# Multi-Objective Security Constrained Optimal Active and Reactive Power Dispatch Using Hybrid Particle Swarm Optimization and Differential Evolution

Dieu Ngoc Vo, Tri Phuoc Nguyen, and Khoa Dang Nguyen

***Abstract—*** *The secure operation of power systems is always the first aid in the power system operation. However, an economic operation of power systems in both the normal and contingency cases is always a goal to achieve for electric power system operators. This paper is dealing with the multi-objective security-constrained optimal active and reactive power dispatch (MO-SCOARPD) problem in power systems considering different objectives such as fuel cost, power losses, stability index, and voltage deviation with the worst scenarios of contingency analysis for transmission line outage to determine the best states for operation. The MO-SCOARPD is a very complex and large-scale problem due to handling many control variables in both normal and contingency cases. In this paper, a hybrid particle swarm optimization and differential evolution (HPSO-DE) has been implemented for solving the problem. The proposed HPSO-DE is a hybrid method to utilize the advantages of both PSO and DE methods for solving the complex and large-scale optimization problems. Consequently, the new hybrid method is more effective than the DE and PSO in obtaining the optimal solution for the optimization problems. The effectiveness of the proposed HPSO-DE has been verified on the IEEE 30 bus system for different objectives and various scenarios of line outages. The obtained results have indicated that the proposed HPSO-DE method can find better solution quality than both DE and PSO methods for all cases. Therefore, the proposed HPSO-DE can be a very favorable and promising method for dealing with the complex and large-scale optimization problem in power systems such as the MO-SCOARPD problem.*

***Keywords—*** **Differential evolution, contingency analysis, hybrid particle swarm optimization and differential evolution, optimal active and reactive power dispatch, fuel cost, stability index, voltage deviation.**

## 1. INTRODUCTION

Optimal active and reactive power dispatch (OARPD) is considered as an important sub-problem in the operation of the power systems and its solution is closely related to many other important problems in the power system analysis and evaluation. The mathematical model of the OARPD problem was first introduced by Carpentier in 1962 [1]. The OARPD aims to find the optimum settings of control variables such as generator active power outputs and voltages, shunt capacitors/reactors, and transformer tap changing settings in order to minimize total generation cost while satisfying the generator and system constraints [2]. Thus, the OARPD problem has become a powerful tool to assist the system operators in decision making for the planning and operating of their system. However, OARPD is a complex and large-dimension optimization problem because there are many adjustable variables. In addition, the problems of OARPD have a nonlinear characteristic due to the nonlinear objective function and constraints. Despite the suffered difficulties, enthusiastic researchers have made continuous efforts to propose newly robust approaches for solving the problem effectively.

In the early stage of the problem discovery process, traditional optimization methods including Newton-based techniques [3], linear programming [4], non-linear programming [5], quadratic programming [6], and interior point methods [7] were first applied in problem solving and achieved encouraging results. In general, these methods are effective in solving the simple OARPD problems with some theoretical assumptions such as convex, continuous, and differential objective functions [8]. However, the OARPD problem is an optimization problem with non-convex, non-continuous, and non-differentiable objective functions. Consequently, conventional methods may be difficult to cope with such problems. Therefore, the determination of a global optimal solution is not possible with conventional methods.

In the later stage of the discovery, artificial intelligence-based methods have emerged as one of the alternative options for solving the OARPD problem with promising results obtained. The main solution methods include genetic algorithm (GA) [9], evolutionary programming (EP) [10], artificial neural network (ANN) [11], bacteria foraging algorithm (BFA) [12], tabu search (TS) [13], and simulated annealing (SA) [14]. In addition to the single methods, hybrid methods have been also widely implemented for solving the OARPD problem such as a hybrid shuffle frog leaping algorithm and simulated annealing (SFLA-SA) method [15] as well as a hybrid modified imperialist competitive algorithm and teaching learning algorithm (MICA–TLA) [16]. Based on the reports from the dominant studies in solving the traditional OARPD problem, it may be recognized that various optimization methods have achieved promising

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results for the problem. However, the solution of the traditional OARPD only meets the normal operating requirements of the power system. In order to explain more clearly the mentioned issue, the OARPD problem was initially established to guide the system operators toward optimum operation of the power system under normal condition (N-0) without considering contingency conditions (N-1) such as outage of a transmission line or a generator. On the other hand, system security has not been properly evaluated in such situation and limit violation after a credible contingency may therefore occur. To overcome this challenge, the traditional OARPD problem can be corrected with the inclusion of security constraints representing operation of the system after contingency outages. These security constraints allow the OARPD to dispatch the system in a defensive manner. That is, the OARPD now forces the system to be operated so that if a contingency happened, the resulting voltages and flows would still be within limit. This special type of OARPD which is called a security-constrained OARPD (SCOARPD) is a vital research area for industrials to enhance the reliability of practical power systems. Recently, a series of articles have been proposed for solving this problem. In [17], the authors have presented a self-organizing hierarchical particle swarm optimization with time-varying acceleration coefficients (SOHPSO-TVAC) for dealing with the SCOARPD problem to achieve the total fuel cost minimization objective. However, the valve point loading effects characteristic of thermal units is not taken into account in this study, which makes the problem unrealistic. Xu et al [18] have introduced a contingency partitioning approach for preventive-corrective security-constrained optimal power flow computation. However, the authors have used a DC model instead of a AC model for calculating power flow and not taken into account the valve loading effects of units when evaluating the cost objective function, which make the problem unrealistic. A modified bacteria foraging algorithm (MBFA) has been proposed in [19] to determine the optimal operating conditions with the aims of minimizing the cost of wind-thermal generation system and reducing the active power loss while maintaining a voltage secure operation. Although the authors have introduced a detailed cost model for the SCOARPD problem that considers the generation cost of different types of generators, the generation cost component of thermal units in the proposed cost model does not include the valve loading effects. In [20], the authors have proposed a fuzzy harmony search algorithm (FHSA) to find out the optimal solution for OARPD problem for power system security enhancement. However, in this study, the valve point loading effects of units, which causes the high non-convexity of the problem, is not considered. In [21], a new planning strategy based on adaptive flower pollination algorithm (APFPA) has been applied to tackle the SCOARPD problem with the objectives of fuel cost, power losses and voltage deviation at normal and critical conditions such as severe faults in generation units. However, this study does not evaluate the different serious scenarios of credible contingencies, such as loss of transmission lines, to select the best post-contingency operating states. An improved version of conventional PSO, namely pseudo-gradient based PSO (PG-PSO), has been proposed [22] to find out the solution of SCOARPD problem with the aim of minimizing the total fuel cost of thermal units. However, there are no specific criteria for assessing the severity level of an outage contingency in this research. In [23], the authors have developed a multi-objective model for SCOARPD problem in which a twin objective of generation cost and voltage stability margin is minimized through a robust differential evolution algorithm (RDEA) - based optimization tool. However, this study does not evaluate the different serious scenarios of credible contingencies, such as loss of transmission lines, to select the best post-contingency operating states. It is worth mentioning again that the SCOARPD problem is inherently highly non-convex, since the considered problem model is related to valve point loading effects and AC power flow equations. Normally, previous authors have endured this challenge and tried to apply adaptive optimization algorithms to deal with it. However, in a recent study [24], Attarha and Amjady have completely changed this point of view by proposing a new technique based on Taylor series and power transformation techniques to convert highly non-convex SCOARPD problem to a convex one. The authors have considered generation cost of thermal units as the objective of this study. In [25], Marcelino et al have proposed the application of a new hybrid canonical differential evolutionary particle swarm optimization (hC-DEEPSO)-based hybrid approach for coping with the problem to minimize the two different mono-objective functions of total operating cost and total active power losses.

From the literature survey, it can be observed that the SCOARPD problem is approached in different ways according to research objectives. Various techniques have been used to solve single-objective SCOARPD problem and the total fuel cost of thermal units is the main objective considered. Only a few studies have examined more than one objective when solving the problem, for example, the further consideration of power loss, voltage deviation or voltage stability. However, in previous studies, the authors only treat these objectives separately without considering combinations of several objectives through a multi-objective optimization framework. In addition, in most previous studies, there are no specific criteria for assessing both the severity level of an outage contingency and the resiliency of power system with corresponding corrective control actions. This is probably the study gap that has been found in previous studies and motivated us to conduct this study.

The main ideas of the study can be expressed as follows: with fuel cost mentioned as a key objective, there may be several different pairs of two objectives including fuel cost and power losses, fuel cost and voltage deviation and fuel cost and voltage stability for further analysis. It is worth noting that in the SCOARPD problem with combined objectives, the obtained operating instructions help not only to reduce the generation cost, but also to improve on a related technical objective even in outage contingency cases.

Further, a specific criteria for ranking the severity cases of outage contingency is necessary.

In this paper, a multi-objective SCOARPD (MO-SCOARPD) framework is formulated and a hybrid particle swarm optimization and differential evolution (HPSO-DE) [26] is also proposed for solving the MO-SCOARPD problem with non-smooth cost functions such as quadratic cost function and fuel cost with valve point effects for both the normal case and selected outage cases considering different objectives of fuel cost, power losses, stability index, and voltage deviation. For the contingency analysis, the outage cases are considered by calculating the severity index (SI) using N-1 criteria. The value of SI is used to rank the severity cases of outage contingency. The outage case corresponding to the high SI value will be selected for inclusion in the problem together with the normal case. Further, in this study, the multi-objective problem considers two objectives for each operating case including fuel cost and power losses, fuel cost and stability index, as well as fuel cost and voltage deviation. For multi-objective problem, a price penalty factor based technique has been proposed to convert the multi-objective problem to a single-objective problem for a direct determination of the best solution for the problem. Regarding the proposed hybrid approach, HPSO-DE, it combines differential information obtained by DE with the memory information extracted by PSO to create the promising solution. The proposed method is tested on IEEE 30-bus system and their results are compared with conventional PSO and DE methods.

## 2. PROBLEM FORMULATION

The SCOARPD is really a very complex and large-scale optimization problem in power system operation. The objective of this problem is to determine the control variables in both normal and contingency cases including voltage magnitude at generation buses, reactive power generation of switchable capacitors, and position of transformer tap changers so as the objective of fuel cost, power losses, stability index, or voltage deviation is minimized satisfying the active and reactive power balance, real and reactive power generation limits, bus voltage limits, reactive power limits of shunt capacitors, transformer tap changer limits, and power limits in transmission lines. The multi-objective SCOARPD problem is a combination of different objective functions in the SCOARPD problem. Consequently, the considered MO-SCOARPD problem is a very complex and large-scale problem with several cases to be calculated. In this paper, the considered multi-objective cases include the fuel cost with the quadratic function or valve point loading effects combined with another objective of power losses, stability index, or voltage deviation. On the other hand, the contingency analysis applied in this problem is based on the severity index (SI) which is used to determine the worst cases of line outages in the system.

In general, the mathematical model of the SCOARPD problem is formulated as follows:

$$\text{Min } [F_1(X, U), F_2(X, U)] \tag{1}$$

subject to the equality and inequality constraints of the normal case:

$$g(X, U) = 0 \tag{2}$$

$$h(X, U) \le 0 \tag{3}$$

and the equality and inequality constraints of the outage case:

$$g(X^S, U^S) = 0 \tag{4}$$

$$h(X^S, U^S) \le 0 \tag{5}$$

where $F_1(X,U)$ is the first objective of fuel cost from thermal generating units, $F_2(X,U)$ is the second objective of power losses, stability index, or voltage deviation from the system, $X$ is the vector of control variables, $U$ is the vector of state variables, $g(.)$ is the set of equality constraints, $h(.)$ is the set of the inequality constraints, and $S$ is the set of outage lines.

### *2.1 Objective functions*

- *Fuel cost:* This objective is to minimize the total fuel cost of all thermal generating units injecting real power into the system:

$$\text{Min}\, F_1 = \text{Min} \sum_{i=1}^{N_g} F_i(P_{gi}) \tag{6}$$

where $F_i(P_{gi})$ is the fuel cost function of thermal unit $i$ represented whether by a quadratic function

$$F_i(P_{gi}) = a_i + b_i P_{gi} + c_i P_{gi}^2 \tag{7}$$

or by a sinusoidal function added to the quadratic function representing valve point loading effects:

$$F_i(P_{gi}) = a_i + b_i P_{gi} + c_i P_{gi}^2 + | e_i \times \sin( f_i \times (P_{gi,\min} - P_{gi})) | \tag{8}$$

in which, $P_{gi}$ is the power output of thermal unit $i$, $P_{gi,min}$ is the minimum power output of thermal unit $i$, and $a_i$, $b_i$, $c_i$, $e_i$ and $f_i$ are fuel cost coefficients, and $N_g$ is the number of generation buses.

- *Power losses:* This objective is to minimize the total power losses of all lines in the system as follows:

$$\begin{aligned} \text{Min}\, F_2 &= \text{Min} \sum_{l=1}^{N_l} P_{loss,l} \\ &= \text{Min} \sum_{l=1}^{N_l} g_l \left[ |V_i|^2 + |V_j|^2 - 2V_i |V_j| \cos(\delta_i - \delta_j) \right] \end{aligned} \tag{9}$$

where $g_l$ is the conductance of line $l$; $N_l$ is the number of lines; $|V_i|$ and $|V_j|$ are the voltage magnitude at buses $i$ and $j$, respectively; $\delta_i$ and $\delta_j$ are the voltage angle at buses $i$ and, $j$, respectively.

- *Stability index:* This objective is to improve the voltage stability at load buses by minimizing the maximum voltage stability index $L_{i,max}$ obtained among the load buses [27]. The objective is expressed follows:

$$\text{Min}\, F_3 = \text{Min}\, L_{\max} = \text{Min}\left\{\max\{L_i\}\right\}, i = 1, 2, ..., N_d \tag{10}$$

where $L_i$ is the stability index at bus $i$; $L_{max}$ is the global stability index of the system; $N_d$ is the number of load buses.

The stability index at a load bus is calculated as follows.

The injected currents at buses are calculated based on the bus admittance matrix $Y_{bus}$ and bus voltage $V_{bus}$ given by:

$$I_{bus} = Y_{bus} V_{bus} \tag{11}$$

The above equation is rewritten by separating the generation and load buses as:

$$\begin{bmatrix} I_G \\ I_L \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \tag{12}$$

where $I_G$ and $V_G$ are the current and voltage at generation buses, respectively; $I_L$ and $V_L$ are the current and voltage at load buses, respectively; $Y_{GG}$ is the admittance related among generation buses; $Y_{LL}$ is the admittance related among load buses; and $Y_{GL}$ and $Y_{LG}$ are the admittance matrix related to both generation and load buses.

The above equation can be rewritten by:

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = \begin{bmatrix} Z_{LL} & F_{LG} \\ K_{GL} & Y_{GG} \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \tag{13}$$

where the sub-matrix $F_{LG}$ is represented by:

$$F_{LG} = -[Y_{LL}]^{-1}[Y_{LG}] \tag{14}$$

Therefore, the $L$-index of load bus $i$ is defined as:

$$L_i = \left| 1 - \frac{\sum_{j=1}^{N_g} F_{ij} \, |V_{gj}|}{|V_{li}|} \right| ; \; i = 1, 2, \ldots, N_d \tag{15}$$

where $V_{gi}$ is the voltage magnitude at generation bus $i$, $V_{li}$ is the voltage magnitude at load bus $i$, and $N_d$ is the number of load buses.

- *Voltage deviation:* This objective is to minimize the total voltage magnitude deviation at load buses expressed by:

$$\mathrm{Min}\, F_4 = \mathrm{Min}\, VD = \sum_{i=1}^{N_d} \left\| V_{li} \right| - \left| V_{li}^{(0)} \right\| \tag{16}$$

where $V_i^{(0)}$ is the pre-specified voltage magnitude at load bus $i$, which is set to 1.0 p.u. in this study.

### *2.2 Equality and Inequality Constraints*

The problem is subject to the equality and inequality constraints for the normal and outage cases as follows.

- *Real and reactive power balance*: The real and reactive balance at each bus in the system is represented as follows.

$$P_{gi} - P_{di} = |V_i| \sum_{j=1}^{N_b} |Y_{ij}| \, |V_j| \cos(\delta_i - \delta_j - \theta_{ij}), \; i = 1, 2, \ldots, N_b \tag{17}$$

$$Q_{gi} - Q_{di} = |V_i| \sum_{j=1}^{N_b} |Y_{ij}| \, |V_j| \sin(\delta_i - \delta_j - \theta_{ij}), \; i = 1, 2, \ldots, N_b \tag{18}$$

where $P_{gi}$ and $Q_{gi}$ are the real and reactive power outputs of thermal unit $i$, respectively; $P_{di}$ and $Q_{di}$ are the real and reactive power demands at load bus $i$, respectively; $N_b$ is the number of buses in the system, $|V_i|\angle\delta_i$ and $|V_j|\angle\delta_j$ are the voltages at buses $i$ and $j$, respectively, and $|Y_{ij}|\angle\theta_{ij}$ is an element in $Y_{bus}$ matrix related to buses $i$ and $j$.

- *Real and reactive power generation limits*: The limits of real and reactive power outputs of thermal units are represented as:

$$P_{gi,\min} \le P_{gi} \le P_{gi,\max}, \; i = 1, 2, \ldots, N_g \tag{19}$$

$$Q_{gi,\min} \le Q_{gi} \le Q_{gi,\max}, \; , \; i = 1, 2, \ldots, N_g \tag{20}$$

where $P_{gi,min}$ and $P_{gi,max}$ are the minimum and maximum real power outputs of thermal unit $i$, respectively; $Q_{gi,min}$ and $Q_{gi,max}$ are the minimum and maximum reactive power outputs of thermal unit $i$, respectively.

- *Bus voltage limits*: The generation and load bus voltages are limited within their upper and lower limits described by:

$$V_{gi,\min} \le V_{gi} \le V_{gi,\max}, \; i = 1, 2, \ldots, N_g \tag{21}$$

$$V_{li,\min} \le V_{li} \le V_{li,\max}, \; , \; i = 1, 2, \ldots, N_d \tag{22}$$

where $V_{gi}$ is the voltage at generation bus $i$; $V_{li}$ is the voltage at load bus $i$; $V_{gi,max}$ and $V_{gi,min}$ are the maximum and minimum voltages at generation bus $i$, respectively; $V_{li,max}$ and $V_{li,min}$ are the maximum and minimum voltages at load bus $i$, respectively.

- *Capacity limits of switchable capacitors:* The capacity of switchable capacitor banks should be limited in their upper and lower boundaries.

$$Q_{ci,\min} \le Q_{ci} \le Q_{ci,\max}, \; i = 1, 2, \ldots, N_c \tag{23}$$

where $Q_{ci}$ is the capacity of switchable capacitor bank at bus $i$; $Q_{ci,max}$ and $Q_{ci,min}$ are the maximum and minimum capacity of switchable capacitor banks; and $N_c$ is the number of buses with switchable capacitor bank.

- *Limits of transformer tap changer:* The transformer tap changers should be within their lower and upper limits as.

$$T_{k,\min} \le T_k \le T_{k,\max}, \; k = 1, 2, \ldots, N_t \tag{24}$$

where $T_k$ is the value of the transformer tap changer $k$; $T_{k,min}$ and $T_{k,max}$ are the minimum and maximum values of transformer tap changer $i$, respectively; and $N_t$ is the number of transformer tap changers.

- *Transmission line limits:* The apparent power flow in transmission lines should be limited in their capacity.

$$S_l \le S_{l,\max}, \; l = 1, 2, \ldots, N_l \tag{25}$$

where $S_l$ is the apparent power flow in line $l$, $S_{l,max}$ is maximum capacity of transmission line $l$, and $N_l$ is the number of transmission lines.

For the security constraint, the outage cases are considered by calculating the severity index (SI) using N-1 criteria. The value of SI used to rank the severity cases of line outage is calculated by:

$$SI = \sum_{l=1}^{N_l} \left( \frac{S_l}{S_{l,\max}} \right)^2 \tag{26}$$

In this research, three cases will be considered in this paper including a combination of fuel cost and power loss, fuel cost and stability index, and fuel cost and voltage deviation. In each case, the number of control and state variables will depend on the used objectives.

## 3. IMPLEMENTATION OF HPSO-DE FOR SOLVING THE PROBLEM

### *3.1 Particle Swarm Optimization Method*

The particle swarm optimization (PSO) method is a population based meta-heuristic method based on the movement organization of a bird flock or a fish school developed by *Kennedy and Eberhart* in 1995 [28]. The main advantages of the PSO are very simple and easy for implementation and applicable to large-scale problem with fast convergence.

In the PSO algorithm, a population (swarm) includes individuals (particles) where each particle contains two parameters of position and velocity that means each particle has its own position and moves from a position to another with a certain velocity. However, the position and velocity of each particle in the swarm should not exceed their limits to guarantee the intake of swarm.

Consider a population with $N_p$ particles where each particle $d$ ($d$ = 1, 2, …, $N_p$) is represented by a position $X_{id}$ and a velocity $V_{id}$, in which $i$ = 1, 2, …, $N$ is the dimension in the position of each particle representing the dimension of a problem. The velocity of each particle is calculated by:

$$\begin{aligned} V_{id}^{(n)} &= \omega V_{id}^{(n-1)} + c_1 * rand_3 * (Pbest_d - X_{id}^{(n-1)}) \\ &+ c_2 * rand_4 * (Gbest - X_{id}^{(n-1)}) \end{aligned} \tag{27}$$

and the position of particles is updated by:

$$X_{id}^{(n)} = X_{id}^{(n-1)} + V_{id}^{(n-1)} \tag{28}$$

where $\omega$ the inertia weight parameter; $n$ is the current iteration; $c_1$ and $c_2$ are the individual and social cognitive factors, respectively; $Pbest_d$ is the best position of individual $d$ up to iteration $n$-1, and $Gbest$ is the best position among positions of particles up to iteration $n$-1.

In addition, the conventional PSO method can be improved to enhance its search ability for complex optimization problems, a constriction factor has been added by Clerc in 1999 [29]. Therefore, the velocity of particles of PSO with constriction factor is calculated by:

$$V_{id}^{(n)} = \chi \begin{pmatrix} \omega V_{id}^{(n-1)} + c_1 * rand_3 * (Pbest_d - X_{id}^{(n-1)}) \\ + c_2 * rand_4 * (Gbest - X_{id}^{(n-1)}) \end{pmatrix} \tag{29}$$

in which, the constriction factor $\chi$ is defined by:

$$\chi = \frac{2}{\left| 2 - \varphi - \sqrt{\varphi^2 - 4\varphi} \right|}, \ \varphi = c1 + c2, \ \varphi > 4 \tag{30}$$

Beside the improvement in the velocity of particles, the position update of particles can be also improved to enhance search ability of the method. One of the methods for updating the position of particles is a concept of pseudo-gradient [30]. The pseudo-gradient is used for determining the best search direction in the search space of non-differentiable problems. Suppose that a function $f(x)$ is minimized, the pseudo-gradient $g_p(x)$ from a point $x_k$ moving to another one $x_l$ is determined as follows [31]:

i) If $f(x_k) \geq f(x_l)$: The direction is good and the particle should continue to follow on this one. Consequently, the pseudo-gradient at point $l$ is nonzero, *e.g.* $g_p(x_l) \neq 0$.

ii) If $f(x_k) \geq f(x_l)$: The direction is not good and the particle should change to another one. Therefore, the pseudo-gradient at point $l$ is zero, *e.g.* $g_p(x_l) = 0$.

Based on the rules, the new position of particles is updated using the pseudo-gradient as follows:

$$X_{id}^{(n)} = \begin{cases} X_{id}^{(n-1)} + g_p(X_{id}^{(n)}) * |V_{id}^{(n)}| & \text{if } g_p(X_{id}^{(n)}) \neq 0 \\ X_{id}^{(n-1)} + V_{id}^{(n)} & \text{otherwise} \end{cases} \tag{31}$$

In this paper, the pseudo-gradient based PSO with constriction factor is used in the proposed hybrid method with the velocity and position of particles are calculated from (29) and (31), respectively.

### *3.2 Differential Evolution Method*

The DE developed by Storn and Price in 1995 [32] is also a simple and effective population based method for solving complex optimization problems. In the DE method, there are three main stages for generating a new population from the parent population including mutation, crossover, and selection as follows.

- *Mutation stage*: This stage is to create a new population by using a base individual added by a difference of other random individuals to effectively explore the search space. In this paper, the DE/rand/1 mutation scheme is selected among the mutation schemes as follows:

$$X_{id}^{'(n)} = X_{r1d}^{(n)} + F * (X_{r2d}^{(n)} - X_{r3d}^{(n)}) \tag{32}$$

where $r1$, $r2$, and $r3$ are differently integer random numbers in the range [1,$N_p$], $X_{id}^{'(n)}$ is the newly created individual based on other individuals, and $F$ is the mutation factor in the range [0,1].

- *Crossover stage:* This stage is also referred as the recombination stage, which is activated to increase the diversity of the perturbed individuals. This stage creates new individuals by mixing the successful individuals from the previous generation with the newly created individuals as:

$$X_{id}^{''(n)} = \begin{cases} X_{id}^{'(n)} & \text{if } rand_5 \le CR \text{ or } d = D_{rand} \\ X_{id}^{(n)} & otherwise \end{cases} \quad (33)$$

where $rand_5$ is a random number in [0,1], $D_{rand}$ is a integer random number in the range [1,$N_p$], and $CR$ is the crossover rate in the range [0,1].

- *Selection stage:* This stage is to determine that whether an individual is selected for the next generation or not by comparing the best individuals from the previous generation with the new created ones in the current generation. The better ones will be selected for the next generation.

### 3.3. The Hybrid PSO and DE Method

Although PSO and DE are efficient methods for dealing with different optimization problems, they still suffer difficulties when dealing with large-scale and complex problems. The PSO method can quickly obtain the optimal solution for a problem but the high solution quality for optimization problems is not always guaranteed. In the contrary, the DE method is very effective for small-scale problems but it may suffer difficulties of long computational time, low solution quality, or infeasible solution when dealing with large-scale problems. In this paper, a hybrid of PSO and DE methods is proposed by utilizing their advantages to form a more powerful method for dealing with large-scale and complex optimization problems. Therefore, the proposed hybrid PSO and DE (HPSO-DE) method is a very effective method for dealing with a very large-scale and complex optimization problem of MO-SCOARPD in power systems. The proposed method consists of the main steps for solving optimization problems as follows:

- *Initialization:* An initial population of $Np$ individuals is randomly initialized in their lower and upper limits.
- *Creation of the first new generation:* The first new generation in this step is created using the mechanism of the PSO method based on the initialized one. The new generated individuals are then evaluated to select the best ones for the next generation.
- *Creation of the second new generation:* The mechanism of this step is from the DE method to create the second new generation. The newly created individuals are also evaluated to select the best ones for the next iteration.

### 3.4. Implementation of the Hybrid PSO and DE Method

*3.4.1 Price penalty factor*

For dealing with a multi-objective optimization problem, there are usually two approaches used to convert the multi-objective problem to a single-objective problem including the weighting factor method to form a Pareto optimal front where the best compromise solution can be obtained [33] and the price penalty factor for a direct determination of the best solution for the problem [34]. In this paper, the second one is used since the first one is time consuming and it is not appropriate for this study with several scenarios to be considered.

In this study, three cases are investigated where each case includes a pair of two objectives as follows:

- *Fuel cost and power losses:*

$$\begin{aligned} F_I &= \mathrm{Min}\{F_1 + h_1 * F_2\} \\ &= \mathrm{Min}\left\{\sum_{i=1}^{N_g} F_i(P_{gi}) + h_1 * \sum_{l=1}^{N_l} P_{loss,l}\right\} \end{aligned} \quad (34)$$

- *Fuel cost and stability index:*

$$\begin{aligned} F_{II} &= \mathrm{Min}\{F_1 + h_2 * F_3\} \\ &= \mathrm{Min}\left\{\sum_{i=1}^{N_g} F_i(P_{gi}) + h_2 * \max\{L_i\}\right\} \end{aligned} \quad (35)$$

- *Fuel cost and voltage deviation:*

$$\begin{aligned} F_{III} &= \mathrm{Min}\{F_1 + h_3 * F_4\} \\ &= \mathrm{Min}\left\{\sum_{i=1}^{N_g} F_i(P_{gi}) + h_2 * VD\right\} \end{aligned} \quad (36)$$

where the penalty factors $h_1$ , $h_2$, and $h_3$ corresponding to the combined objectives $F_I$, $F_{II}$, and $F_{III}$ are respectively determined based on the obtained solution from the power flow problem in the base case as follows:

$$h_1 = \sum_{i=1}^{N_g} F_i(P_{gi}) \Big/ \sum_{l=1}^{N_l} P_{loss,l} \quad (37)$$

$$h_2 = \sum_{i=1}^{N_g} F_i(P_{gi}) \Big/ \max\{L_i\} \quad (38)$$

$$h_3 = \sum_{i=1}^{N_g} F_i(P_{gi}) \Big/ VD \quad (39)$$

*3.4.2 Implementation of HPSO-DE*

The overall procedure of the proposed HPSO-DE applied for solving the MO-SCOARPD problem includes the steps as follows:

***Step 1***: Select control parameters for the method including the population size $N_p$, maximum number of iterations $N_{max}$, individual and social cognitive coefficients $c_1$ and $c_2$, mutation factor $F$, crossover ratio $CR$, and penalty factors.

Perform the contingency analysis, calculate the SI value, and select the most severe cases corresponding to the high SI value for inclusion together with the normal case.

***Step 2***: *Initialization*

An initial population with $N_p$ individuals, where each individual $d$ ($d$ = 1, 2, …, $N_p$) contains a vector of control variables represented by $X_{id} = [P_{g2},...,P_{gN_g}, V_{g1d}, V_{g2d},..., V_{gN_gd}, Q_{c1d}, Q_{c2d}, ..., Q_{N_cd}, T_{1d}, T_{2d},..., T_{N_td}]$, where bus 1 is selected as the slack bus, $i$ = 1, 2, …, $N$ with $N = 2N_g + N_c + N_d$ -1. The state variables represented by $U = [P_{g1}, Q_{g1}, Q_{g2}, ..., Q_{gN_g}, V_{l1}, V_{l2}, ..., V_{lN_d}, S_{l1}, S_{l2}, ..., S_{lN_l}]$ are used to evaluate the feasible solution provided from the individuals.

For each individual $d$ in the population, its position is initialized by:

$$X_{id}^{(0)} = X_{id}^{\min} + rand_1 * (X_{id}^{\max} - X_{id}^{\min}) \quad (40)$$

In addition, the velocity of each individual $d$ is also initialized similar to its position:

$$V_{id}^{(0)} = V_{id}^{\min} + rand_2 * (V_{id}^{\max} - V_{id}^{\min}) \quad (41)$$

where $X_{id}^{max}$ and $X_{id}^{min}$ are the upper and lower limits for individual $d$, respectively; $V_{id}^{max}$ and $V_{id}^{min}$ are the upper and lower bounces of velocity for individual $d$, respectively; $rand_1$ and $rand_2$ are the random numbers in the range [0,1]; and the maximum and minimum limits for the velocity of individuals are determined by:

$$V_{id}^{\max} = R * (X_{id}^{\max} - X_{id}^{\min}) \quad (42)$$

$$V_{id}^{\min} = -V_{id}^{\max} \quad (43)$$

where $R$ is the scale factor for the velocity limits from the positions.

***Step 3***: *Evaluate the initial population*

The power flow problem is solved based on the initial population to evaluate the quality of individuals. The result from the obtained solution from the power flow problem is used to include in the fitness function for each individual in the normal case. Moreover, the initial population is also used to solve the power flow problem for the severe case to evaluate the quality of individual for line outage case. The fitness function for each individual consisting of the results from the normal and severe cases is calculated by:

$$\begin{aligned} FT_d^{(0)} &= F + K_{p0} * (P_{g1} - P_{g1}^{\lim})^2 \\ &+ K_{q0} * \sum_{i=1}^{N_g} (Q_{gi} - Q_{gi}^{\lim})^2 + K_{v0} * \sum_{i=1}^{N_d} (V_{li} - V_{li}^{\lim})^2 \\ &+ K_{s0} * \sum_{l=1}^{N_l} (S_l - S_{l,\max})^2 + K_q * \sum_{i=1}^{N_g} (Q_{gi}^s - Q_{gi}^{\lim})^2 \\ &+ K_v * \sum_{i=1}^{N_d} (V_{li}^s - V_{li}^{\lim})^2 + K_s * \sum_{l=1}^{N_l} (S_l^s - S_{l,\max})^2 \end{aligned} \quad (44)$$

where $F$ is one of the combined objectives as defined in (34)-(36); $K_{p0}$, $K_{q0}$, $K_{v0}$, and $K_{s0}$ are the penalty factors for real power at the slack bus, reactive power at generation buses, voltage at load buses, and apparent power flow in transmission lines the normal case, respectively; $K_q$, $K_v$, and $K_s$ are the penalty factors for the outage case, $P_{g1}^{\lim}$ is the power limits at the slack bus; $Q_{gi}^{\lim}$ is the reactive power limits at generation buses; $V_{li}^{\lim}$ is the voltage limits at load buses, $Q^s_{gi}$ is the reactive power at generation bus $i$ in the outage case; $V^s_{li}$ is the voltage at load bus $i$ in the outage case; $S^s_l$ is the apparent power flow in transmission line $l$ in the outage case.

The limits of the state variables consisting of the real power output at the slack bus, reactive power at generation buses, and voltage at load buses for both normal, and outage cases are defined by:

$$X^{\lim} = \begin{cases} X_{\max} & \text{if } X > X_{\max} \\ X_{\min} & \text{if } X < X_{\min} \\ X & \text{otherwise} \end{cases} \quad (45)$$

where $X$ represents $P_{g1}$, $Q_{gi}$, and $V_{li}$.

The initial population is set to the best position $Pbest_d$ of each particle and the corresponding best fitness function is set to $FT_d^{best}$. The position of particle having the best fitness function value among particles in the population is set to *Gbest*.

Set the iteration counter $k$ = 1.

***Step 4***: *Generate a first new population*

The first new population in this step is created using the mechanism of the PSO method. Firstly, the new velocity of particles in the population is calculated by using (29). New created velocity of particle is checked with their upper and lower limits and if violations are found, a repair action is used as follows:

$$V_{id}^{(k)} = \begin{cases} V_{id}^{\max} & \text{if } V_{id}^{(k)} > V_{id}^{\max} \\ V_{id}^{\min} & \text{if } V_{id}^{(k)} < V_{id}^{\min} \end{cases} \quad (46)$$

The new generated population is then updated by using (31) and the new obtained position of particles is also need to be checked with their limits and a repair action is applied if there are any violations as follows:

$$X_{id}^{(k)} = \begin{cases} X_{id}^{\max} & \text{if } X_{id}^{(k)} > X_{id}^{\max} \\ X_{id}^{\min} & \text{if } X_{id}^{(k)} < X_{id}^{\min} \end{cases} \quad (47)$$

***Step 5***: *Evaluate the first created population*

The new generated population is used to run the power flow problem in the normal and outage cases and the obtained results from the problem are used to calculate the fitness function (44) to evaluate the quality of individuals.

***Step 6***: *Mutation for the second created generation*

The mutation stage is to create a second new population using mechanism of the DE method. The individuals $X_{id}^{'(k)}$ in the second new population are determined from the first generated population $X_{id}^{(k)}$ by the PSO method as in (32).

The new created position $X_{id}^{'(k)}$ is checked with their limits and a repair action is used as in (45) if any limit violations found.

***Step 7***: *Crossover for the second created generation*

The purpose of the crossover process in the DE method is to provide new individuals $X_{id}^{''(k)}$ from the second new created population $X_{id}^{'(k)}$ by using (33).

***Step 8***: *Evaluation for the second created population*

The newly generated individuals from the crossover stage is used to solve the power flow problem in the normal and outage cases and the obtained results are applied to calculate the fitness function in (44).

***Step 9***: *Selection for the second created population*

The selection process in this step is to choose the best individuals for the next generation by comparing the values of the fitness function from individuals from the first and second generated populations. The individual corresponding to the lower the value of the fitness function will be selected for the next population as follows:

$$X_{id}^{new(k)} = \begin{cases} X_{id}^{''(k)} & \text{if } FT_d^{''(k)} \le FT_d^{(k)} \\ X_{id}^{(k)} & otherwise \end{cases} \tag{48}$$

The new fitness function value $FT_d^{new(d)}$ and the corresponding individual $X_{id}^{new(n)}$ are updated accordingly.

***Step 10***: *Update the best population*

The best selected individuals from the first generated population by PSO and the second generated population be DE in this iteration is compared to the best one from the previous iteration to choose the best individual so far for the next iteration. The better individuals between the two populations will be selected and will be stored as the best individual so far. The update process is performed as follows:

$$Pbest_d = \begin{cases} X_{id}^{new(k)} & \text{if } FT_d^{new(k)} \le FT_d^{best} \\ Pbest_d & otherwise \end{cases} \tag{49}$$

The corresponding better fitness function $FT_d^{best}$ is also updated for comparison in the next iteration and the best position among $Pbest_d$ is updated to *Gbest*.

***Step 11***: *Stopping criteria*

Only the number of iterations is controlled in this study. If $k < N_{max}$, $k = k + 1$ and return to Step 4, Otherwise, stop.

## 4. NUMERICAL RESULTS

The proposed HPSO-DE has been verified on the IEEE 30-bus system with quadratic fuel cost function and valve point loading effects for both the normal case and selected outage cases considering different objectives of fuel cost, power losses, stability index, and voltage deviation. In this study, the multi-objective problem considers two objectives for each case including fuel cost and power losses, fuel cost and stability index, and fuel cost and voltage deviation.

The test system comprises six generators at buses 1, 2, 5, 8, 11, and 13 where bus 1 is selected as the slack bus, 41 transformers and transmission lines, and two switchable capacitor banks located at buses 10 and 24. The data of this system is given in [35] and the fuel cost data for generators with quadratic function and valve point loading effects is given in Tables 1 and 2, respectively. The other data for the system such as the bus voltage and transformer tap changer limits is given in Table 3 and the reactive power limits at generation and compensated buses given in Table 4. For the base case, the real power outputs at generation buses 2, 5, 8, 11, and 13 are set to 80 MW, 50 MW, 20 MW, 20 MW, and 20 MW, respectively. The limits of apparent power in transmission lines are given in Appendix.

**Table 1: Data of generators with quadratic cost function of the IEEE 30-bus system**

| Unit | $a_i$ ($/h) | $b_i$ ($/MWh) | $c_i$ ($/MW$^2$h) | $P_{i,max}$ (MW) | $P_{i,min}$ (MW) |
|---|---|---|---|---|---|
| 1 | 0 | 2.00 | 0.00375 | 200 | 50 |
| 2 | 0 | 1.75 | 0.01750 | 80 | 20 |
| 5 | 0 | 1.00 | 0.06250 | 50 | 15 |
| 8 | 0 | 3.25 | 0.00834 | 35 | 10 |
| 11 | 0 | 3.00 | 0.02500 | 30 | 10 |
| 13 | 0 | 3.00 | 0.02500 | 40 | 12 |

**Table 2: Data of generators with valve point effects of the IEEE 30-bus system**

| Unit | $a_i$ ($/h) | $b_i$ ($/MWh) | $c_i$ ($/MW$^2$h) | $e_i$ ($/h) | $f_i$ (1/MW) | $P_{i,max}$ (MW) | $P_{i,min}$ (MW) |
|---|---|---|---|---|---|---|---|
| 1 | 150 | 2.00 | 0.00160 | 50 | 0.063 | 200 | 50 |
| 2 | 25 | 2.50 | 0.01000 | 40 | 0.098 | 80 | 20 |
| 5 | 0 | 1.00 | 0.06250 | 0 | 0 | 50 | 15 |
| 8 | 0 | 3.25 | 0.00834 | 0 | 0 | 35 | 10 |
| 11 | 0 | 3.00 | 0.02500 | 0 | 0 | 30 | 10 |
| 13 | 0 | 3.00 | 0.02500 | 0 | 0 | 40 | 12 |

**Table 3: Bus voltage and transformer tap changer limits of the IEEE 30-bus system**

| | Lower limit (pu) | Upper limit (pu) |
|---|---|---|
| Slack bus voltage | 0.95 | 1.05 |
| Gen. bus voltage | 0.95 | 1.10 |
| Load bus voltage | 0.95 | 1.05 |
| Trans. tap changer | 0.90 | 1.10 |

**Table4: Reactive power limits at generation and compensated buses of the IEEE 30-bus system**

| | | Lower limit (MVAr) | Upper limit (MVAr) |
|---|---|---|---|
| Generation buses | Bus 1 | -20 | 200 |
| | Bus 2 | -20 | 100 |
| | Bus 5 | -15 | 80 |
| | Bus 8 | -15 | 60 |
| | Bus 11 | -10 | 50 |
| | Bus 13 | -15 | 60 |
| Compensated buses | Bus 10 | 0 | 19 |
| | Bus 24 | 0 | 4.3 |

For the contingency analysis, the SI is calculated for each N-1 outage line and five outage cases including lines 1-2, 1-3, 3-4, 2-5 and 4-6 are selected as the most severe outage cases due to the highest SI value, where each of these five outage cases is considered in one contingency case.

The conventional PSO and DE methods have been also implemented for solving the problem and run on the same computer for a result comparison. For implementation of the methods, their control parameters are generally selected as follows. The number of individuals $N_p$ in the population is set to 10 and all penalty factors are set to $10^6$ for all implemented methods. The cognitive coefficients $c_1$ and $c_2$ for the proposed HPSO-DE are set to 2.05. The mutation factor $F$ and the crossover ratio $CR$ for the proposed HPSO-DE and DE methods are set to 0.7 and 0.5, respectively. The cognitive coefficients $c_1$ and $c_2$ for the PSO are set to 2.0. The maximum number of iterations is set to 150 for the normal case with quadratic cost function, and 200 for the normal case with valve point loading effects. For the contingency cases, the maximum number of iterations is set to 200 for the cases with quadratic cost function and 300 for the cases with valve point loading effects for all implemented methods. All these methods are coded in Matlab and run 50 independent trials for each case in a CPU E5-1620@3.5 GHz. In this paper, the Newton-Raphson method in Matpower toolbox [35] is used to solve the power flow problem.

### *4.1 Base case*

In the base case, the methods have been implemented for solving the multi-objective OARPD problem with the quadratic cost function and valve loading effects of generators. The three considered multi-objective cases are including the fuel cost and power losses, fuel cost and stability index, and fuel cost and voltage deviation. For obtaining different solutions, the methods have been implemented for solving the problem with single and multiple objectives.

#### *4.1.1 Quadratic fuel cost function*

The results obtained by the methods for the three cases with different objectives are given in Tables 5-7. In each table, the best solution from DE, PSO, and HPSO-DE for single and multiple objectives are provided.

For the fuel cost objective only, the DE method has provided three different best results for the three combinations of objectives while the best results for the three cases by PSO and HPSO-DE are not much different. In fact, the three cases of the problem with three combinations of objectives are the same. The standard deviation of DE, PSO, and HPSO-DE from the three cases of combined objectives are 3.0723 $/h, 0.1452 $/h, and 0.0012 $/h, respectively. As observed from the standard deviation, the DE method has lower solution quality than PSO and HPSO-DE and the proposed has the highest solution quality for these cases. The best total cost obtained by the proposed HPSO-DE from the three combined objectives is better than that from DE and PSO. For the power loss objective only, the total power loss obtained by the proposed method is much lower than that from DE and slightly lower than that from the PSO. For the case with the objective of the stability index, the stability indices obtained by the three methods are approximately together. For the case with only the voltage deviation objective, the proposed method can obtain a better result than both DE and PSO methods. Therefore, the proposed HPSO-DE method is can obtain better results than both DE and PSO for the cases with single objective.

For the three cases with combined objectives, the proposed HPSO-DE method can obtain dominant solutions compared to those by the DE and PSO methods. It has indicated that the proposed method dominate DE and PSO methods for dealing with the considered multi-objective problem.

For the computational time, the DE is faster than the other methods while the proposed HPSO-DE method is slower than the others for all cases. It is easy to explain that the proposed HPSO-DE method combines both the PSO and DE methods, thus it is always slower either one of them when dealing with the same optimization problem. However, the effectiveness of the proposed HPSO-DE method is always higher than that from the PSO and DE. Therefore, the hybrid method is better than the single methods for the test cases in this section.

The convergence characteristics of the best result from the DE, PSO, and HPSO-DE methods for the three cases with the combined objectives including the fuel cost and power losses, fuel cost and stability index, and fuel cost an d voltage deviation are given in Figures 1 to 3, respectively. As observed from the curves, all the methods have reached the stability of the fitness function after 10 iterations.

**Table 5: The best result for the base case with quadratic fuel cost and power losses**

| Method | | Min. fuel cost | Min. power loss | Min. combined fuel cost and power loss |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **808.4815** | 907.2449 | **858.8653** |
| | Power losses (MW) | 9.5466 | **4.9723** | **5.4654** |
| | Avg. CPU (s) | 2.6598 | 2.5715 | 2.5806 |
| PSO | Fuel cost ($/h) | **802.5413** | 968.0259 | **845.1316** |
| | Power losses (MW) | 9.5035 | **3.2523** | **5.2201** |
| | Avg. CPU (s) | 4.6641 | 5.0329 | 4.9537 |
| HPSO-DE | Fuel cost ($/h) | **802.2482** | 967.9584 | **844.8537** |
| | Power losses (MW) | 9.4507 | **3.2240** | **5.1915** |
| | Avg. CPU (s) | 6.7505 | 7.3286 | 7.4612 |

**Table 6: The best result for the base case with quadratic fuel cost and stability index**

| Method | | Min. fuel cost | Min. stability index | Min. combined fuel cost and stability index |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **813.7452** | 833.1655 | **811.2307** |
| | Stability index (pu) | 0.1732 | **0.1378** | **0.1383** |
| | Avg. CPU (s) | 3.0015 | 2.7862 | 2.8948 |
| PSO | Fuel cost ($/h) | **802.8308** | 838.2087 | **802.9334** |
| | Stability index (pu) | 0.1494 | **0.1375** | **0.1386** |
| | Avg. CPU (s) | 5.2279 | 5.3368 | 5.5814 |
| HPSO-DE | Fuel cost ($/h) | **802.2503** | 948.8882 | **802.4336** |
| | Stability index (pu) | 0.1386 | **0.1368** | **0.1374** |
| | Avg. CPU (s) | 7.7314 | 8.2025 | 8.1195 |

**Table 7: The best result for the base case with quadratic fuel cost and voltage deviation**

| Method | | Min. fuel cost | Min. voltage deviation | Min. combined fuel cost and voltage deviation |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **808.3679** | 844.3073 | **821.0422** |
| | Voltage deviation (pu) | 0.5202 | **0.2405** | **0.4124** |
| | Avg. CPU (s) | 2.4289 | 2.4720 | 2.5728 |
| PSO | Fuel cost ($/h) | **802.7055** | 843.2551 | **804.6858** |
| | Voltage deviation (pu) | 0.3436 | **0.1557** | **0.1597** |
| | Avg. CPU (s) | 4.8523 | 4.9555 | 4.8307 |
| HPSO-DE | Fuel cost ($/h) | **802.2482** | 826.3338 | **804.0284** |
| | Voltage deviation (pu) | 0.7549 | **0.1399** | **0.1468** |
| | Avg. CPU (s) | 6.7614 | 6.8924 | 7.0372 |

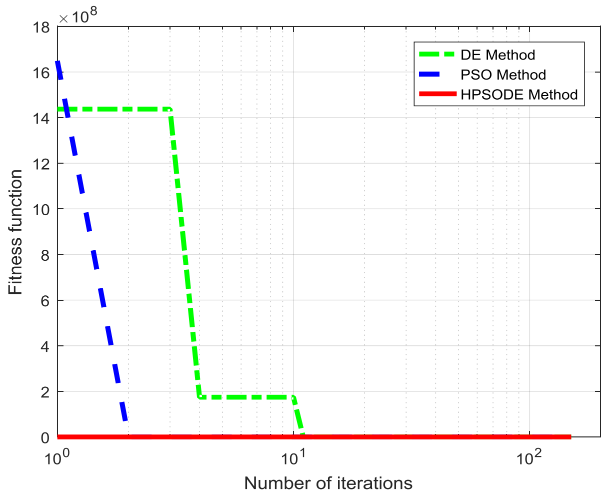


**Fig. 1: Convergence characteristic of DE, PSO, and HPSO-DE for the base case with quadratic fuel cost and power losses.**

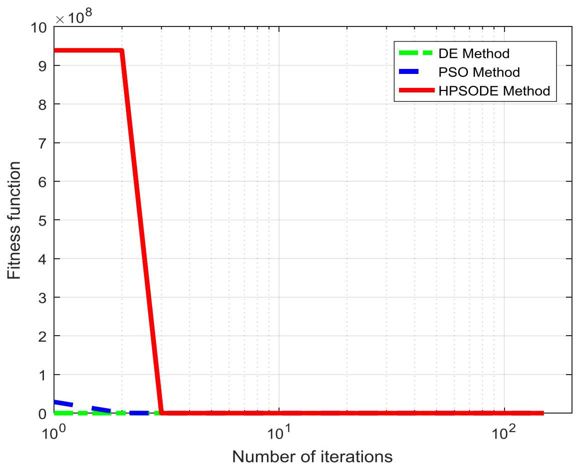


**Fig. 2: Convergence characteristic of DE, PSO, and HPSO-DE for the base case with quadratic fuel cost and stability index.**

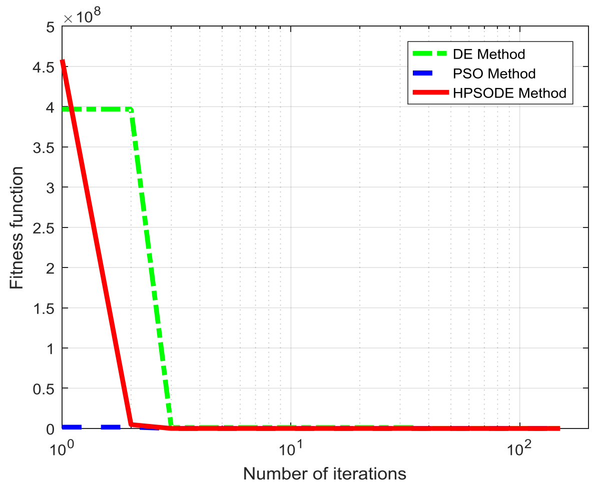


**Fig. 3: Convergence characteristic of DE, PSO, and HPSO-DE for the base case with quadratic fuel cost and voltage deviation.**

*4.1.2 Valve point loading effects*

For the bases case with the valve point loading effects, the investigation is also performed similar to the bases case with quadratic fuel cost. In the three cases with single objective of fuel cost, the DE has obtained different results with large deviation while the PSO and HPSO-DE methods can obtain results with smaller deviation. The standard deviations for the total fuel cost by the three methods for the three cases are 29.1265 $/h, 3.1357 $/h, and 0.5453 $/h, respectively. Among the three methods, the HPSO-DE method can provide the highest solution quality than the others due to obtaining the lowest standard deviation. For the cases with single objective of power losses and voltage deviation, the total power loss and voltage deviation from the proposed method are slightly lower than those from PSO and DE methods, respectively while the stability index obtained the proposed method is approximate to that from the DE and PSO methods for the single stability index objective. For the combined objectives between fuel cost and power losses, fuel cost and stability index, and fuel cost and voltage deviation, the proposed HPSO-DE method has always obtained dominant solutions to the DE and

PSO have done. In fact, the proposed method has achieved better solution quality than the DE and PSO methods for all the considered cases with valve point loading effects. The proposed HPSO-DE method is also very effective for dealing with the complex and non-convex problem.

The computational time in this case is also similar to the base case with the quadratic cost function. The average CPU time from the proposed method is approximate thrice compared to that from the DE method and twice compared to that from the DE method for all the three combinations of objectives. The convergence characteristics of the three methods for the three combinations of objectives are given in Figures 4-6. Obviously, the fitness function of all methods can reach a stable state before 10 iterations. From 10 to 200 iterations, there are not any further changes from the fitness functions from the methods.

**Table 8: The best result for the base case with valve point loading effects of fuel cost and power losses**

| Method | | Min. fuel cost | Min. power loss | Min. combined fuel cost and power loss |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **984.7631** | 1.1553e+03 | **1.0561e+03** |
| | Power losses (MW) | 8.0540 | **4.5373** | **5.8433** |
| | Avg. CPU (s) | 3.3606 | 3.2935 | 3.4183 |
| PSO | Fuel cost ($/h) | **928.2641** | 1.1702e+03 | **1.0432e+03** |
| | Power losses (MW) | 10.6817 | **3.2437** | **4.6960** |
| | Avg. CPU (s) | 6.0432 | 6.6893 | 6.4101 |
| HPSO-DE | Fuel cost ($/h) | **922.1135** | 1.1700e+03 | **1.0411e+03** |
| | Power losses (MW) | 10.5262 | **3.2238** | **4.5415** |
| | Avg. CPU (s) | 9.9803 | 10.2265 | 9.4259 |

**Table 9: The best result for the base case with valve point loading effects of fuel cost and stability index**

| Method | | Min. fuel cost | Min. stability index | Min. combined fuel cost and stability index |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **928.6662** | 1.0986e+03 | **973.7027** |
| | Stability index (pu) | 0.1498 | **0.1350** | **0.1487** |
| | Avg. CPU (s) | 3.6570 | 3.6401 | 3.8860 |
| PSO | Fuel cost ($/h) | **922.5905** | 1.0809e+03 | **924.2321** |
| | Stability index (pu) | 0.1558 | **0.1375** | **0.1387** |
| | Avg. CPU (s) | 6.8653 | 7.1470 | 7.2266 |
| HPSO-DE | Fuel cost ($/h) | **922.9096** | 1.0210e+03 | **921.6729** |
| | Stability index (pu) | 0.1396 | **0.1371** | **0.1383** |
| | Avg. CPU (s) | 10.3984 | 10.7956 | 11.1463 |

**Table 10: The best result for the base case with valve point loading effects of fuel cost and voltage deviation**

| Method | | Min. fuel cost | Min. voltage deviation | Min. combined fuel cost and voltage deviation |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **974.7877** | 1.0626e+03 | **949.2665** |
| | Voltage deviation (pu) | 1.0777 | **0.2568** | **0.3619** |
| | Avg. CPU (s) | 2.4286 | 2.4533 | 2.5834 |
| PSO | Fuel cost ($/h) | **923.1131** | 1.0600e+03 | **930.7257** |
| | Voltage deviation (pu) | 0.3502 | **0.1570** | **0.2065** |
| | Avg. CPU (s) | 4.5225 | 4.5903 | 4.7362 |
| HPSO-DE | Fuel cost ($/h) | **921.8661** | 1.0660e+03 | **922.7598** |
| | Voltage deviation (pu) | 0.2317 | **0.1404** | **0.1683** |
| | Avg. CPU (s) | 6.7162 | 6.7577 | 6.9673 |

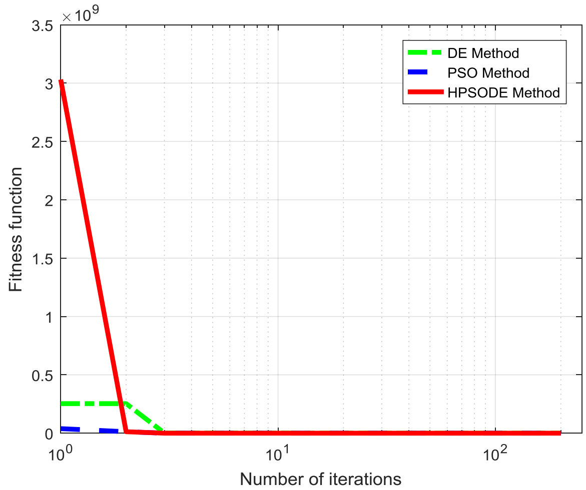


**Fig. 4: Convergence characteristic of DE, PSO, and HPSO-DE for valve point loading effects of fuel cost and power losses.**

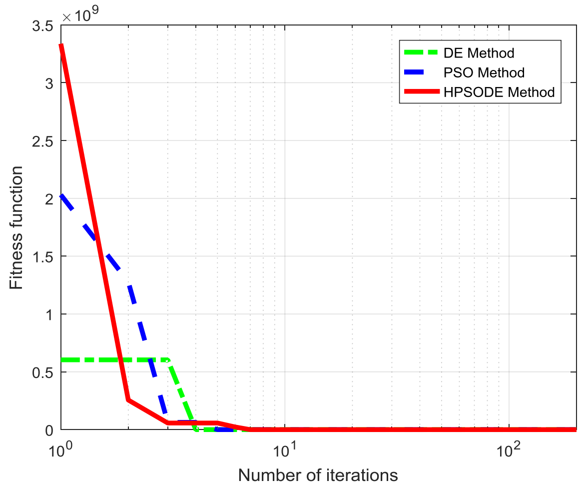


**Fig. 5: Convergence characteristic of DE, PSO, and HPSO-DE for valve point loading effects of fuel cost and stability index.**

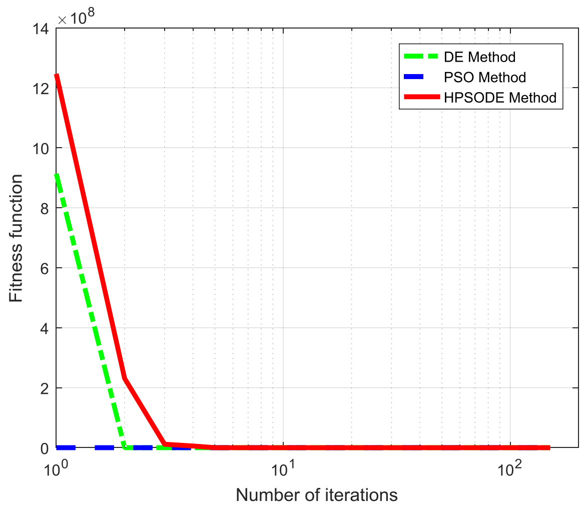


**Figure 6: Convergence characteristic of DE, PSO, and HPSO-DE for valve point loading effects of fuel cost and voltage deviation.**

### *4.2 Outage cases*

A contingency analysis is performed before solving the outage cases for the problem. The contingency analysis is based on the N-1 criteria and the outage case corresponding to the high SI value will be selected for inclusion in the problem together with the normal case. The most severe cases from the analysis for the IEEE 30-bus system are given in Table 11. Among the outage cases, the outage lines 1-2, 1-3, 3-4, 2-5, and 4-6 have a higher SI value compared to the other cases and each of them is selected for consideration in the problem. Therefore, the study in this section will include the normal case and one outage line for each the combination of objectives those are fuel cost and power losses, fuel cost and stability index, and fuel coat and voltage deviation for quadratic fuel cost function and valve loading effects.

#### *4.2.1 Quadratic fuel cost*

The problem with the quadratic fuel cost is considered for the outage lines as mentioned with three different combinations of objectives of fuel cost and power losses, fuel cost and stability index, and fuel cost and voltage deviation. For each case of line outage, the results for single objective and combined objectives are also provided.

##### *4.2.1.1 Fuel cost and power loss objective*

The best results obtained by the methods for the five cases of line outage for the combined objective of fuel cost and power losses are given in Tables 12 to 16. For the fuel cost objective only, the proposed method can obtain much better total cost than that from DE and also slightly better than that from PSO for all five outage cases. This manner is also similar for the case with single objective of power losses, where the total power loss provided by the HPSO-DE method is much lower than that from the DE method and slightly lower than that from the PSO method. For the combined objectives, the proposed method only dominate the DE method for the case with line 1-2 outage while the solutions for other cases of line outage do not dominate each other.

On the other hand, the successful rate of the DE method among the independent runs is generally much lower than that from the other methods while the rate of success from the proposed method is slightly higher than that of the PSO method for both single objective and combined objectives. For the CPU time, the proposed method is generally thrice slower than the DE method and twice slower than the SPO method for all outage lines.

The convergence characteristics of the DE, PSO, and HPSO-DE methods for the problem with the combined fuel cost and power loss objective for five outage cases are given in Figure 7 to 11 and the successful rate of these methods for the five outage cases is also depicted in Figure 12. As seen from the curves, the fitness function of PSO and HPSO-DE can reach the stable state less than 10 iterations while that from the DE method sometimes reaches the stable state after 10 iterations. Moreover, the successful rate as observed from Figure 12 is much lower than that of PSO and HPSO-DE methods while the proposed HPSO-DE method can reach the highest rate of success among the three methods. Therefore, the proposed method is very effective for solving the problem with the two objective of fuel cost and power losses for five outage cases.

**Table 11: Contingency analysis of the IEEE 30 bus system**

| Outage line | Overload line | Line flow (MVA) | Line flow limit (MVA) | Overload rate (%) | Severity index |
|---|---|---|---|---|---|
| 1-2 | 2 | 307.0136 | 130 | 236.1643 | 16.3035 |
| | 4 | 281.3522 | 130 | 216.4248 | |
| | 7 | 178.4014 | 90 | 198.2238 | |
| | 10 | 46.5144 | 32 | 145.3575 | |
| 1-3 | 1 | 274.0264 | 180 | 152.2369 | 7.3218 |
| | 3 | 86.1203 | 65 | 132.4928 | |
| | 6 | 92.7203 | 65 | 142.6466 | |
| | 10 | 35.2567 | 32 | 110.1773 | |
| 3-4 | 1 | 271.0750 | 180 | 150.5972 | 7.1590 |
| | 3 | 84.8816 | 65 | 130.5871 | |
| | 6 | 91.7672 | 65 | 141.1803 | |
| | 10 | 34.9449 | 32 | 109.2027 | |
| 2-5 | 3 | 74.6652 | 65 | 114.8695 | 6.9418 |
| | 6 | 102.9619 | 65 | 158.4030 | |
| | 7 | 123.6755 | 90 | 137.4172 | |
| | 10 | 35.4150 | 32 | 110.6719 | |
| 4-6 | 1 | 200.5759 | 180 | 111.4311 | 4.6212 |
| | 6 | 98.5645 | 65 | 151.6377 | |
| | 15 | 67.5536 | 65 | 103.9286 | |

**Table 12: The best result for line 1-2 outage case with quadratic fuel cost and power losses**

| Method | | Min. fuel cost | Min. power loss | Min. combined fuel cost and power loss |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **838.1276** | 899.8429 | **868.4918** |
| | Power losses (MW) | 6.2099 | **5.0426** | **5.8822** |
| | Avg. CPU (s) | 7.0076 | 6.9083 | 7.0019 |
| | Rate of success (%) | 22 | 20 | 18 |
| PSO | Fuel cost ($/h) | **826.3915** | 967.9847 | **846.5717** |
| | Power losses (MW) | 6.5656 | **3.2350** | **5.1752** |
| | Avg. CPU (s) | 12.8295 | 13.4017 | 13.4816 |
| | Rate of success (%) | 98 | 96 | 96 |
| HPSO-DE | Fuel cost ($/h) | **825.3446** | 967.9579 | **844.9736** |
| | Power losses (MW) | 6.2735 | **3.2238** | **5.1872** |
| | Avg. CPU (s) | 19.4792 | 20.1553 | 19.9610 |
| | Rate of success (%) | 98 | 100 | 98 |

**Table 13: The best result for line 1-3 outage case with quadratic fuel cost and power losses**

| Method | | Min. fuel cost | Min. power loss | Min. combined fuel cost and power loss |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **820.0893** | 871.6405 | **815.6695** |
| | Power losses (MW) | 8.2336 | **5.4651** | **7.7767** |
| | Avg. CPU (s) | 6.8023 | 7.1795 | 6.8940 |
| | Rate of success (%) | 14 | 30 | 18 |
| PSO | Fuel cost ($/h) | **803.1417** | 968.0002 | **838.1468** |
| | Power losses (MW) | 9.3406 | **3.2415** | **5.4960** |
| | Avg. CPU (s) | 11.9665 | 13.0108 | 13.0386 |
| | Rate of success (%) | 98 | 96 | 98 |
| HPSO-DE | Fuel cost ($/h) | **802.5571** | 967.9600 | **845.1425** |
| | Power losses (MW) | 9.2018 | **3.2247** | **5.1819** |
| | Avg. CPU (s) | 18.5158 | 20.6704 | 19.7703 |
| | Rate of success (%) | 98 | 100 | 100 |

**Table 14: The best result for line 3-4 outage case with quadratic fuel cost and power losses**

| Method | | Min. fuel cost | Min. power loss | Min. combined fuel cost and power loss |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **811.7975** | 869.5471 | **880.2518** |
| | Power losses (MW) | 9.6164 | **5.0592** | **4.9663** |
| | Avg. CPU (s) | 7.3221 | 6.8247 | 7.0238 |
| | Rate of success (%) | 22 | 24 | 30 |
| PSO | Fuel cost ($/h) | **803.0869** | 967.8676 | **845.0038** |
| | Power losses (MW) | 9.4465 | **3.2440** | **5.2554** |
| | Avg. CPU (s) | 12.6111 | 12.9425 | 13.3858 |
| | Rate of success (%) | 96 | 96 | 96 |
| HPSO-DE | Fuel cost ($/h) | **802.4731** | 967.9645 | **845.0078** |
| | Power losses (MW) | 9.3108 | **3.2266** | **5.1904** |
| | Avg. CPU (s) | 19.2162 | 20.1968 | 19.8262 |
| | Rate of success (%) | 98 | 100 | 98 |

**Table 15: The best result for line 2-5 outage case with quadratic fuel cost and power losses**

| Method | | Min. fuel cost | Min. power loss | Min. combined fuel cost and power loss |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **818.1612** | 869.5423 | **844.9030** |
| | Power losses (MW) | 9.0420 | **5.6705** | **6.5657** |
| | Avg. CPU (s) | 6.6594 | 6.7271 | 6.9271 |
| | Rate of success (%) | 20 | 22 | 12 |
| PSO | Fuel cost ($/h) | **809.0476** | 968.0114 | **850.6773** |
| | Power losses (MW) | 8.1112 | **3.2462** | **5.0337** |
| | Avg. CPU (s) | 12.2645 | 13.5531 | 12.8342 |
| | Rate of success (%) | 100 | 100 | 96 |
| HPSO-DE | Fuel cost ($/h) | **808.2097** | 967.9586 | **844.9093** |
| | Power losses (MW) | 7.7863 | **3.2241** | **5.1894** |
| | Avg. CPU (s) | 18.6019 | 19.6511 | 19.2724 |
| | Rate of success (%) | 100 | 98 | 100 |

**Table 16: The best result for line 4-6 outage case with quadratic fuel cost and power losses**

| Method | | Min. fuel cost | Min. power loss | Min. combined fuel cost and power loss |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **822.0595** | 864.2526 | **852.8589** |
| | Power losses (MW) | 9.8934 | **5.3472** | **6.3168** |
| | Avg. CPU (s) | 6.8297 | 7.6075 | 6.8912 |
| | Rate of success (%) | 22 | 20 | 34 |
| PSO | Fuel cost ($/h) | **803.7308** | 968.0040 | **849.9696** |
| | Power losses (MW) | 9.0994 | **3.2431** | **5.0628** |
| | Avg. CPU (s) | 12.0118 | 13.0676 | 13.2663 |
| | Rate of success (%) | 96 | 96 | 98 |
| HPSO-DE | Fuel cost ($/h) | **803.2353** | 967.9698 | **845.2778** |
| | Power losses (MW) | 8.8824 | **3.2288** | **5.1824** |
| | Avg. CPU (s) | 18.2974 | 20.7060 | 19.6908 |
| | Rate of success (%) | 100 | 100 | 100 |

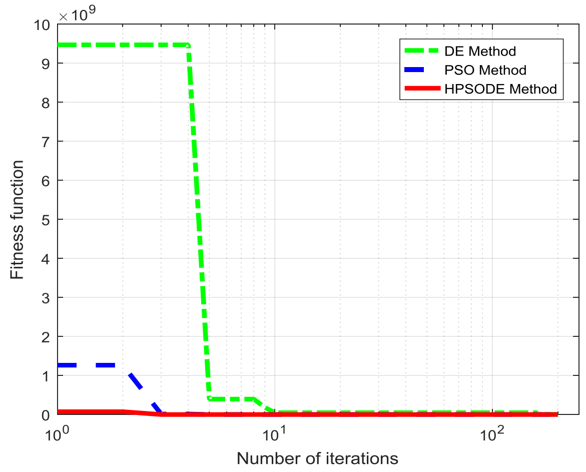


**Fig. 7: Convergence characteristic of DE, PSO, and HPSO-DE for line 1-2 outage case with quadratic fuel cost and power losses.**

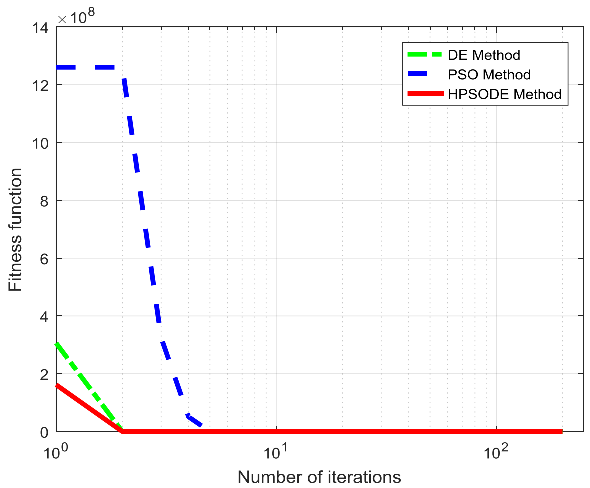


**Fig. 8: Convergence characteristic of DE, PSO, and HPSO-DE for line 1-3 outage case with quadratic fuel cost and power losses.**

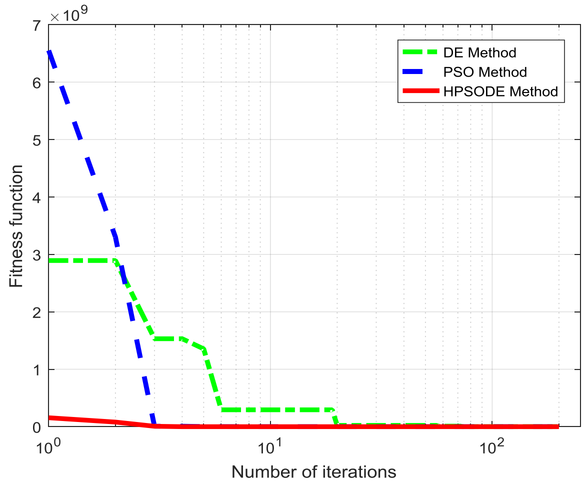


**Fig. 9: Convergence characteristic of DE, PSO, and HPSO-DE for line 3-4 outage case with quadratic fuel cost and power losses.**

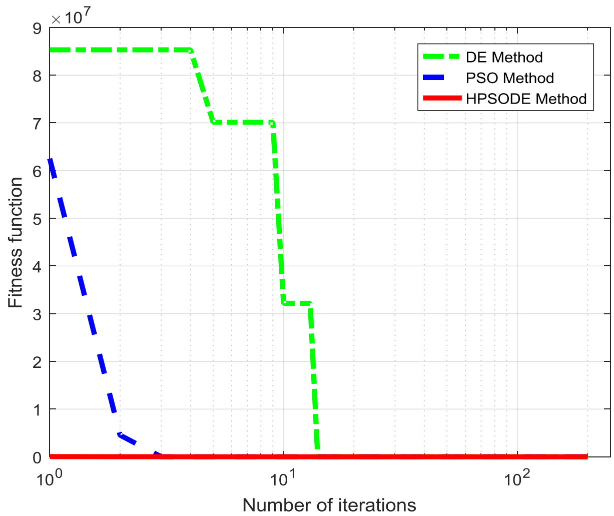


**Fig. 10: Convergence characteristic of DE, PSO, and HPSO-DE for line 2-5 outage case with quadratic fuel cost and power losses.**

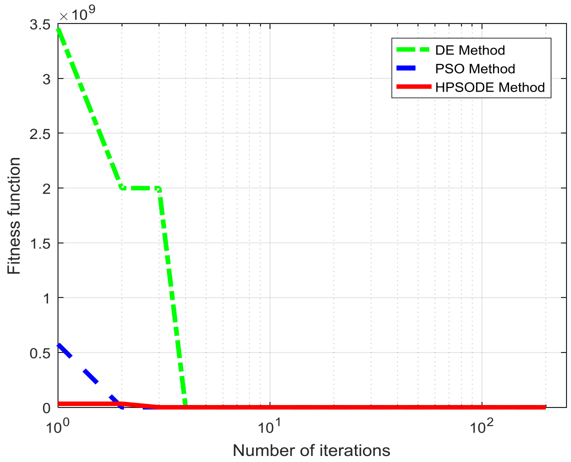


**Fig. 11: Convergence characteristic of DE, PSO, and HPSO-DE for line 4-6 outage case with quadratic fuel cost and power losses.**

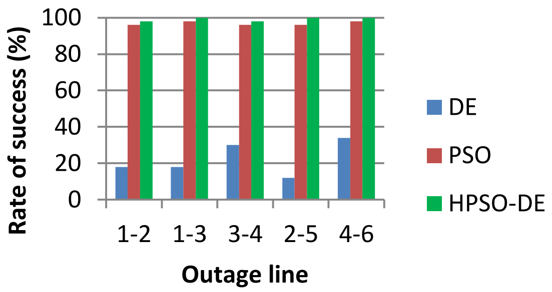


**Fig. 12: The successful rate of DE, PSO, and HPSO-DE methods for the outage cases with combined quadratic fuel cost and power losses.**

*4.2.1.2 Fuel cost and stability index objective*

The obtained best results from DE, PSO, and HPSO-DE methods for different outage cases with this combined objective are given in Tables 17 to 21. In these tables, the results including fuel cost, stability index, average CPU time and rate of success for each single objective and the combined objective are presented. For the single objective of fuel cost, the proposed method has obtained better total cost than both DE and PSO methods for all the outage cases, where the DE has obtained much higher total cost than both PSO and HPSO-DE while the total cost from the PSO is slightly higher than that of HPSO-DE. For the single objective of stability index, the proposed HPSO-DE method has also provided much better stability index than that of the DE method and slightly better than that from PSO method. For the combined objective, the best compromise solution from the proposed has also dominated that from DE and PSO methods.

In terms of the computational time, the DE is faster than both PSO and HPSO-DE methods while the proposed method is the slowest one among the three methods. However, the successful rate from the DE method is much lower than that from PSO and HPSO-DE for all outage cases. The convergence characteristics by DE, PSO, and HPSO-DE methods for the problem with different outage lines are given in Figures 13 to 17 and the rate of success of these methods for the corresponding outage lines is also given in Figure 18. For the characteristic curves, the fitness function from PSO and HPSO-DE methods can reach a stable state after 10 iterations while the DE method may need up to 100 iterations for the stability. Therefore, the proposed HPSO-DE method is effective for the problem with objectives of fuel cost and stability index for different severe outage cases.

**Table 17: The best result for line 1-2 outage case with quadratic fuel cost and stability index**

| Method | | Min. fuel cost | Min. stability index | Min. combined fuel cost and stability index |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **842.7381** | 884.4817 | **847.7666** |
| | Stability index (pu) | 0.1437 | **0.1427** | **0.1420** |
| | Avg. CPU (s) | 8.0809 | 7.7951 | 7.3164 |
| | Rate of success (%) | 18 | 20 | 12 |
| PSO | Fuel cost ($/h) | **826.1567** | 896.5642 | **826.7181** |
| | Stability index (pu) | 0.1467 | **0.1376** | **0.1382** |
| | Avg. CPU (s) | 13.5630 | 13.8872 | 14.2875 |
| | Rate of success (%) | 92 | 100 | 100 |
| HPSO-DE | Fuel cost ($/h) | **825.4490** | 897.0565 | **825.9634** |
| | Stability index (pu) | 0.1427 | **0.1367** | **0.1370** |
| | Avg. CPU (s) | 20.4049 | 21.7915 | 20.8458 |
| | Rate of success (%) | 96 | 96 | 96 |

**Table 18: The best result for line 1-3 outage case with quadratic fuel cost and stability index**

| Method | | Min. fuel cost | Min. stability index | Min. combined fuel cost and stability index |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **813.5007** | 900.6135 | **807.7377** |
| | Stability index (pu) | 0.1463 | **0.1416** | **0.1446** |
| | Avg. CPU (s) | 7.3402 | 7.1735 | 7.2899 |
| | Rate of success (%) | 20 | 20 | 14 |
| PSO | Fuel cost ($/h) | **803.0136** | 890.5964 | **804.8835** |
| | Stability index (pu) | 0.1460 | **0.1375** | **0.1381** |
| | Avg. CPU (s) | 13.6398 | 14.0597 | 13.7128 |
| | Rate of success (%) | 96 | 98 | 100 |
| HPSO-DE | Fuel cost ($/h) | **802.5372** | 907.0473 | **802.7189** |
| | Stability index (pu) | 0.1391 | **0.1368** | **0.1377** |
| | Avg. CPU (s) | 20.0127 | 20.0386 | 19.6259 |
| | Rate of success (%) | 100 | 94 | 100 |

**Table 19: The best result for line 3-4 outage case with quadratic fuel cost and stability index**

| Method | | Min. fuel cost | Min. stability index | Min. combined fuel cost and stability index |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **822.6512** | 848.5848 | **815.8437** |
| | Stability index (pu) | 0.1539 | **0.1395** | **0.1462** |
| | Avg. CPU (s) | 7.7298 | 7.8000 | 7.3408 |
| | Rate of success (%) | 20 | 24 | 16 |
| PSO | Fuel cost ($/h) | **802.7692** | 855.5184 | **803.8044** |
| | Stability index (pu) | 0.1402 | **0.1379** | **0.1382** |
| | Avg. CPU (s) | 13.5727 | 13.8847 | 14.3989 |
| | Rate of success (%) | 94 | 98 | 92 |
| HPSO-DE | Fuel cost ($/h) | **802.4723** | 836.2112 | **802.7179** |
| | Stability index (pu) | 0.1394 | **0.1374** | **0.1376** |
| | Avg. CPU (s) | 20.4274 | 20.7874 | 20.6358 |
| | Rate of success (%) | 98 | 94 | 100 |

**Table 20: The best result for line 2-5 outage case with quadratic fuel cost and stability index**

| Method | | Min. fuel cost | Min. stability index | Min. combined fuel cost and stability index |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **827.0310** | 823.4742 | **827.3048** |
| | Stability index (pu) | 0.1521 | **0.1394** | **0.1438** |
| | Avg. CPU (s) | 7.8082 | 7.3489 | 7.2388 |
| | Rate of success (%) | 18 | 22 | 26 |
| PSO | Fuel cost ($/h) | **809.0015** | 860.4109 | **810.6360** |
| | Stability index (pu) | 0.1474 | **0.1373** | **0.1380** |
| | Avg. CPU (s) | 13.0261 | 13.3833 | 13.8663 |
| | Rate of success (%) | 92 | 98 | 96 |
| HPSO-DE | Fuel cost ($/h) | **808.1932** | 879.4510 | **808.7812** |
| | Stability index (pu) | 0.1414 | **0.1368** | **0.1377** |
| | Avg. CPU (s) | 19.2168 | 20.3850 | 20.0542 |
| | Rate of success (%) | 98 | 94 | 100 |

**Table 21: The best result for line 4-6 outage case with quadratic fuel cost and stability index**

| Method | | Min. fuel cost | Min. stability index | Min. combined fuel cost and stability index |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **814.6387** | 819.9251 | **810.3613** |
| | Stability index (pu) | 0.1443 | **0.1412** | **0.1397** |
| | Avg. CPU (s) | 7.7801 | 8.8141 | 7.3579 |
| | Rate of success (%) | 14 | 34 | 10 |
| PSO | Fuel cost ($/h) | **803.8655** | 833.8059 | **804.7025** |
| | Stability index (pu) | 0.1449 | **0.1378** | **0.1379** |
| | Avg. CPU (s) | 14.6101 | 14.6756 | 13.7312 |
| | Rate of success (%) | 92 | 98 | 94 |
| HPSO-DE | Fuel cost ($/h) | **803.2421** | 858.3530 | **803.8445** |
| | Stability index (pu) | 0.1413 | **0.1370** | **0.1373** |
| | Avg. CPU (s) | 19.1688 | 20.2920 | 19.7878 |
| | Rate of success (%) | 94 | 100 | 96 |

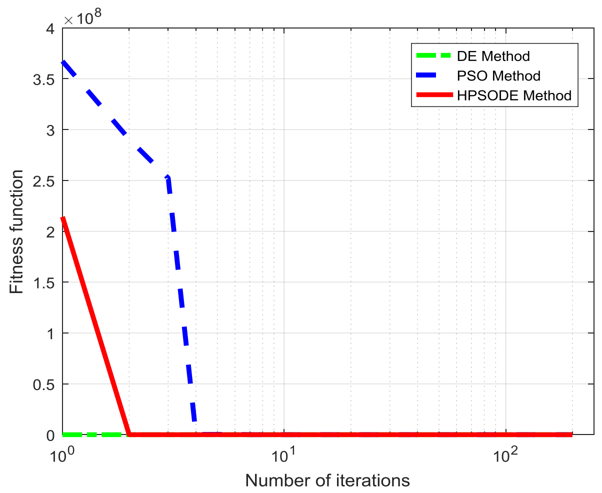


**Fig. 13: Convergence characteristic of DE, PSO, and HPSO-DE for line 1-2 outage case with quadratic fuel cost and stability index.**

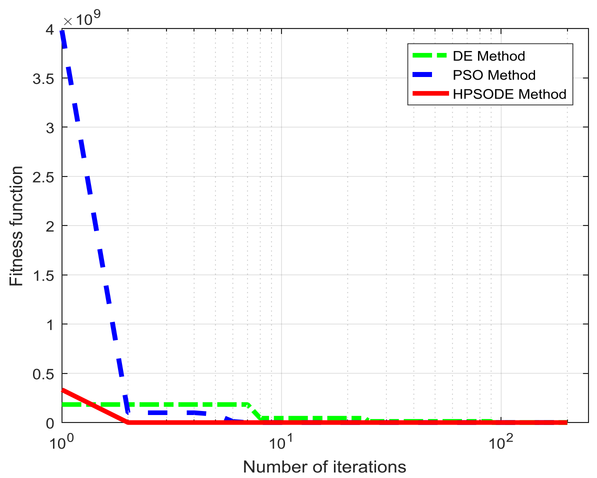


**Fig. 14: Convergence characteristic of DE, PSO, and HPSO-DE for line 1-3 outage case with quadratic fuel cost and stability index.**

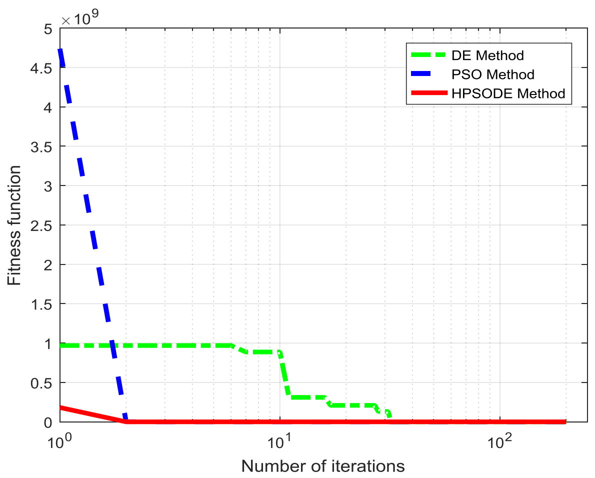


**Fig. 15: Convergence characteristic of DE, PSO, and HPSO-DE for line 3-4 outage case with quadratic fuel cost and stability index.**

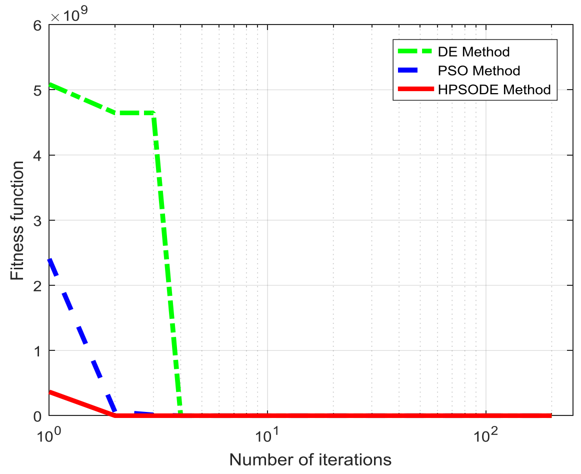


**Fig. 16: Convergence characteristic of DE, PSO, and HPSO-DE for line 2-5 outage case with quadratic fuel cost and stability index.**

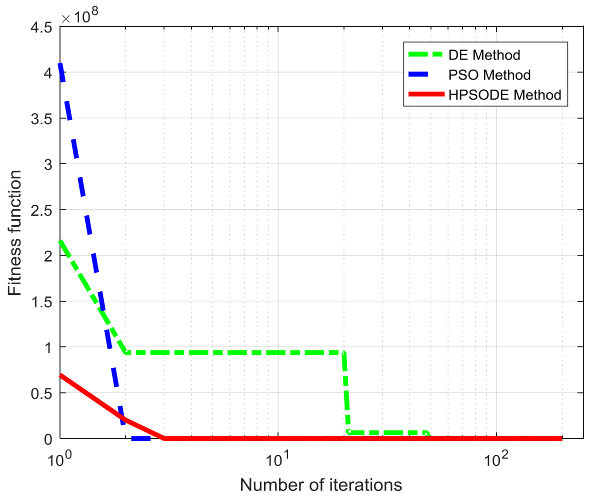


**Fig. 17: Convergence characteristic of DE, PSO, and HPSO-DE for line 4-6 outage case with quadratic fuel cost and stability index**

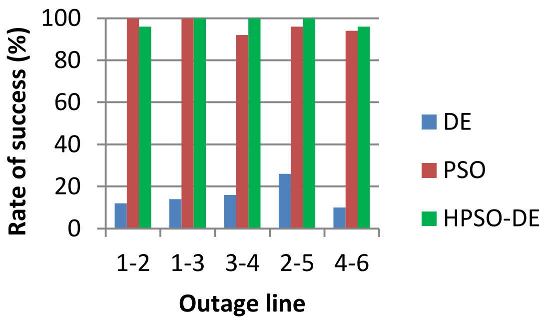


**Fig. 18: The successful rate of DE, PSO, and HPSO-DE methods for the outage cases with combined quadratic fuel cost and stability index.**

*4.2.1.3 Fuel cost and voltage deviation objective*

The two objectives including fuel cost and voltage deviation are considered for the multi-objective problem in this section. The outage cases considered here are also similar to those from the previous cases. The best results from DE, PSO, and HPSO-DE methods for the outage lines 1-2, 1-3, 3-4, 2-5, and 4-6 with the objective of fuel cost, voltage deviation, and combined fuel cost and voltage deviation are shown in Tables 22 to 26. As observed from the tables, the proposed HPSO-DE method can obtain much better fuel cost and voltage deviation than those from DE and PSO methods for the single objective of fuel cost and voltage deviation corresponding to all outage cases, respectively. For the combined objective, the solutions for all outage cases obtained by the proposed method are also dominating those from DE and PSO methods.

For the computational time, the DE method is always faster than the other methods and the proposed method is always slower than the others. However, the rate of success from the DE is very low for all cases compared to the other methods while the proposed method has better the rate of success than that from the others. The convergence characteristics by DE, PSO, and HSPO-DE methods for the problem with five outage cases are shown in Figures 19 to 23 and the successful rate of the methods corresponding to the outage cases is also depicted in Figure 24. As observed from the figures, the fitness function from the methods can reach the stable state after 20 iterations. Therefore, the proposed HPSO-DE method is very effective for dealing with the problem with two objectives of fuel cost and voltage deviation for the most severe five outage cases of the system.

**Table 22: The best result for line 1-2 outage case with quadratic fuel cost and voltage deviation**

| Method | | Min. fuel cost | Min. voltage deviation | Min. combined fuel cost and voltage deviation |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **847.5497** | 924.5120 | **850.3111** |
| | Voltage deviation (pu) | 0.4394 | **0.2916** | **0.3850** |
| | Avg. CPU (s) | 8.0809 | 7.5071 | 6.9046 |
| | Rate of success (%) | 22 | 14 | 20 |
| PSO | Fuel cost ($/h) | **825.8534** | 904.2731 | **829.1851** |
| | Voltage deviation (pu) | 0.3826 | **0.1573** | **0.1761** |
| | Avg. CPU (s) | 12.7431 | 12.7044 | 13.3022 |
| | Rate of success (%) | 94 | 100 | 96 |
| HPSO-DE | Fuel cost ($/h) | **825.4980** | 901.3550 | **828.2059** |
| | Voltage deviation (pu) | 0.3438 | **0.1397** | **0.1467** |
| | Avg. CPU (s) | 19.2296 | 19.3685 | 19.3279 |
| | Rate of success (%) | 96 | 98 | 100 |

**Table 23: The best result for line 1-3 outage case with quadratic fuel cost and voltage deviation**

| Method | | Min. fuel cost | Min. voltage deviation | Min. combined fuel cost and voltage deviation |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **819.0703** | 860.9857 | **845.7150** |
| | Voltage deviation (pu) | 0.4636 | **0.2592** | **0.3101** |
| | Avg. CPU (s) | 7.3264 | 7.2793 | 6.8082 |
| | Rate of success (%) | 20 | 14 | 22 |
| PSO | Fuel cost ($/h) | **803.2157** | 881.3245 | **807.2743** |
| | Voltage deviation (pu) | 0.2145 | **0.1661** | **0.1623** |
| | Avg. CPU (s) | 12.1266 | 12.1384 | 12.5213 |
| | Rate of success (%) | 94 | 90 | 96 |
| HPSO-DE | Fuel cost ($/h) | **802.5491** | 875.1800 | **804.0308** |
| | Voltage deviation (pu) | 0.7325 | **0.1415** | **0.1483** |
| | Avg. CPU (s) | 17.9008 | 17.6986 | 18.2187 |
| | Rate of success (%) | 100 | 100 | 96 |

**Table 24: The best result for line 3-4 outage case with quadratic fuel cost and voltage deviation**

| Method | | Min. fuel cost | Min. voltage deviation | Min. combined fuel cost and voltage deviation |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **823.3570** | 841.1731 | **816.0523** |
| | Voltage deviation (pu) | 0.5570 | **0.2545** | **0.3919** |
| | Avg. CPU (s) | 7.8927 | 7.6977 | 6.8531 |
| | Rate of success (%) | 8 | 12 | 16 |
| PSO | Fuel cost ($/h) | **803.0765** | 836.6660 | **806.5116** |
| | Voltage deviation (pu) | 0.2645 | **0.1632** | **0.1698** |
| | Avg. CPU (s) | 13.8292 | 13.7456 | 13.2286 |
| | Rate of success (%) | 96 | 94 | 96 |
| HPSO-DE | Fuel cost ($/h) | **802.4870** | 912.7295 | **804.0239** |
| | Voltage deviation (pu) | 0.6917 | **0.1413** | **0.1481** |
| | Avg. CPU (s) | 18.8643 | 19.1607 | 19.2995 |
| | Rate of success (%) | 94 | 98 | 98 |

**Table 25: The best result for line 2-5 outage case with quadratic fuel cost and voltage deviation**

| Method | | Min. fuel cost | Min. voltage deviation | Min. combined fuel cost and voltage deviation |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **830.1180** | 903.3103 | **828.0206** |
| | Voltage deviation (pu) | 0.4285 | **0.2408** | **0.2253** |
| | Avg. CPU (s) | 7.8465 | 7.8540 | 6.8971 |
| | Rate of success (%) | 12 | 12 | 22 |
| PSO | Fuel cost ($/h) | **809.0451** | 869.9984 | **813.1933** |
| | Voltage deviation (pu) | 0.2830 | **0.1531** | **0.1623** |
| | Avg. CPU (s) | 12.1646 | 13.2838 | 12.7456 |
| | Rate of success (%) | 100 | 98 | 98 |
| HPSO-DE | Fuel cost ($/h) | **808.2091** | 887.6686 | **810.5690** |
| | Voltage deviation (pu) | 0.6052 | **0.1395** | **0.1494** |
| | Avg. CPU (s) | 18.6973 | 18.2206 | 18.4303 |
| | Rate of success (%) | 96 | 100 | 98 |

**Table 26: The best result for line 4-6 outage case with quadratic fuel cost and voltage deviation**

| Method | | Min. fuel cost | Min. voltage deviation | Min. combined fuel cost and voltage deviation |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **817.2350** | 935.0229 | **824.9980** |
| | Voltage deviation (pu) | 0.3202 | **0.2402** | **0.3688** |
| | Avg. CPU (s) | 7.7657 | 7.7829 | 6.7948 |
| | Rate of success (%) | 18 | 30 | 26 |
| PSO | Fuel cost ($/h) | **803.7879** | 820.6546 | **806.3401** |
| | Voltage deviation (pu) | 0.3825 | **0.1611** | **0.1571** |
| | Avg. CPU (s) | 12.7868 | 13.0301 | 12.6311 |
| | Rate of success (%) | 98 | 98 | 98 |
| HPSO-DE | Fuel cost ($/h) | **803.2338** | 870.7119 | **804.7736** |
| | Voltage deviation (pu) | 0.5562 | **0.1399** | **0.1462** |
| | Avg. CPU (s) | 19.0349 | 18.2780 | 18.6643 |
| | Rate of success (%) | 100 | 98 | 96 |

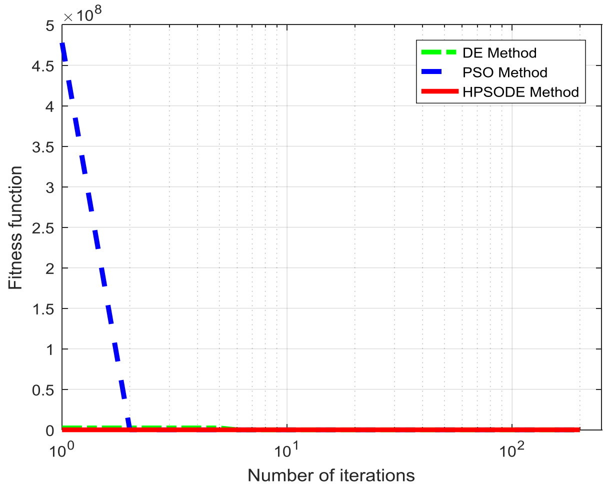


**Fig. 19: Convergence characteristic of DE, PSO, and HPSO-DE for line 1-2 outage case with quadratic fuel cost and voltage deviation.**

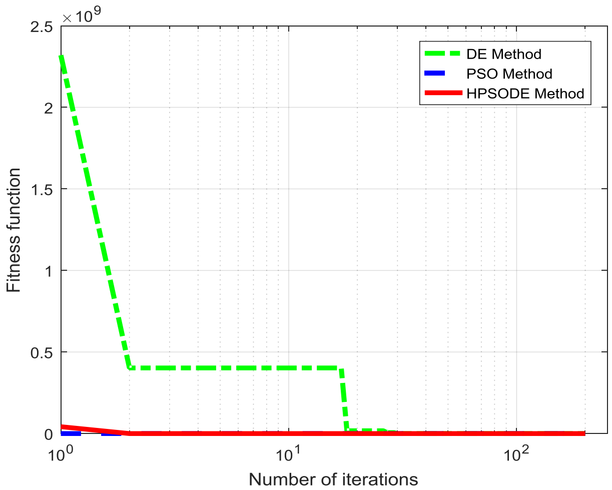


**Fig. 20: Convergence characteristic of DE, PSO, and HPSO-DE for line 1-3 outage case with quadratic fuel cost and voltage deviation.**

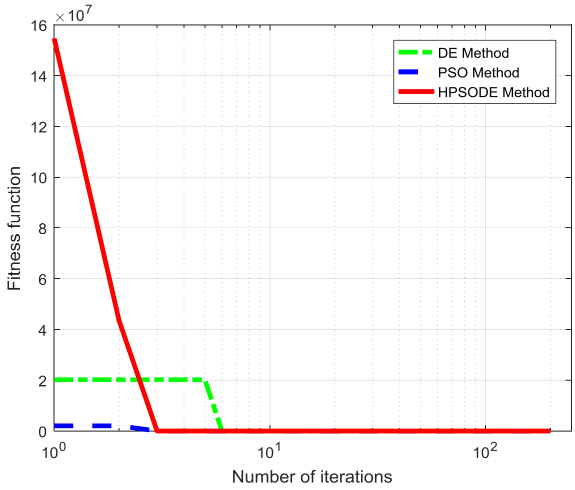


**Fig. 21: Convergence characteristic of DE, PSO, and HPSO-DE for line 3-4 outage case with quadratic fuel cost and voltage deviation.**

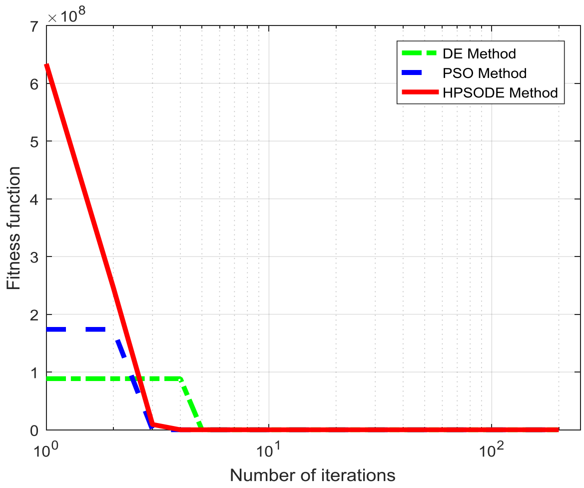


**Fig. 22: Convergence characteristic of DE, PSO, and HPSO-DE for line 2-5 outage case with quadratic fuel cost and voltage deviation.**

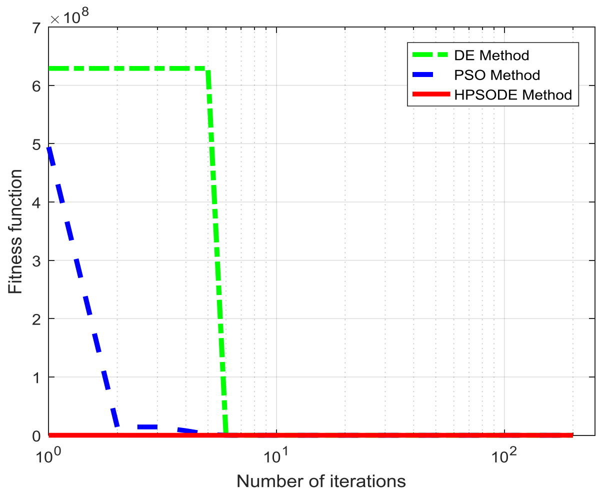


**Fig. 23: Convergence characteristic of DE, PSO, and HPSO-DE for line 4-6 outage case with quadratic fuel cost and voltage deviation.**

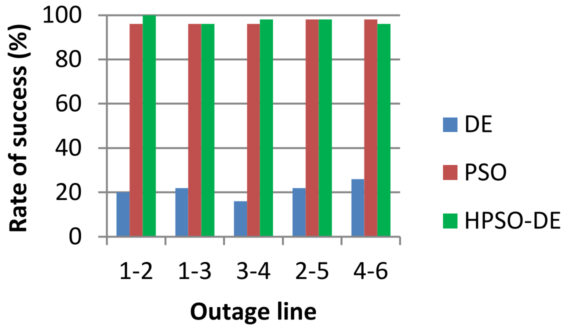


**Fig. 24: The successful rate of DE, PSO, and HPSO-DE methods for the outage cases with combined quadratic fuel cost and voltage deviation.**

*4.2.2 Valve point loading effects*

When the valve point loading effects are considered for the fuel cost of generators, the considered cases are also the same to those considered for the quadratic fuel cost of generators. However, the problem has become more complex due to the non-differentiable fuel cost function which leads to more difficult to find the optimal solution. Therefore, the problem with the valve point loading effects is more challenge than that with the quadratic fuel cost function for generating units.

*4.2.2.1 Fuel cost and power loss objective*

The best results obtained by DE, PSO, and HPSO-DE methods including fuel cost, power losses, average CPU time, and rate of success for the problem with single objectives and combined objective are given in Tables 27 to 31 corresponding to the outage lines of 1-2, 1-3, 3-4, 2-5, and 46, respectively. As shown in the tables, the proposed HPSO-DE method can obtain better total cost than the DE and PSO do for the single objective of fuel cost and better power loss than the others do for the single objective of power losses for all the outage cases. For the combined objective of fuel cost and power loss, the proposed method can obtain best compromise solutions dominating those from DE and PSO methods for most of outage cases except PSO for outage line 1-3, DE for outage line 3-4, DE and PSO for outage line 2-5, and DE for outage line 4-6, where there is a trade-off between the total fuel cost and power losses from the provided solutions.

**Table 27: The best result for line 1-2 outage case with valve point loading effects of fuel cost and power losses**

| Method | | Min. fuel cost | Min. power loss | Min. combined fuel cost and power loss |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **1.0599e+03** | 1.0833e+03 | **1.0787e+03** |
| | Power losses (MW) | 7.4777 | **5.2735** | **5.3012** |
| | Avg. CPU (s) | 10.2920 | 10.6299 | 10.2792 |
| | Rate of success (%) | 22 | 16 | 16 |
| PSO | Fuel cost ($/h) | **1.0364e+03** | 1.1701e+03 | **1.0444e+03** |
| | Power losses (MW) | 5.7086 | **3.2387** | **4.6412** |
| | Avg. CPU (s) | 19.2680 | 20.5060 | 19.9413 |
| | Rate of success (%) | 94 | 96 | 96 |
| HPSO-DE | Fuel cost ($/h) | **1.0359e+03** | 1.1700e+03 | **1.0403e+03** |
| | Power losses (MW) | 5.5740 | **3.2237** | **4.5576** |
| | Avg. CPU (s) | 29.6857 | 30.0698 | 30.5949 |
| | Rate of success (%) | 98 | 100 | 98 |

**Table 28: The best result for line 1-3 outage case with valve point loading effects of fuel cost and power losses**

| Method | | Min. fuel cost | Min. power loss | Min. combined fuel cost and power loss |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **978.0553** | 1.1241e+03 | **1.0657e+03** |
| | Power losses (MW) | 9.3330 | **6.3157** | **6.0973** |
| | Avg. CPU (s) | 10.2075 | 9.7513 | 10.0808 |
| | Rate of success (%) | 30 | 24 | 16 |
| PSO | Fuel cost ($/h) | **955.3976** | 1.1701e+03 | **1.0402e+03** |
| | Power losses (MW) | 7.6920 | **3.2446** | **4.7754** |
| | Avg. CPU (s) | 18.4440 | 19.5145 | 18.7641 |
| | Rate of success (%) | 94 | 100 | 96 |
| HPSO-DE | Fuel cost ($/h) | **953.8790** | 1.1700e+03 | **1.0413e+03** |
| | Power losses (MW) | 7.7058 | **3.2241** | **4.5301** |
| | Avg. CPU (s) | 26.9002 | 29.6021 | 28.8365 |
| | Rate of success (%) | 100 | 94 | 98 |

**Table 29: The best result for line 3-4 outage case with valve point loading effects of fuel cost and power losses**

| Method | | Min. fuel cost | Min. power loss | Min. combined fuel cost and power loss |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **992.1392** | 1.1127e+03 | **1.0284e+03** |
| | Power losses (MW) | 8.8199 | **4.7438** | **7.5135** |
| | Avg. CPU (s) | 9.8833 | 9.8795 | 10.2958 |
| | Rate of success (%) | 18 | 22 | 14 |
| PSO | Fuel cost ($/h) | **956.7896** | 956.7896 | **1.0470e+03** |
| | Power losses (MW) | 7.9006 | **7.9006** | **4.6003** |
| | Avg. CPU (s) | 19.7497 | 19.7497 | 19.5578 |
| | Rate of success (%) | 94 | 94 | 98 |
| HPSO-DE | Fuel cost ($/h) | **953.6279** | 1.1700e+03 | **1.0413e+03** |
| | Power losses (MW) | 7.6013 | **3.2264** | **4.5335** |
| | Avg. CPU (s) | 28.4231 | 30.2839 | 30.1881 |
| | Rate of success (%) | 98 | 96 | 98 |

**Table 30: The best result for line 2-5 outage case with valve point loading effects of fuel cost and power losses**

| Method | | Min. fuel cost | Min. power loss | Min. combined fuel cost and power loss |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **1.0082e+03** | 1.1735e+03 | **1.0216e+03** |
| | Power losses (MW) | 8.1609 | **5.0484** | **7.2747** |
| | Avg. CPU (s) | 9.6730 | 9.7170 | 10.0942 |
| | Rate of success (%) | 22 | 14 | 20 |
| PSO | Fuel cost ($/h) | **970.4957** | 1.1701e+03 | **985.7324** |
| | Power losses (MW) | 7.9976 | **3.2379** | **6.3845** |
| | Avg. CPU (s) | 19.0583 | 20.4421 | 18.8927 |
| | Rate of success (%) | 94 | 94 | 92 |
| HPSO-DE | Fuel cost ($/h) | **965.8093** | 1.1700e+03 | **1.0409e+03** |
| | Power losses (MW) | 8.3106 | **3.2239** | **4.5541** |
| | Avg. CPU (s) | 27.2964 | 29.3332 | 28.8633 |
| | Rate of success (%) | 94 | 98 | 100 |

**Table 31: The best result for line 4-6 outage case with valve point loading effects of fuel cost and power losses**

| Method | | Min. fuel cost | Min. power loss | Min. combined fuel cost and power loss |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **1.0479e+03** | 1.1040e+03 | **995.5160** |
| | Power losses (MW) | 7.1423 | **5.1848** | **8.3728** |
| | Avg. CPU (s) | 10.0705 | 10.0181 | 10.0711 |
| | Rate of success (%) | 16 | 14 | 30 |
| PSO | Fuel cost ($/h) | **923.5486** | 1.1701e+03 | **1.0456e+03** |
| | Power losses (MW) | 10.7852 | **3.2416** | **4.6142** |
| | Avg. CPU (s) | 18.4780 | 19.8942 | 19.3176 |
| | Rate of success (%) | 96 | 100 | 96 |
| HPSO-DE | Fuel cost ($/h) | **922.3533** | 1.1701e+03 | **1.0397e+03** |
| | Power losses (MW) | 10.5272 | **3.2288** | **4.5823** |
| | Avg. CPU (s) | 27.7176 | 30.0108 | 29.5859 |
| | Rate of success (%) | 100 | 100 | 98 |

For the CPU time, the DE always provides a solution with a faster manner than both PSO and HPSO-DE methods for the problem with single and combined objectives in all outage cases. However, the successful rate of the DE method is very low compared to that from the PSO and HPSO-DE methods for all cases of line outage in both single and combined objective of the problem. As observed from the tables, the successful rate from DE is not higher than 30% while that from PSO and HPSO-DE is not lower than 94%. The convergence characteristics of DE, PSO, and HPSO-DE methods for the combined objective with the five outage cases are given in Figures 17 to 23 and the corresponding successful rate of the methods is also shown in Figure 24. As shown in the figures, the fitness function from PSO and HPSO-DE methods usually reaches the stabile state earlier than that from DE method. Therefore, the proposed method is rather effective for the non-convex problem in these cases.

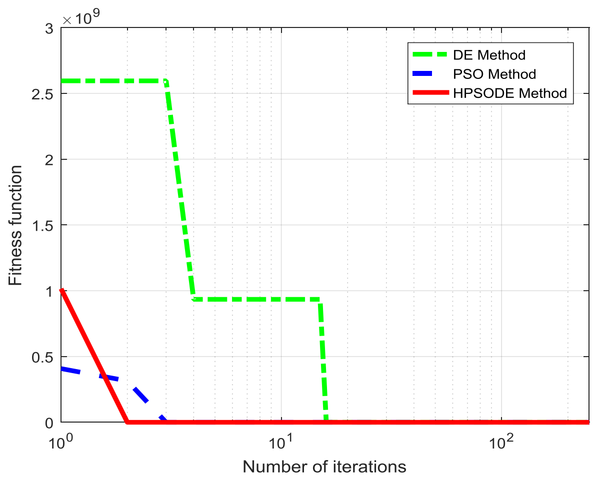


**Fig. 19: Convergence characteristic of DE, PSO, and HPSO-DE for line 1-2 outage case with valve point loading effects of fuel cost and power losses.**

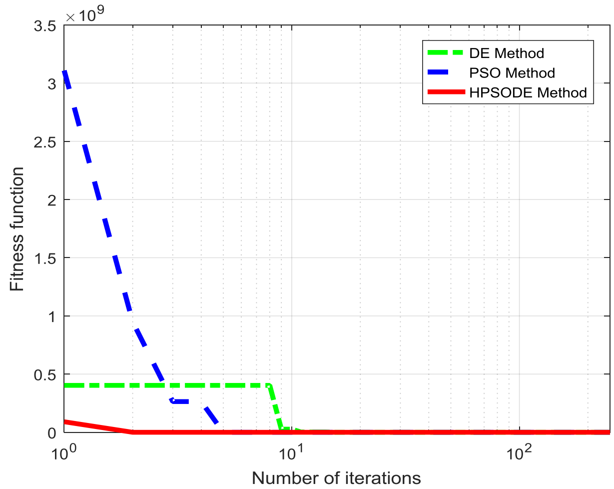


**Fig. 20: Convergence characteristic of DE, PSO, and HPSO-DE for line 1-3 outage case with valve point loading effects of fuel cost and power losses.**

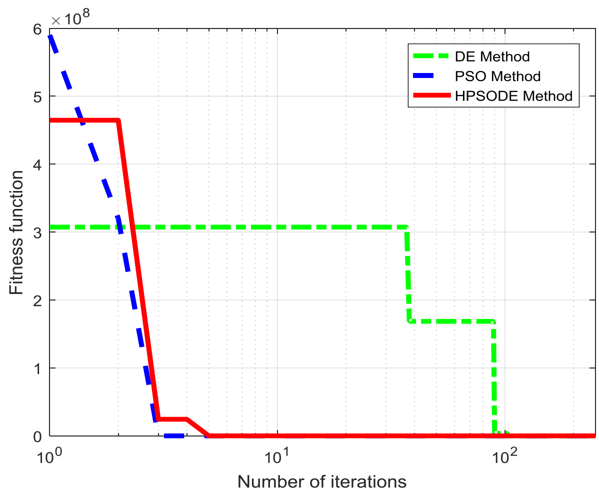


**Fig. 21: Convergence characteristic of DE, PSO, and HPSO-DE for line 3-4 outage case with valve point loading effects of fuel cost and power losses.**

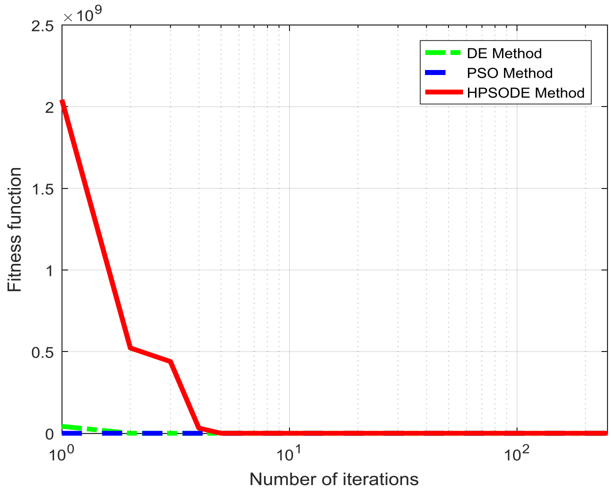


**Fig. 22: Convergence characteristic of DE, PSO, and HPSO-DE for line 2-5 outage case with valve point loading effects of fuel cost and power losses.**

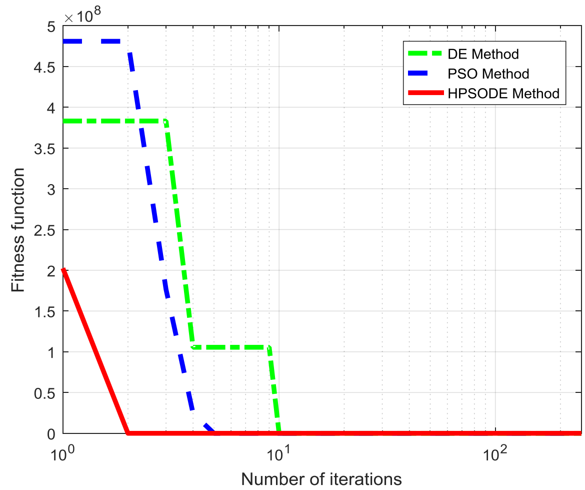


**Fig. 23: Convergence characteristic of DE, PSO, and HPSO-DE for line 4-6 outage case with valve point loading effects of fuel cost and power losses.**

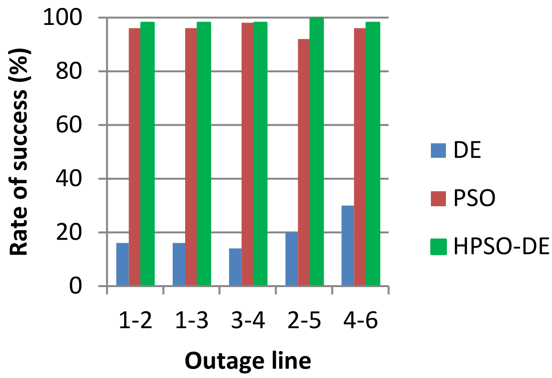


**Fig. 24: Successful rate of DE, PSO, and HPSO-DE for outage cases with combined valve point loading effects of fuel cost and power losses.**

*4.2.2.2 Fuel cost and stability index objective*

The combined objective of fuel cost and stability index is also considered for the mentioned five outage cases as previous sections. The results including fuel cost, stability index, average CPU time, and successful rate of DE, PSO, and HPSO-DE methods for five different outage lines are depicted in Tables 32 to 36. For the single objective, the proposed method can obtain better total cost and stability index than that from DE and PSO methods for all the outage cases. Moreover, the best compromise solution from the proposed method for the combined objective is also dominating that from DE and PSO for all the outage cases.

**Table 32: The best result for line 1-2 outage case with valve point loading effects of fuel cost and stability index**

| Method | | Min. fuel cost | Min. stability index | Min. combined fuel cost and stability index |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **1.0617e+03** | 1.0846e+03 | **1.0684e+03** |
| | Stability index (pu) | 0.1426 | **0.1395** | **0.1405** |
| | Avg. CPU (s) | 11.1862 | 10.3881 | 10.8854 |
| | Rate of success (%) | 30 | 18 | 16 |
| PSO | Fuel cost ($/h) | **1.0369e+03** | 1.0870e+03 | **1.0374e+03** |
| | Stability index (pu) | 0.1386 | **0.1374** | **0.1375** |
| | Avg. CPU (s) | 20.9116 | 21.4380 | 21.3799 |
| | Rate of success (%) | 98 | 100 | 100 |
| HPSO-DE | Fuel cost ($/h) | **1.0358e+03** | 1.1159e+03 | **1.0362e+03** |
| | Stability index (pu) | 0.1403 | **0.1366** | **0.1368** |
| | Avg. CPU (s) | 29.9918 | 31.4966 | 32.4289 |
| | Rate of success (%) | 100 | 98 | 98 |

**Table 33: The best result for line 1-3 outage case with valve point loading effects of fuel cost and stability index**

| Method | | Min. fuel cost | Min. stability index | Min. combined fuel cost and stability index |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **995.8053** | 1.0761e+03 | **998.5109** |
| | Stability index (pu) | 0.1445 | **0.1401** | **0.1495** |
| | Avg. CPU (s) | 10.5809 | 10.3507 | 10.7659 |
| | Rate of success (%) | 16 | 26 | 18 |
| PSO | Fuel cost ($/h) | **956.7811** | 1.0212e+03 | **956.7622** |
| | Stability index (pu) | 0.1458 | **0.1374** | **0.1388** |
| | Avg. CPU (s) | 19.7357 | 20.5806 | 20.3837 |
| | Rate of success (%) | 98 | 92 | 96 |
| HPSO-DE | Fuel cost ($/h) | **953.8760** | 1.0927e+03 | **954.8022** |
| | Stability index (pu) | 0.1411 | **0.1366** | **0.1379** |
| | Avg. CPU (s) | 28.7591 | 30.3200 | 30.4813 |
| | Rate of success (%) | 100 | 98 | 98 |

**Table 34: The best result for line 3-4 outage case with valve point loading effects of fuel cost and stability index**

| Method | | Min. fuel cost | Min. stability index | Min. combined fuel cost and stability index |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **993.6127** | 1.0675e+03 | **1.0017e+03** |
| | Stability index (pu) | 0.1436 | **0.1403** | **0.1455** |
| | Avg. CPU (s) | 10.3647 | 10.7663 | 10.7550 |
| | Rate of success (%) | 20 | 20 | 26 |
| PSO | Fuel cost ($/h) | **956.4927** | 1.0806e+03 | **956.6893** |
| | Stability index (pu) | 0.1435 | **0.1379** | **0.1387** |
| | Avg. CPU (s) | 20.8096 | 20.5191 | 20.9750 |
| | Rate of success (%) | 94 | 92 | 96 |
| HPSO-DE | Fuel cost ($/h) | **954.1112** | 1.0809e+03 | **954.2330** |
| | Stability index (pu) | 0.1449 | **0.1372** | **0.1377** |
| | Avg. CPU (s) | 31.2376 | 31.2673 | 31.5724 |
| | Rate of success (%) | 98 | 98 | 100 |

**Table 35: The best result for line 2-5 outage case with valve point loading effects of fuel cost and stability index**

| Method | | Min. fuel cost | Min. stability index | Min. combined fuel cost and stability index |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **987.8837** | 1.0612e+03 | **987.9997** |
| | Stability index (pu) | 0.1505 | **0.1396** | **0.1545** |
| | Avg. CPU (s) | 10.9968 | 10.9828 | 10.6252 |
| | Rate of success (%) | 28 | 28 | 40 |
| PSO | Fuel cost ($/h) | **968.4729** | 1.0628e+03 | **971.7530** |
| | Stability index (pu) | 0.1392 | **0.1373** | **0.1396** |
| | Avg. CPU (s) | 20.6252 | 20.4271 | 20.5382 |
| | Rate of success (%) | 98 | 98 | 96 |
| HPSO-DE | Fuel cost ($/h) | **964.8336** | 1.1085e+03 | **968.7722** |
| | Stability index (pu) | 0.1547 | **0.1368** | **0.1383** |
| | Avg. CPU (s) | 29.6701 | 30.9893 | 30.5459 |
| | Rate of success (%) | 94 | 100 | 100 |

**Table 36: The best result for line 4-6 outage case with valve point loading effects of fuel cost and stability index**

| Method | | Min. fuel cost | Min. stability index | Min. combined fuel cost and stability index |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **1.0074e+03** | 1.0046e+03 | **988.1132** |
| | Stability index (pu) | 0.1443 | **0.1386** | **0.1467** |
| | Avg. CPU (s) | 10.5266 | 10.4979 | 10.5135 |
| | Rate of success (%) | 22 | 32 | 28 |
| PSO | Fuel cost ($/h) | **923.6711** | 1.0524e+03 | **922.9424** |
| | Stability index (pu) | 0.1511 | **0.1378** | **0.1399** |
| | Avg. CPU (s) | 20.4321 | 20.9644 | 20.5001 |
| | Rate of success (%) | 96 | 96 | 100 |
| HPSO-DE | Fuel cost ($/h) | **921.7702** | 1.0582e+03 | **922.6878** |
| | Stability index (pu) | 0.1451 | **0.1369** | **0.1385** |
| | Avg. CPU (s) | 30.4177 | 30.7990 | 31.1294 |
| | Rate of success (%) | 98 | 96 | 100 |

Similar to other cases, the average CPU time from DE method is also faster than that of PSO and HPSO-DE for all the considered cases. In the contrary, the successful rate of DE method is very low compared to that from PSO and HPSO-DE while the HPSO-DE method usually reaches the highest rate of success among the methods for all cases. The convergence curves of DE, PSO, and HPSO-DE methods for the outage cases with the combined objective are given in Figures 25 to 29 and the rate of success of DE, PSO, and HPSO-DE methods are also shown in Figure 30. As observed from the figures, the proposed method is stable during convergence process to the optimal solution. Therefore, the proposed method is also very effective for combined objective of fuel cost and stability index accompanying with different outage lines.

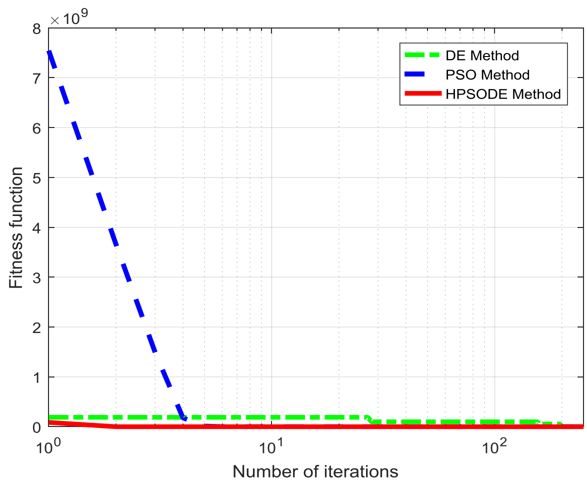


**Fig. 25: Convergence characteristic of DE, PSO, and HPSO-DE for line 1-2 outage case with valve point loading effects of fuel cost and stability index.**

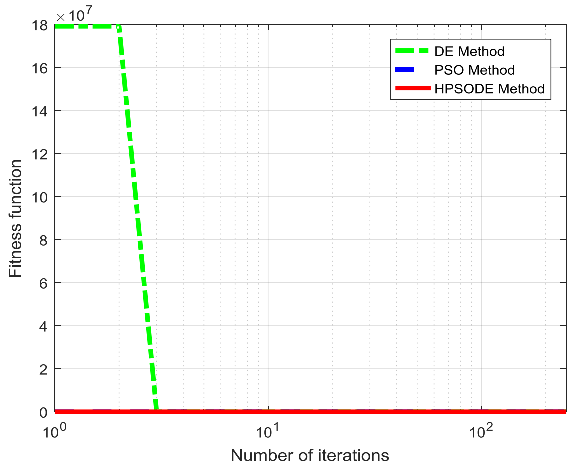


**Fig. 26: Convergence characteristic of DE, PSO, and HPSO-DE for line 1-3 outage case with valve point loading effects of fuel cost and stability index.**

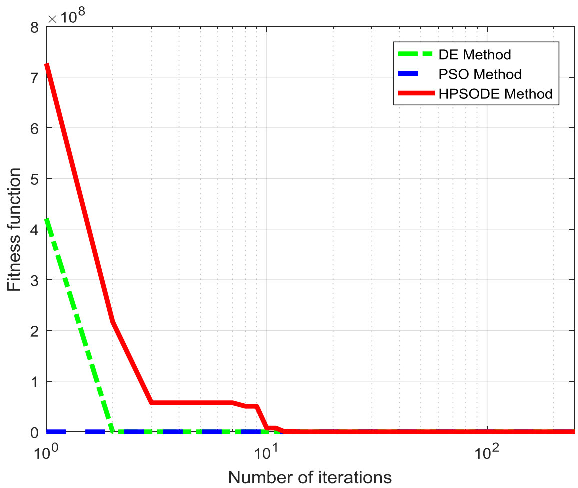


**Fig. 27: Convergence characteristic of DE, PSO, and HPSO-DE for line 3-4 outage case with valve point loading effects of fuel cost and stability index.**

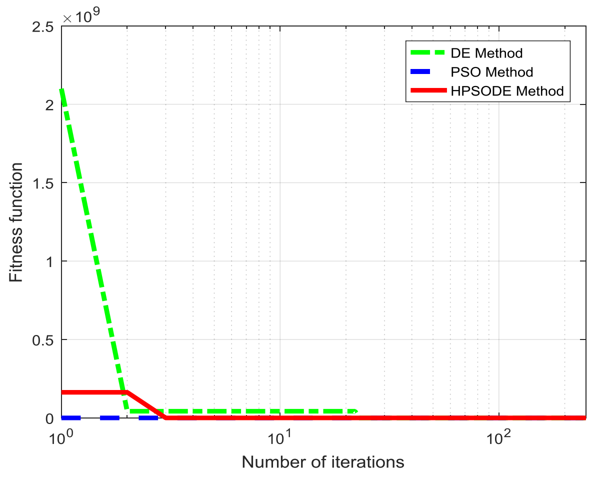


**Fig. 28: Convergence characteristic of DE, PSO, and HPSO-DE for line 2-5 outage case with valve point loading effects of fuel cost and stability index.**

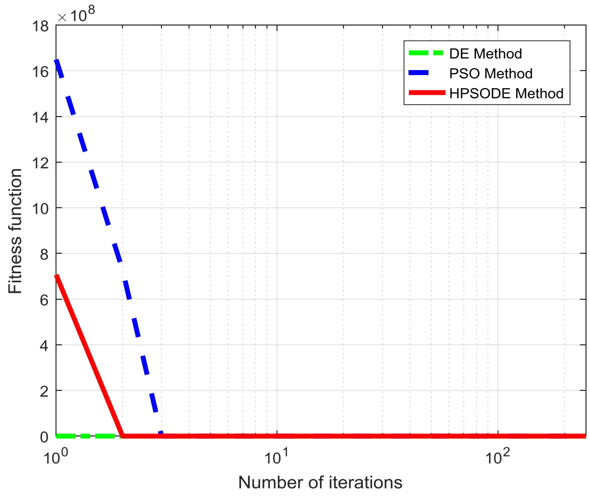


**Fig. 29: Convergence characteristic of DE, PSO, and HPSO-DE for line 4-6 outage case with valve point loading effects of fuel cost and stability index.**

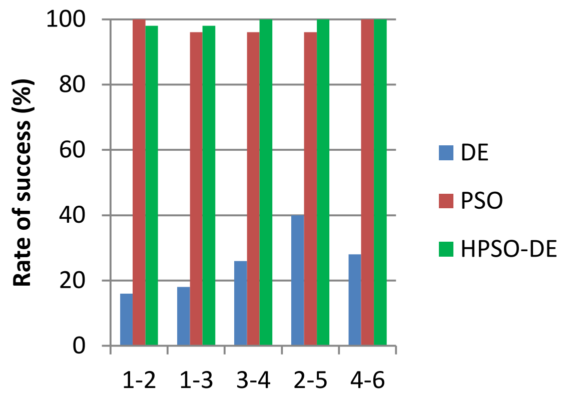


**Fig. 30: Successful rate of DE, PSO, and HPSO-DE for outage cases with combined valve point loading effects of fuel cost and stability index.**

*4.2.2.3 Fuel cost and voltage deviation objective*

The problem with the objective of fuel cost and voltage deviation is considered with five different outage cases in this section. The best results from each of DE, PSO, and HSPO-DE corresponding to the outage cases are given Tables 37 to 41. As seen from these tables, the proposed method has obtained better total cost than that from DE and PSO for the case with single objective of fuel cost and better voltage deviation than that from DE and PSO methods for the single objective of voltage deviation. For the combined objective of fuel cost and voltage deviation, the proposed method has also obtained dominated solutions to those from DE and PSO methods except for the PSO method for the outage case of line 4-6. Like many previous cases, the computational time from DE method is usually faster than that from PSO and HPSO-DE methods for all cases with different objectives. The successful rate of DE for all cases is less than 30% while that of PSO is from 92% and HPSO-DE from 96%.

**Table 37: The best result for line 1-2 outage case with valve point loading effects of fuel cost and voltage deviation**

| Method | | Min. fuel cost | Min. voltage deviation | Min. combined fuel cost and voltage deviation |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **1.0566e+03** | 1.1062e+03 | **1.0637e+03** |
| | Voltage deviation (pu) | 0.3915 | **0.2654** | **0.3679** |
| | Avg. CPU (s) | 10.4583 | 10.3167 | 10.0262 |
| | Rate of success (%) | 18 | 22 | 30 |
| PSO | Fuel cost ($/h) | **1.0366e+03** | 1.1253e+03 | **1.0387e+03** |
| | Voltage deviation (pu) | 0.3362 | **0.1546** | **0.1718** |
| | Avg. CPU (s) | 19.8174 | 19.2430 | 19.6034 |
| | Rate of success (%) | 94 | 92 | 94 |
| HPSO-DE | Fuel cost ($/h) | **1.0358e+03** | 1.1210e+03 | **1.0392e+03** |
| | Voltage deviation (pu) | 0.7852 | **0.1383** | **0.1418** |
| | Avg. CPU (s) | 29.4957 | 28.4939 | 29.0920 |
| | Rate of success (%) | 100 | 98 | 98 |

**Table 38: The best result for line 1-3 outage case with valve point loading effects of fuel cost and voltage deviation**

| Method | | Min. fuel cost | Min. voltage deviation | Min. combined fuel cost and voltage deviation |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **978.1773** | 1.0665e+03 | **1.0125e+03** |
| | Voltage deviation (pu) | 0.4364 | **0.2788** | **0.4213** |
| | Avg. CPU (s) | 9.7513 | 10.5241 | 9.9541 |
| | Rate of success (%) | 14 | 22 | 8 |
| PSO | Fuel cost ($/h) | **958.0084** | 1.0521e+03 | **957.1436** |
| | Voltage deviation (pu) | 0.2829 | **0.1576** | **0.1894** |
| | Avg. CPU (s) | 18.4702 | 18.7410 | 18.5754 |
| | Rate of success (%) | 100 | 96 | 100 |
| HPSO-DE | Fuel cost ($/h) | **953.8069** | 1.1351e+03 | **956.5392** |
| | Voltage deviation (pu) | 0.3491 | **0.1402** | **0.1494** |
| | Avg. CPU (s) | 27.6970 | 26.2671 | 27.4268 |
| | Rate of success (%) | 98 | 100 | 98 |

**Table 39: The best result for line 3-4 outage case with valve point loading effects of fuel cost and voltage deviation**

| Method | | Min. fuel cost | Min. voltage deviation | Min. combined fuel cost and voltage deviation |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **996.0560** | 1.0928e+03 | **1.0561e+03** |
| | Voltage deviation (pu) | 0.2665 | **0.2613** | **0.3050** |
| | Avg. CPU (s) | 10.0156 | 9.9666 | 10.1061 |
| | Rate of success (%) | 26 | 28 | 16 |
| PSO | Fuel cost ($/h) | **957.3019** | 1.0772e+03 | **964.6208** |
| | Voltage deviation (pu) | 0.4128 | **0.1590** | **0.1786** |
| | Avg. CPU (s) | 18.9685 | 18.6967 | 19.5070 |
| | Rate of success (%) | 98 | 96 | 94 |
| HPSO-DE | Fuel cost ($/h) | **953.6579** | 1.1187e+03 | **958.1908** |
| | Voltage deviation (pu) | 0.5951 | **0.1402** | **0.1490** |
| | Avg. CPU (s) | 29.3273 | 28.5644 | 28.7697 |
| | Rate of success (%) | 96 | 98 | 100 |

**Table 40: The best result for line 2-5 outage case with valve point loading effects of fuel cost and voltage deviation**

| Method | | Min. fuel cost | Min. voltage deviation | Min. combined fuel cost and voltage deviation |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **1.0102e+03** | 1.0276e+03 | **1.0271e+03** |
| | Voltage deviation (pu) | 0.2909 | **0.2682** | **0.2805** |
| | Avg. CPU (s) | 10.5132 | 9.6930 | 10.0355 |
| | Rate of success (%) | 24 | 28 | 20 |
| PSO | Fuel cost ($/h) | **970.4563** | 1.0885e+03 | **980.2773** |
| | Voltage deviation (pu) | 0.3092 | **0.1566** | **0.1759** |
| | Avg. CPU (s) | 18.1719 | 18.1610 | 18.7498 |
| | Rate of success (%) | 96 | 98 | 98 |
| HPSO-DE | Fuel cost ($/h) | **965.3996** | 1.1536e+03 | **974.8435** |
| | Voltage deviation (pu) | 0.4246 | **0.1386** | **0.1545** |
| | Avg. CPU (s) | 28.1485 | 26.5077 | 27.1863 |
| | Rate of success (%) | 100 | 100 | 100 |

**Table 41: The best result for line 4-6 outage case with valve point loading effects of fuel cost and voltage deviation**

| Method | | Min. fuel cost | Min. voltage deviation | Min. combined fuel cost and voltage deviation |
|---|---|---|---|---|
| DE | Fuel cost ($/h) | **1.0086e+03** | 1.0766e+03 | **988.6897** |
| | Voltage deviation (pu) | 0.2892 | **0.3026** | **0.5128** |
| | Avg. CPU (s) | 9.7663 | 9.8203 | 9.9120 |
| | Rate of success (%) | 16 | 14 | 20 |
| PSO | Fuel cost ($/h) | **926.1876** | 1.0857e+03 | **927.1626** |
| | Voltage deviation (pu) | 0.3299 | **0.1541** | **0.1810** |
| | Avg. CPU (s) | 19.4090 | 19.0683 | 18.7922 |
| | Rate of success (%) | 96 | 94 | 86 |
| HPSO-DE | Fuel cost ($/h) | **922.5569** | 1.1678e+03 | **926.1991** |
| | Voltage deviation (pu) | 0.3084 | **0.1379** | **0.1868** |
| | Avg. CPU (s) | 27.1333 | 26.5935 | 29.1853 |
| | Rate of success (%) | 96 | 98 | 98 |

The convergence characteristics of DE, PSO, and HPSO-DE methods for the problem with the combined objective for the five outage cases are given in Tables 31 to 35 and the successful rate from these methods from many independent runs is also given in Figure 36. For the convergence process, the value of fitness function from these methods can reach a stable state in early iterations. Therefore, the proposed HPSO-DE method is also very effective for dealing with the non-convex problem with two objectives of fuel cost and voltage deviation for different outage cases.

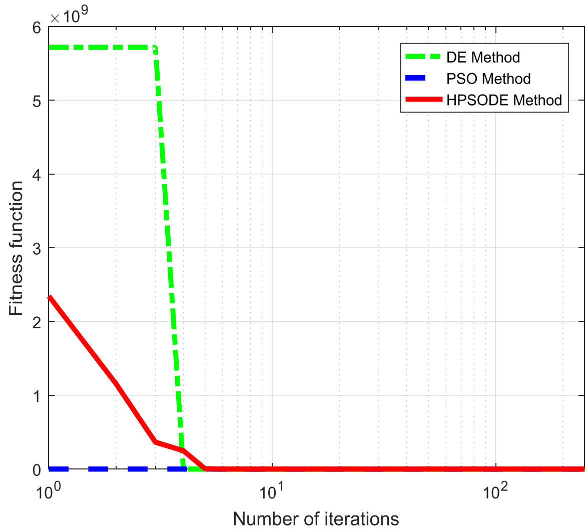


**Fig. 31: Convergence characteristic of DE, PSO, and HPSO-DE for line 1-2 outage case with valve point loading effects of fuel cost and voltage deviation.**

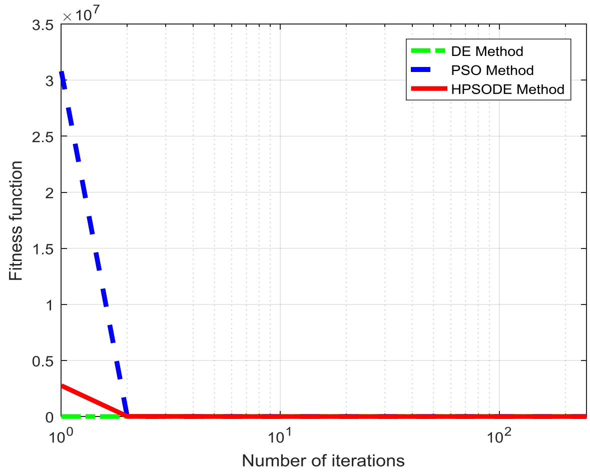


**Fig. 32: Convergence characteristic of DE, PSO, and HPSO-DE for line 1-3 outage case with valve point loading effects of fuel cost and voltage deviation.**

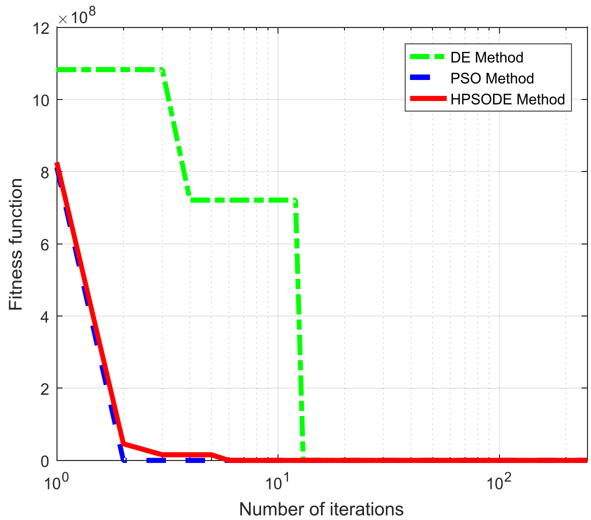


**Fig. 33: Convergence characteristic of DE, PSO, and HPSO-DE for line 3-4 outage case with quadratic fuel cost and voltage deviation.**

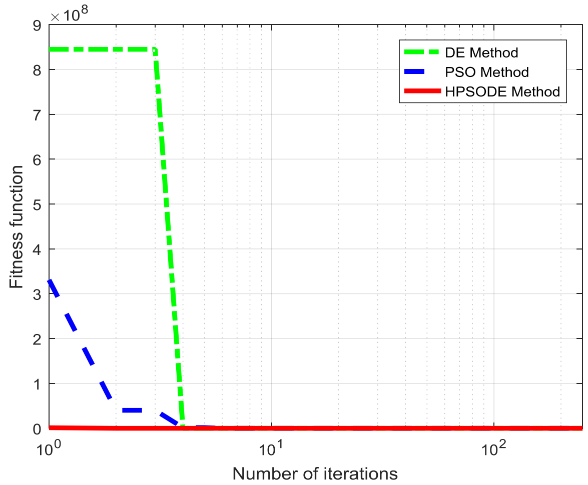


**Fig. 34: Convergence characteristic of DE, PSO, and HPSO-DE for line 2-5 outage case with valve point loading effects of fuel cost and voltage deviation.**

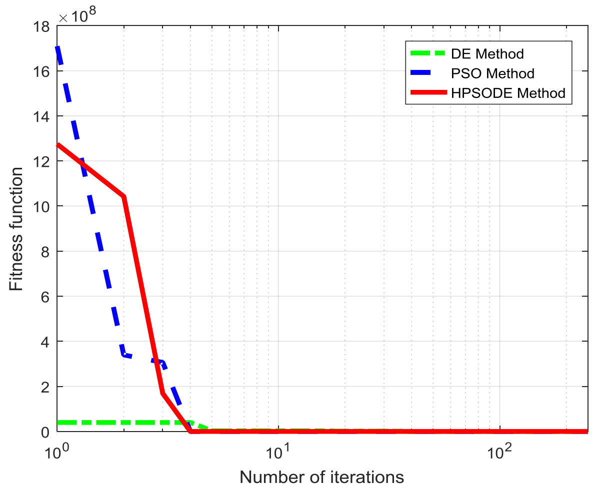


**Fig. 35: Convergence characteristic of DE, PSO, and HPSO-DE for line 4-6 outage case with valve point loading effects of fuel cost and voltage deviation.**

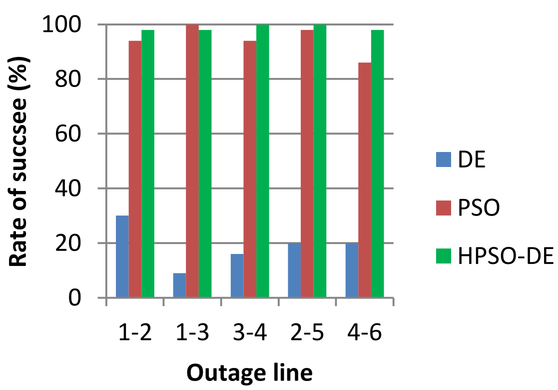


**Fig. 36: Successful rate of DE, PSO, and HPSO-DE for outage cases with combined valve point loading effects of fuel cost and voltage deviation..**

## 5. CONCLUSION

In this paper, the proposed HPSO-DE method has been effectively implemented for solving a very complex MO-SCOARPD problem in power systems. The considered problem is a non-linear large-scale problem of multiple objectives for both normal and outage cases satisfying several constraints. This paper is a real challenge for solution methods to deal with it. The proposed HPSO-DE is hybrid between PSO and DE methods to utilize the advantages of each method for effectively dealing with large-scale and complex optimization problems. The proposed method has been tested on the IEEE 30-bus system for many cases. The considered test cases on the system include quadratic cost and valve point loading effects of fuel cost in different two-objective cases consisting of fuel cost and power losses, fuel cost and stability index, and fuel cost and voltage deviation associated with one of five most serve outage lines. The results from the test cases have indicated that the proposed HPSO-DE method is more effective than both DE and PSO methods with better solution quality with a trade-off for computational time. Therefore, the proposed HPSO-DE method can be very favorable method for solving the MO-SCOARPD problem as well as other large-scale and complex problems in power systems.

## ACKNOWLEDGMENT

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## APPENDIX

The power flow solution for the base case and transmission limits of the IEEE 30-bus system are given in Table A1.

**Table A1: Power flow solution for the IEEE 30-bus system in the base case**

| Line No. | From | To | Line flow (MVA) | Line flow limit (MVA) |
|---|---|---|---|---|
| 1 | 1 | 2 | 175.0588 | 180 |
| 2 | 1 | 3 | 87.7545 | 130 |
| 3 | 2 | 4 | 43.9103 | 65 |
| 4 | 3 | 4 | 82.2323 | 130 |
| 5 | 2 | 5 | 82.4083 | 130 |
| 6 | 2 | 6 | 60.3956 | 65 |
| 7 | 4 | 6 | 73.8616 | 90 |
| 8 | 5 | 7 | 19.8974 | 130 |
| 9 | 6 | 7 | 38.2334 | 130 |
| 10 | 6 | 8 | 30.4264 | 32 |
| 11 | 6 | 9 | 29.3751 | 65 |
| 12 | 6 | 10 | 15.8775 | 32 |
| 13 | 9 | 11 | 16.0574 | 65 |
| 14 | 9 | 10 | 28.3384 | 65 |
| 15 | 4 | 12 | 46.4832 | 65 |
| 16 | 12 | 13 | 10.4507 | 65 |
| 17 | 12 | 14 | 8.2160 | 32 |
| 18 | 12 | 15 | 19.1368 | 32 |
| 19 | 12 | 16 | 7.9804 | 32 |
| 20 | 14 | 15 | 1.7098 | 16 |
| 21 | 16 | 17 | 3.9594 | 16 |
| 22 | 15 | 18 | 6.2247 | 16 |
| 23 | 18 | 19 | 2.8459 | 16 |
| 24 | 19 | 20 | 7.3125 | 32 |
| 25 | 10 | 20 | 9.7580 | 32 |
| 26 | 10 | 17 | 6.9315 | 32 |
| 27 | 10 | 21 | 18.6923 | 32 |
| 28 | 10 | 22 | 8.8994 | 32 |
| 29 | 21 | 22 | 2.3194 | 32 |
| 30 | 15 | 23 | 5.8147 | 16 |
| 31 | 22 | 24 | 6.5049 | 16 |
| 32 | 23 | 24 | 2.1916 | 16 |
| 33 | 24 | 25 | 2.3476 | 16 |
| 34 | 25 | 26 | 4.2621 | 16 |
| 35 | 25 | 27 | 4.8051 | 16 |
| 36 | 28 | 27 | 18.7576 | 65 |
| 37 | 27 | 29 | 6.4110 | 16 |
| 38 | 27 | 30 | 7.2843 | 16 |
| 39 | 29 | 30 | 3.7529 | 16 |
| 40 | 8 | 28 | 3.8422 | 32 |
| 41 | 6 | 28 | 18.6739 | 32 |