d,new} = \begin{cases} X_{d,new} - X_{d,\max} & \text{If } X_{d,new} > X_{d,\max} \\ X_{d,\min} - X_{d,new} & \text{If } X_{d,new} < X_{d,\min} \\ 0 & \text{Otherwise} \end{cases} \quad (27)$$

The new solution is used for solving the power flow problem and then the fitness function (18) is used for evaluation of the new solution.

For each nest, the current fitness value is compared to that from the best previous one and the nest corresponding to the best fitness function value is set to $X_{d,best}$ for the next step of calculation. The nest corresponding to the best fitness function so far is set to G_{best}.

3.3 New Solution Generation via the Alien Egg Discovery

For the cuckoo eggs, some of them may be discovered by the host bird and the host bird either abandons its nest and builds a new one or destroys the eggs. The solutions corresponding to the discovered eggs are considered the low quality ones and they should be replaced by the new ones. The new solution generated via the probability of an alien egg discovered in the host bird's nets is calculated by:

$$X_{d,dis} = X_{d,best} + K \times \Delta X_{d,dis} \quad (28)$$

in which, K is the update step size determined as follows:

$$K = \begin{cases} 0 & \text{if } rand_3 < p_a \\ 1 & \text{if } rand_3 \geq p_a \end{cases} \quad (29)$$

and the step value $\Delta X_{d,dis}$ is determined by

$$\Delta X_{d,dis} = rand_4 \times \left[randp_1(X_{d,best}) - randp_2(X_{d,best})\right] \quad (30)$$

where $rand_3$ and $rand_4$ are the random number in the range [0,1], p_a is the probability of an alien egg discovered in the host bird's nest, $randp_1(X_{d,best})$ và $randp_2(X_{d,best})$ are the random perturbation for positions of nests in $X_{d,best}$.

In this step, the obtained value of $X_{d,dis}$ is also adjusted if it violates its limits similar to (27). The power flow problem is performed using Newton-Raphson method [21] and the fitness function (18) is used to evaluate the quality of the new obtained solution. By comparing the value of the fitness function at the current step and the previous one to choose $X_{d,best}$ for each nest and G_{best} for all the nests.

3.4 Stooping Criteria

In the proposed method, the stopping criteria are based on the maximum number of iterations. The algorithm will stop as the maximum number of iterations is reached.

4. NUMERICAL RESULTS

The proposed method has been tested on the IEEE 30 and 118 bus systems. For the test systems, the control parameters such as the number of nests, maximum number of iterations, and probability p_a are selected by experiments. By tuning, these values are selected as follows: N_p = 10, N_{max} = 100 and p_a = 0.5. The proposed algorithm is coded in Matlab and run on a 2.0 GHz PC with 2GB of RAM.

4.1 The IEEE 30-Bus System

The IEEE bus system from [21] is a part of American Electric Power System (in the Midwestern US) in 1961 having six generators and 41 branches including four transformers. The system configuration is given in Figure 1.

The result obtained by the proposed CSA has been compared to that from multi agent-based hybrid particle swarm optimization (MAHPSO) [15] differential evolution (DE) [19] and hybrid differential evolution with particle swarm optimization (HDEPSO) [19] as in Table 1. As observed from the table, the proposed CSA has obtained the highest total real power load among the compared methods. The maximum voltage amplitude and angle obtained by CSA for the IEEE 30 bus system is given in Table 2. The convergence characteristic of the proposed CSA for the test system is given in Figure 2.

Table 1. Result comparison for the IEEE 14-bus system

Method	Max total real power load (pu)	CPU time (s)
MAHPSO [15]	2.6081	-
DE [19]	2.6709	-
HDEPSO [19]	2.6974	-
CSA	2.8396	85.301

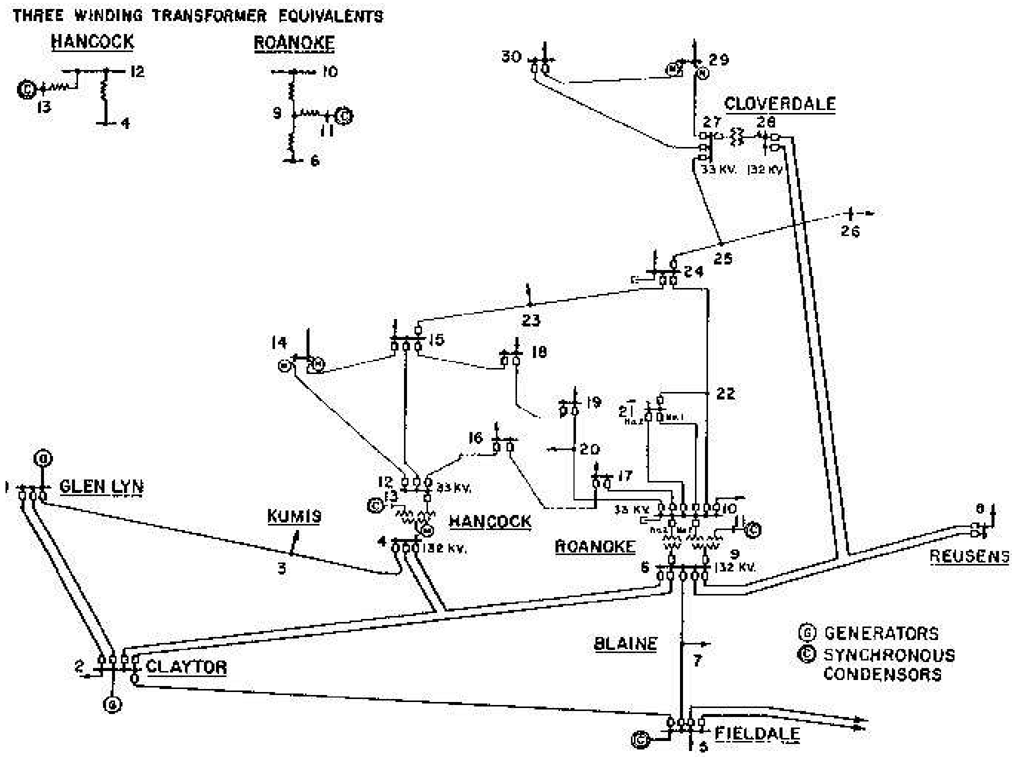

Fig. 1. The IEEE 30-bus system.

Table 2. The maximum voltage amplitude and angle by CSA for the IEEE 30 bus system

Bus	Voltage (pu)	Angle (degree)	Bus	Voltage (pu)	Angle (degree)
1	1.0000	0.0000	16	0.9631	-11.3313
2	1.0000	-2.9802	17	0.9632	-12.2847
3	0.9626	-5.4061	18	0.9488	-12.9567
4	0.9569	-6.5069	19	0.9441	-13.5354
5	0.9646	-6.1111	20	0.9505	-13.2798
6	0.9451	-7.6755	21	0.9897	-12.8340
7	0.9368	-7.9491	22	1.0000	-12.8265
8	0.9240	-8.5290	23	1.0000	-11.1806
9	0.9645	-10.6437	24	0.9812	-12.1774
10	0.9757	-12.1492	25	0.9840	-10.8271
11	0.9645	-10.6437	26	0.9550	-11.5530
12	0.9748	-9.8114	27	1.0000	-9.5531
13	1.0000	-6.7653	28	0.9460	-8.1693
14	0.9615	-11.2216	29	0.9667	-11.6301
15	0.9670	-11.4058	30	0.9476	-13.1195

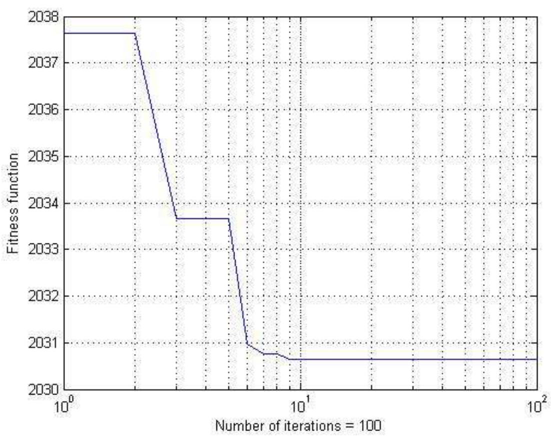

Fig. 2. The convergence characteristic of CSA for the IEEE 30 bus system.

4.2 The IEEE 118-Bus System

The IEEE 118 bus system from [21], [23] representing a portion of the American Electric Power System (in the Midwestern US) in 1962 includes 54 generation buses, 64 load buses, and 186 branches where 9 transformers are located at branches 8, 32, 36, 51, 93, 95, 102, 107, and 127 and 14 switchable capacitor bank located at buses 5, 34, 37, 44, 45, 46, 48, 74, 79, 82, 105, 107, and 110. The configuration of the system is given in Figure 3.

The result obtained by the proposed CSA for the system has been compared to that from other methods including MAHPSO [15], DE [19], HDEPSO [19], and hybrid PSO [17] as in Table 3. The result comparison has indicated that the proposed CSA is the most efficient methods for dealing with this system. The convergence characteristic of the proposed CSA for the test system is in given in Figure 4.

Table 3. Result comparison for the IEEE 118-bus system

Method	Max total real power load (pu)	CPU time (s)
MAHPSO [15]	56.450	-
DE [19]	56.543	-
HDEPSO [19]	57.0156	-
HPSO [17]	55.146	-
CSA	62.5671	806.556

5. CONCLUSION

In this paper, the proposed CSA has been effectively implemented for solving the maximum loadability of power systems. The proposed CSA method is effective for solving optimization problem with few control variables. The proposed CSA has been tested on the IEEE 30 and 118 bus systems and the obtained results have been compared to those from other methods in the literature. The result comparisons have shown that the proposed CSA is more efficient than other methods for the test systems. Therefore, the proposed CSA can be a very favorable method for solving the maximum loadability of power systems.

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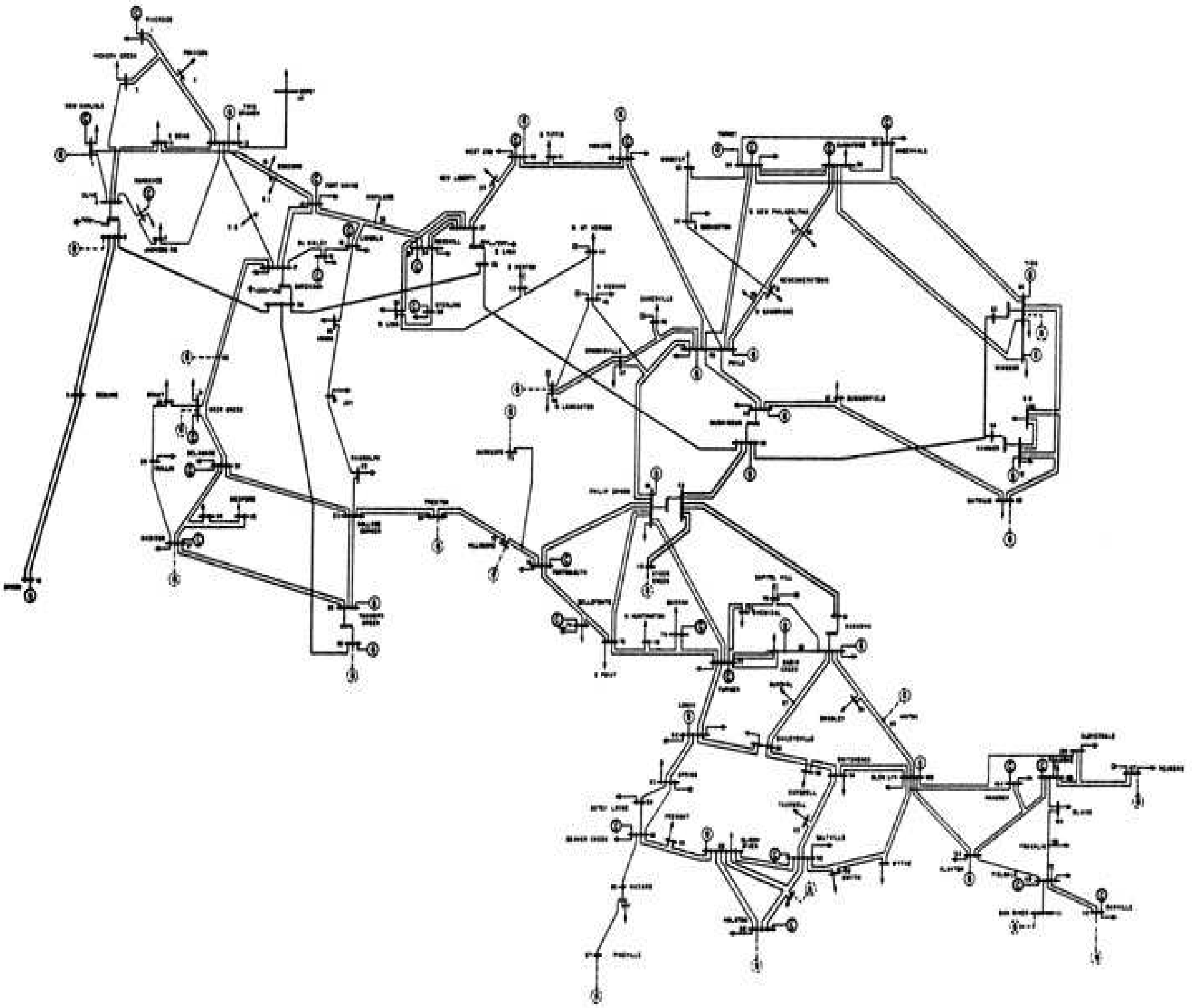

Fig. 3. The IEEE 118-bus system.

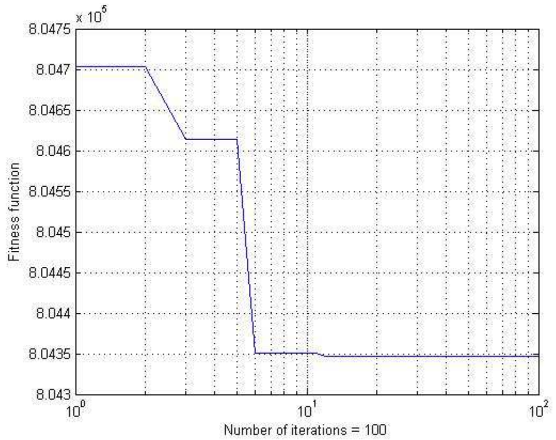

Fig. 4. The convergence characteristic of CSA for the IEEE 118 bus system.

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