Self-Learning Cuckoo Search Algorithm for Optimal Power Flow Considering Tie-Line Constraints in Large-Scale Systems

Khai Phuc Nguyen* and Goro Fujita

Abstract— *This study proposes the Self-Learning Cuckoo search algorithm (SLCSA) and applies it for solving optimal power flow problems in large-scale power systems. The proposed method is an improvement of the Cuckoo search algorithm by employing a new strategy to focus Cuckoo eggs on the global optima. Cuckoo eggs have to learn and modify themselves to enhance their performance. The learning strategy of Cuckoo eggs is also controlled by a learning factor to prevent the search engine falling into local optima. The proposed method is applied for solving optimal power flow problems to figure the effectiveness out. The aim of the problem is to determine the minimized fuel cost while satisfying equal and unequal operating constraints of elements. The proposed SLCSA is also evaluated the problem on three IEEE 57-, 118- and 300-bus systems. According to numerical results, the proposed method is more efficient than the conventional Cuckoo search algorithm and other compared algorithms in literature.*

Keywords— **Cuckoo search algorithm, optimal power flow, teaching-learning based optimization, tie-line constraints.**

1. INTRODUCTION

Optimal power flow (OPF) is a conventional and useful tool to analyze the system. This problem focuses on controlling the power flow to minimize the total operation costs of the power system. The OPF is really a non-convex problem. because its controlled variables consist of continuous discrete or binary values. Nodal voltage and generating power are usually continuous variables, while tap changers of transformers or shunt capacitors can be discrete or binary values. On another hand, the solution of the OPF has to satisfy many operating constraints to keep the power system working in stable. Some frequent constraints needed to be handled are the balance of real and reactive powers, limitation of equipments, for instance: generators, transformers, transmission lines... In addition, when the power system is much more interconnected, the OPF is also more complicated.

In literature, many proposed methods are applied to solve the OPF problems. Since 1973, O. Alsac and B. Scott employed the gradient method to solve the problem on the 30-bus system [1], they also considered the system in normal case and in contingent case. Later works, Yuryevich J. and Wong K. P. proposed the OPF problems considering different fuel cost functions and evaluated it on the 30-bus system by the Evolutionary Programming (EP) [2]. Since the development of computer science, heuristic methods has skyrocketed to employ for the OPF problems and the scale of the problem is also expended. In 2012, Duman S. et al. solved the optimal power flow problem on the 57-bus system by the Gravitational Search Algorithm (GSA). On another hand, Bouchekara, H.R.E.H et al. proposed the Teaching-learning based optimization (TLBO) for the problem on the 118-bus system [3]. However, they neglected the controlled VAR compensators on the evaluated case studies. As an expansion of the OPF problem, R.H. Liang et al. proposed the optimal power flow combining with the emission of thermal units and solved it by the Fuzzy based hybrid Particles Swarm optimization [4]. All mentioned methods have been successful in solving the OPF problems with various types of objective functions and scales of systems. However, most of case studies have been evaluated on the 118-bus or smaller systems. Hence, the require to develop a powerful computation tool to apply for large-scale systems continues increasingly.

Since 2009, the Cuckoo Search Algorithm developed by Yang and Deb succeeds in solving many engineering problems [5]. For example, Gandomi A. H. et al. employed the CSA to solve 12 structural problems [6]. The CSA is also used to give the optimal parameters for milling operations [7]. In the power system, many applications has employed the CSA. For instance, V. N. Dieu et al. applied the CSA for the non-convex economic dispatch [8], or Ahmed, J., and Salam, Z. used the CSA to give the solution for a maximum power point tracking of photo-voltaic systems [9]. A deep survey made by Civicioglu, P. and Besdok, E. on 50 different benchmark functions shows that the CSA is better than the Particle Swarm Optimization and the Artificial Bee Colony algorithms [10]. However, the applications of the CSA for the OPF problems have been few.

In this study, we introduce the Self-Learning Cuckoo Search Algorithm (SLCSA) to evaluate the OPF problems in large-scale systems. According to our experiments, the conventional CSA is not effective on OPF problems. Thus, the proposed SLCSA leads Cuckoo eggs to follow the better solutions. The proposed SLCSA

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employs a learning factor p_l to control the rate of Cuckoo eggs following other solutions. In order to investigate the effects of the learning factor p_l, the proposed SLCSA has been evaluated on three IEEE standard 57-, 118- and 300-bus systems. Furthermore, the proposed SLCSA is also tested on various values of the learning factor p_l to identify its effective range. Numerical results on various systems show that the proposed SLCSA is totally better than the conventional CSA and others in literature.

This paper has been divided into six sections. The second section gives the formulas of the optimal power flow problem. The proposed SLCSA has been discussed in the third section. The next section is the implementation of the proposed SLCSA including its overall procedure. Numerical results are given in the fifth section, and the final is the conclusion and future works.

2. PROBLEM FORMULATION

2.1 Fitness function

The essential purpose of the optimal power flow is to minimize total fuel cost of generating units while satisfying operating constraints and limitations of installed elements on the power system. In this study, the fuel cost function is represented by the quadratic function of generating real power. Generally, the mathematical formula and the fuel cost function of the OPF problem as follows:

$$\min F(x,u) \tag{1}$$

$$FC\left(P_i^G\right) = a + b.P_i^G + c\left(P_i^G\right)^2 \tag{2}$$

subject to:

$$g(x,u) = 0 \tag{3}$$

$$h(x,u) \le 0 \tag{4}$$

2.2 Operational constraints

2.2.1 Power balance constraint

As the primary constraint of operating the electric system, both of generating real and reactive powers has to satisfy load powers. This constraint is represented by the equal constraint $g(x,u)$ in the general formulas. The power balance constraints are given by:

$$P_i^G - P_i^D = V_i \sum_{i=1}^{N_b} \left[V_j \left[G_{ij} \cos\left(\delta_i - \delta_j\right) + B_{ij} \sin\left(\delta_i - \delta_j\right) \right] \right] \tag{5}$$

$$Q_i^G - Q_i^D = V_i \sum_{i=1}^{N_b} \left[V_j \left[G_{ij} \sin\left(\delta_i - \delta_j\right) - B_{ij} \sin\left(\delta_i - \delta_j\right) \right] \right] \tag{6}$$

2.2.2 Limited constraints of generators

In order to keep generators work in stable, the terminal voltage V_i^G and generating powers of a generator have to be in a range as follows:

$$V_{i,\min}^G \le V_i^G \le V_{i,\max}^G \tag{7}$$

$$P_{i,\min}^G \le P_i^G \le P_{i,\max}^G \tag{8}$$

$$Q_{i,\min}^G \le Q_i^G \le Q_{i,\max}^G \tag{9}$$

2.2.3 Shunt-VAR compensators capacity

Each shunt-VAR compensator has a limit to inject/absorb reactive power Q_i^C into the system as follow:

$$Q_{i,\min}^C \le Q_i^C \le Q_{i,\max}^C \tag{10}$$

2.2.4 Limitation of tap changers of transformers

The tap changer of a transformer only works in restricted upper and lower limits as shown below:

$$V_{i,\min}^T \le V_i^T \le V_{i,\max}^T \tag{11}$$

2.2.5 Limitation of voltages at load buses

In order to guarantee the quality of system, magnitude voltages at loads must be maintained around nominal values.

$$V_{i,\min}^L \le V_i^L \le V_{i,\max}^L \tag{12}$$

2.2.6 Capacity of transmission lines

All transmission lines have to satisfy limited thermal condition represented by an upper bound as follow:

$$\left|S_{li}\right| \le S_{li}^{\max} \tag{13}$$

3. SELF-LEARNING CUCKOO SEARCH ALGORITHM

3.1 Cuckoo search Algorithm

Since 2009, Yang and Deb have developed the Cuckoo search algorithm, a few-parameter heuristic optimization technique [5, 11]. In nature, Cuckoo species are very lazy to build their own nests and raise up their children. The Cuckoo bird usually lays its eggs into a neighbor's nest by learning and adjusts the pattern and color. However, if the neighbor detects the Cuckoo bird's eggs, she will abandon her own nest.

Basing on the behavior of Cuckoo species, the original Cuckoo search algorithm proposed by Yang and Deb includes two probability-generating stages. In the first stage, the Lévy flight generates random Cuckoo eggs, and then these eggs will be laid into the neighbors nests to create new solutions. On another hand, the second stage describes the action of the host birds to abandon Cuckoo eggs in their nests probably.

3.2 Proposed Self-learning Cuckoo Search Algorithm

The Self-learning Cuckoo search algorithm proposes an improvement to complement the behavior of Cuckoo eggs. The proposed strategy helps the Cuckoo eggs modify themselves and avoid being abandoned by the host bird. The Cuckoo eggs learn from other better solutions and modify to follow them. Following equations describe the proposed idea:

$$X_i^{t+1} = X_i^t + rand().\Delta X_i^{improve} \tag{14}$$

$$\Delta X_i^{improve} = \begin{cases} X_i - X_j, \text{if } f(X_i) < f(X_j) \\ X_j - X_i, \text{ otherwise} \end{cases} \quad (15)$$

The proposed process gives a gradient to let Cuckoo eggs follow the better eggs and helps the search engine converge faster. We employ a learning factor p_l to control the convergence of search engine. If the learning factor p_l is near to 1, the proposed method will converge faster but it may fall into local solutions. If the learning factor p_l is near to 0, the proposed method will become the conventional Cuckoo search algorithm. In this research, the effectiveness of the factor p_l has been investigated.

Cuckoo Search Algorithm is a modern and powerful optimization technique and is the best choice for the problems on small dimension as [13]. However, for the large-scale problems, Cuckoo Search Algorithm can be worse than other methods as the numerical results. The proposed SLCSA forces solutions to follow the better ones and improves the search engine.

4. IMPLEMENTATION OF THE SLCSA FOR THE OPTIMAL POWER FLOW

4.1 Controlled and dependent variables:

Controlled variables x consists of generating power of generators P_i^G, nodal voltages of generators V_i^G, injected reactive powers of shunt VAR compensators Q_i^C and positions of tap changers of transformers V_i^T. In addition, dependent variables u include output of the generator at the slack bus P_1^G, reactive powers of generators Q_i^G, voltages at load buses V_i^L and apparent powers of transmission lines S_i.

$$x = \left[P_2^G ... P_{N_g}^G, V_1^G ... V_{N_g}^G, Q_1^C ... Q_{N_c}^C, V_1^T ... V_{N_t}^T \right] \quad (16)$$

$$u = \left[P_1^G, Q_1^G ... Q_{N_g}^G, V_1^L ... V_{N_l}^L, S_1 ... S_{N_{br}} \right] \quad (17)$$

4.2 Fitness function

According to the objective of OPF problem, the fitness function $F(x,u)$ is a combination of the fuel cost function $FC(P_i^G)$ and operational constraints. The limitations of controllable variables, e.g. (7), (8), (10), (11), are self-modified during the optimizing process. The limited function $X^{\lim}(x)$ written as (19) combines to the fitness function via penalties factors to handle limitations of dependent variables, e.g. (9), (12), (13). The penalty factors K_P,K_Q,K_S are set at 1000. The penalty factor K_V is set at 10^6 for small scale systems and at 10^{10} for the 300-bus system. For the power balance constraints (5), (6), the power flow algorithm completely satisfies them when calculating. Finally, the fitness function can be written as follows:

$$\begin{aligned} F(x,u) &= \sum_{i=1}^{N_g} FC_i\left(P_i^G\right) + K_P\left(P_{slack}^G - P_{slack}^{\lim}\right)^2 + \\ &+ K_Q \sum_{i=1}^{N_g}\left(Q_i^G - Q_i^{\lim}\right)^2 + K_S \sum_{i=1}^{N_{br}}\left(\left|S_{li}\right| - S_{li}^{\max}\right)^2 + \\ &+ K_V \sum_{i=1}^{N_b}\left(V_i^L - V_i^{\lim}\right)^2 \end{aligned} \quad (18)$$

$$X^{\lim}(x) = \begin{cases} x_{\max}, \text{if } x > x_{\max} \\ x, \text{if } x_{\min} \le x \le x_{\max} \\ x_{\min}, \text{if } x < x_{\min} \end{cases} \quad (19)$$

4.3 Overall procedure:

The overall procedure for the implementation of the SLCSA to solve the optimal power flow is following:

Step 1: Choose controlling parameters for algorithm, which include the probability of discovering Cuckoo eggs p_a, the learning factor p_l, the number of nests *NP* and the number of iterations *Itmax*.

Step 2: Create randomly initial nests *X* and evaluate value of the fitness function $F(x,u)$ in (18).

Step 3: Determine the best value of the fitness function F_{best} and the best nest X_{best}. Set the iteration counter *it*=1.

Step 4: Create Cuckoo eggs via Lévy flight and the new nests X_{new}, modify the eggs that violate the limitations.

Step 5: Evaluate the fitness function F_{new} for new nests. Update the solutions *X*, the best value of fitness function F_{best} and the best nest X_{best}.

Step 6: Randomly decide either discovering alien eggs or improving alien eggs. Modify the eggs that violate the limitations.

Step 7: Once again, evaluate the fitness function F_{new} for new nests X_{new}. Update the current nests *X*, the best value of fitness function F_{best} and the best nest X_{best}.

Step 8: Check if the iteration counter *it* is lower than the maximum iteration *Itmax*, increase *it* and return step 4. Otherwise, stop.

5. NUMERICAL RESULTS

The proposed Self-learning Cuckoo search algorithm is evaluated on the IEEE 57-, 118- and 300-bus systems to solve the optimal power flow problems. In the 57- and 118-bus systems, the proposed method are compared with other algorithms in literature; for the 300-bus system, all compared methods are programmed and run on a personal computer with an Intel 3.0GHz Core 2Duo processor and 4Gb RAM. Numerical results of each benchmark are obtained through 50 independent trials in order to compared the effectiveness of the proposed Self-learning Cuckoo search algorithm. The power flow of each benchmark is calculated by the Newton-Raphson method via the MATPOWER toolbox [14].

The optimal power flow is a complex and non-convex problem that combines various types of controllable variables. In the 57- and 118-bus systems, the real powers P_i^G and the terminal voltages V_i^G of generators and the tap changers of transformer V_i^T are continuous values, and the capacities of transmission lines are neglected. In the 300-bus system, tap changers of transformer V_i^T are discrete numbers with $0.01 p.u.$ step size and tie-line constraints are obtained. Reactive powers of shunt-VAR compensators Q_i^C are continuous values for all case studies. The total number of controlled variables and set parameters of the SLCSA for each case study are given in Tab. 1.

Table 1: Number of controlled variables and setting parameters of the SLCSA for evaluated benchmarks

Case study	Total of variables	Factor p_a	Factor p_l	Number of nests *NP*	Number of iteration *Itmax*
1	33	0.3	0.7	50	500
2	128	0.1	0.7	50	1000
3	213	0.2	0.8	150	1000

5.1 Case study 1: IEEE 57-bus system:

The standard IEEE 57-bus system consists of seven generators, 17 transformers and three shunt capacitors. Among the transformers, two parallel transformers in the line (24,25) are fixed taps and others have tap changers. The bus data, line data, fuel cost coefficients and operational constraints are taken from MATPOWER Toolbox [14]. The maximum reactive power of three capacitors is 30 MVar, and the minimum is zero. The numerical results have been compared with other algorithms in literature such as: Improved Teaching-learning based optimization (ITLBO), Gravitational Search Algorithm (GSA) and Artificial Bee Colony algorithm (ABC).

According to Tab. 2, the conventional CSA is worse than other compared methods on search the global solution. When employing the new strategy, the proposed SLCSA improves the search engine and gives the best solution. The best solution of the proposed method is slightly worse than the ABC; however, the mean value of the fitness function is clearly better. The ITLBO gives the best solution, but it violates the limitation of voltage at load buses as Fig. 1.

Table 2: Comparison of numerical results proposed by the proposed SLCSA and other methods for IEEE 57-bus system with continuous values of capacitors

Methods	Best [$]	Mean [$]	Worst [$]	Std. dev.
GSA [18]	41695.9	-	-	-
ABC [19]	41694	41778.7	41867.9	-
ITLBO [20]	41679.5	-	-	-
CSA	41717.4	41740.4	41765.6	16.3616
SLCSA	**41694.2**	**41707.1**	**41721**	**8.1918**

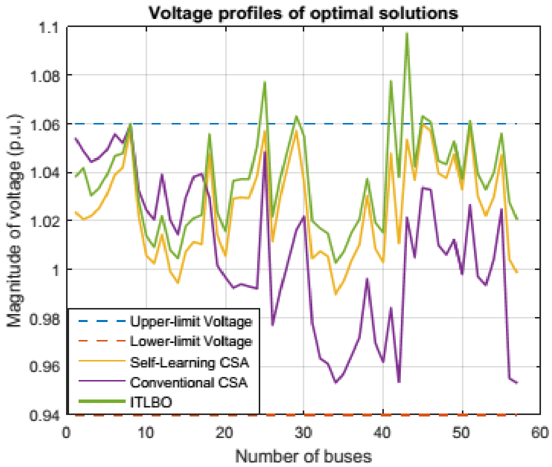

Fig. 1. Examining limits of voltages at load buses the IEEE 57-bus system.

5.2 IEEE 118-bus system:

The IEEE 118-bus system includes 54 generators, 9 transformers with load tap changers and 14 installed shunt VAR compensators. Two of compensators are reactors and the others are capacitors. In this study, we focus on setting up injected reactive powers of capacitors and keep amount of absorbed reactive powers of reactors. The upper and the lower bounds of capacitors are 30MVA and zero, respectively. The upper and limits of magnitude voltages at all buses are 1.1 and 0.95 p.u., respectively. The data of the IEEE 118-bus system, coefficients of fuel cost functions and other operational constraints are also given in MATPOWER Toolbox [14].

The proposed SLCSA has been evaluated on various parameters of the probability p_a and the learning factor p_l to investigate its effectiveness. The conventional CSA only solves the problem successfully when the the probability rate of discovering alien eggs p_a=0.1. When using the learning factor p_l, the search engines has clearly been enhanced. The proposed SLCSA is successful in solving this problem with any setting parameters. However, the SLCSA gives better solutions when the learning factor p_l is over 0.3, and the best performance of the SLCSA is at p_l=0.7.

Table 3: Comparison of numerical results proposed by the proposed SLCSA and other methods for IEEE 118-bus system

Methods	Best solution [$]
SLCSA	**129,536**
CSA	129,746
GSA [22]	129,565
TLBO [3]	129,682
DE [23]	129,582
GWO [23]	129,720

The best solution proposed by SLCSA has been compared with the conventional CSA and other optimization techniques in literature as Tab. 3. Table 3 shows that the proposed method gives better solution and higher performance than both of other methods. On another hand, the TLBO also is better than the conventional CSA on searching global optima. The optimal solution also satisfies limits of voltages at all buses as Fig. 2.

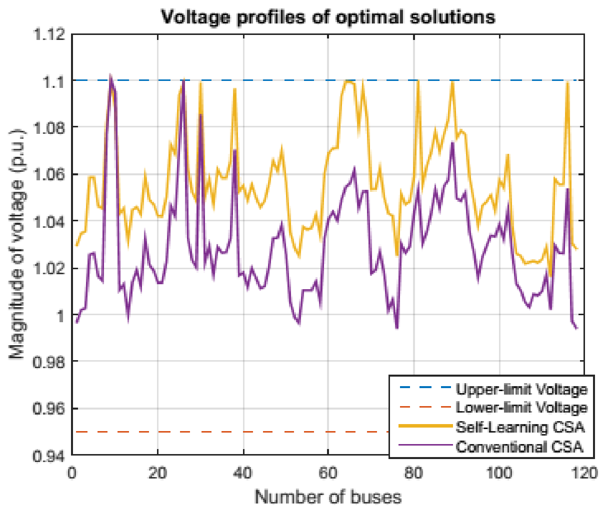

Fig. 2 Examining limits of voltages at load buses the IEEE 118-bus system.

5.3 IEEE 300-bus system:

The last tested system is the huge IEEE 300-bus system, which includes 69 generators and the total of controlled variables is up to 213. Similarly, the data of the IEEE 300-bus system is taken from the MATPOWER Toolbox [14], while the lower bounds of generating real powers and the capacities of transmission lines are conducted from the IEEE testbed [24].

Table 4: Numerical results of the SCLCSA and the conventional CSA for IEEE 300-bus system

Methods	Best [$]	Mean [$]	Worst [$]	Std. dev.
SLCSA	**722,899**	**728,712**	**759,860**	**6,955**
CSA	1,963,015	3,964,877	7,229,361	1,342,516
TLBO	724,166	776,090	1,003,903	96,945

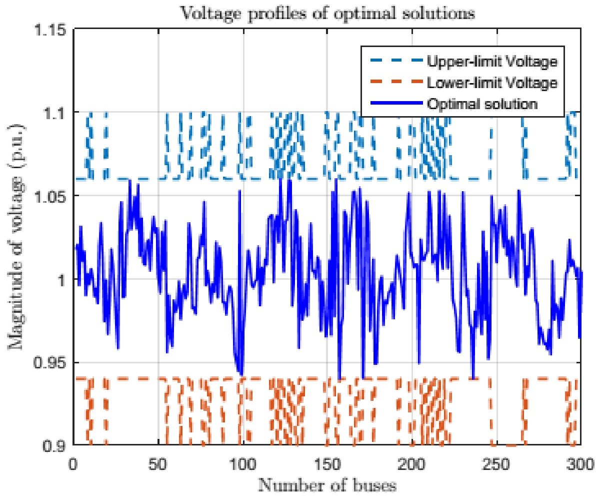

Fig. 3. Voltage profiles of the optimal solution on the IEEE 300-bus system.

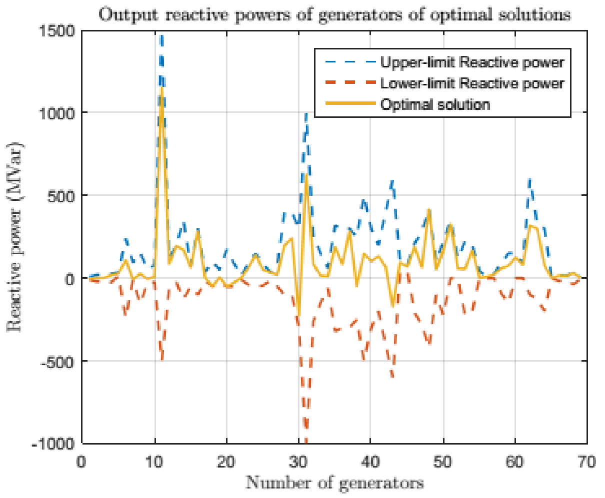

Fig. 4. Generating reactive powers of generators on the IEEE 300-bus system.

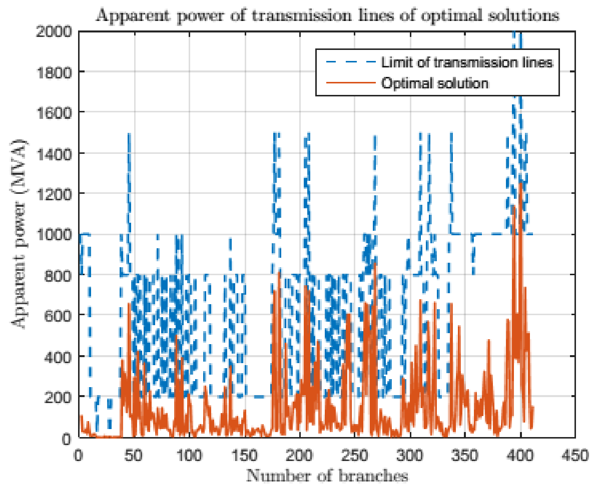

Fig. 5. Apparent power through transmission lines of the optimal solution on the IEEE 300-bus system.

Numerical results in Tab. 4 show that the conventional CSA unsuccessfully solves this problem while the proposed SLCSA succeeds in searching the optimal solution. On the other hand, the Teaching-learning based optimization (TLBO) also gives favorable solutions. However, TLBO has a rate that falls into the local optima. Furthermore, the solution proposed by TLBO is worse than SLCSA's one. The optimal solution is in Appendix, and it also satisfies all of required operating constraints as Fig. 3, 4 and 5.

6. CONCLUSION

The proposed Self-learning Cuckoo search algorithm successfully solves the optimal power flow problems in large-scale power systems. The proposed strategy to enhance Cuckoo eggs is clearly effective. According to the numerical results on four evaluated systems, the SLCSA is much better than the conventional CSA in finding optimal solutions with higher performance. The conventional CSA is unsuccessful in solving the problem in the large-scale 300-bus system, while the proposed SLCSA handles all operating constraints and gives the optimal solution. Comparing with other algorithms in literature, the proposed method is also better than Evolution Programing, Differential Evolution, Gravitation Search Algorithm and Teaching-learning based optimization on the IEEE 57- and 118-bus tested systems. The proposed method also improves the global solutions on the problems, which consist of various types of variables and handle a huge of equal and unequal constraints. Discussing the effectiveness of learning factor p_l, when the factor p_l is over 0.5, the search engine gives better solutions than the lower value. However, when the factor p_l is near to 1.0, the Cuckoo eggs can be too excited and its performance is not good. Thus, we propose the learning factor p_l around 0.8 to give the better solution. On summary, the proposed SLCSA is favorable to non-convex and large-scale problems like the optimal power flow problem. In future, the proposed method should be continued evaluating on various benchmarks to identify its effectiveness on engineering problems.

NOMENCLATURE

$F(x,u)$, $FC(P_i^G)$	the fitness and fuel cost functions, respectively
x, u	controlled and dependent variables, respectively
a, b, c	fuel cost coefficients
P_i^G	output real powers of generators
$g(x,u)$,$h(x,u)$	equal and unequal constraints, respectively
Q_i^G	generating reactive power
P_i^D, Q_i^D	demanded real and reactive powers, respectively
V_i, δ_i	magnitude and angle of voltage, respectively
G_{ij}, B_{ij}	real and imaginary components taken from the admittance matrix, respectively
N_b	number of buses

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APPENDIX

Table A1. Optimal solution for the IEEE 300-bus system

Variables	Solution	Variables	Solution
P_{8}^{G}(MW)	40.3068	V_{8}^{G}(p.u.)	1.0030
P_{10}^{G}(MW)	44.8642	V_{10}^{G}(p.u.)	1.0058
P_{20}^{G}(MW)	44.3019	V_{20}^{G}(p.u.)	0.9991
P_{63}^{G}(MW)	49.5046	V_{63}^{G}(p.u.)	0.9558
P_{76}^{G}(MW)	54.1091	V_{76}^{G}(p.u.)	0.9759
P_{84}^{G}(MW)	373.2180	V_{84}^{G}(p.u.)	1.0234
P_{91}^{G}(MW)	152.0334	V_{91}^{G}(p.u.)	1.0202
P_{92}^{G}(MW)	280.1124	V_{92}^{G}(p.u.)	1.0462
P_{98}^{G}(MW)	87.0508	V_{98}^{G}(p.u.)	0.9965
P_{108}^{G}(MW)	125.4805	V_{108}^{G}(p.u.)	0.9859
P_{119}^{G}(MW)	1867.3113	V_{119}^{G}(p.u.)	1.0527
P_{124}^{G}(MW)	256.5403	V_{124}^{G}(p.u.)	1.0169
P_{125}^{G}(MW)	54.1564	V_{125}^{G}(p.u.)	1.0102
P_{138}^{G}(MW)	31.8129	V_{138}^{G}(p.u.)	1.0384
P_{141}^{G}(MW)	281.7596	V_{141}^{G}(p.u.)	1.0378
P_{143}^{G}(MW)	681.6624	V_{143}^{G}(p.u.)	1.0599
P_{146}^{G}(MW)	91.6161	V_{146}^{G}(p.u.)	1.0348
P_{147}^{G}(MW)	210.4158	V_{147}^{G}(p.u.)	1.0352
P_{149}^{G}(MW)	99.2045	V_{149}^{G}(p.u.)	1.0585
P_{152}^{G}(MW)	322.5976	V_{152}^{G}(p.u.)	1.0409
P_{153}^{G}(MW)	205.7379	V_{153}^{G}(p.u.)	1.0348
P_{156}^{G}(MW)	49.5701	V_{156}^{G}(p.u.)	0.9756
P_{170}^{G}(MW)	187.9672	V_{170}^{G}(p.u.)	0.9655
P_{171}^{G}(MW)	73.1298	V_{171}^{G}(p.u.)	0.9772
P_{176}^{G}(MW)	208.3053	V_{176}^{G}(p.u.)	1.0598
P_{177}^{G}(MW)	90.7954	V_{177}^{G}(p.u.)	1.0132
P_{185}^{G}(MW)	207.9606	V_{185}^{G}(p.u.)	1.0348
P_{186}^{G}(MW)	1174.2478	V_{186}^{G}(p.u.)	1.0521
P_{187}^{G}(MW)	1208.6181	V_{187}^{G}(p.u.)	1.0522

P_{190}^{G}(MW)	487.7060	V_{190}^{G}(p.u.)	1.0544
P_{191}^{G}(MW)	1909.3309	V_{191}^{G}(p.u.)	1.0370
P_{198}^{G}(MW)	452.4902	V_{198}^{G}(p.u.)	1.0119
P_{213}^{G}(MW)	288.7926	V_{213}^{G}(p.u.)	1.0081
P_{220}^{G}(MW)	129.6215	V_{220}^{G}(p.u.)	1.0160
P_{221}^{G}(MW)	499.3277	V_{221}^{G}(p.u.)	1.0125
P_{222}^{G}(MW)	258.6833	V_{222}^{G}(p.u.)	1.0068
P_{227}^{G}(MW)	330.7820	V_{227}^{G}(p.u.)	1.0118
P_{230}^{G}(MW)	360.8058	V_{230}^{G}(p.u.)	1.0165
P_{233}^{G}(MW)	323.1361	V_{233}^{G}(p.u.)	1.0095
P_{236}^{G}(MW)	571.6399	V_{236}^{G}(p.u.)	0.9987
P_{238}^{G}(MW)	242.0424	V_{238}^{G}(p.u.)	1.0161
P_{239}^{G}(MW)	564.0645	V_{239}^{G}(p.u.)	1.0059
P_{241}^{G}(MW)	623.2231	V_{241}^{G}(p.u.)	1.0255
P_{242}^{G}(MW)	176.9308	V_{242}^{G}(p.u.)	1.0063
P_{243}^{G}(MW)	92.2649	V_{243}^{G}(p.u.)	1.0376
P_{7001}^{G}(MW)	440.7553	V_{7001}^{G}(p.u.)	1.0496
P_{7002}^{G} (MW)	575.0587	V_{7002}^{G} (p.u.)	1.0322
P_{7003}^{G} (MW)	1058.4121	V_{7003}^{G} (p.u.)	1.0326
P_{7011}^{G} (MW)	246.8336	V_{7011}^{G} (p.u.)	1.0098
P_{7012}^{G} (MW)	393.7651	V_{7012}^{G} (p.u.)	1.0327
P_{7017}^{G} (MW)	305.5999	V_{7017}^{G} (p.u.)	1.0413
P_{7023}^{G} (MW)	192.6768	V_{7023}^{G} (p.u.)	1.0299
P_{7024}^{G} (MW)	363.7661	V_{7024}^{G} (p.u.)	1.0192
P_{7039}^{G} (MW)	484.5276	V_{7039}^{G} (p.u.)	1.0435
P_{7044}^{G} (MW)	43.9926	V_{7044}^{G} (p.u.)	1.0142

P_{7049}^{G} (MW)	78.5143	V_{7049}^{G} (p.u.)	1.0229
P_{7055}^{G} (MW)	49.3134	V_{7055}^{G} (p.u.)	1.0011
P_{7057}^{G} (MW)	171.5392	V_{7057}^{G} (p.u.)	1.0251
P_{7061}^{G} (MW)	384.3697	V_{7061}^{G} (p.u.)	1.0188
P_{7062}^{G} (MW)	369.2490	V_{7062}^{G} (p.u.)	1.0026
P_{7071}^{G} (MW)	132.6656	V_{7071}^{G} (p.u.)	0.9954
P_{7130}^{G} (MW)	1210.0413	V_{7130}^{G} (p.u.)	1.0530
P_{7139}^{G} (MW)	673.9895	V_{7139}^{G} (p.u.)	1.0402
P_{7166}^{G} (MW)	603.1292	V_{7166}^{G} (p.u.)	1.0182
P_{9002}^{G} (MW)	44.4260	V_{9002}^{G} (p.u.)	0.9907
P_{9051}^{G} (MW)	54.4227	V_{9051}^{G} (p.u.)	1.0050
P_{9053}^{G} (MW)	42.2456	V_{9053}^{G} (p.u.)	1.0076
P_{9054}^{G} (MW)	69.4099	V_{9054}^{G} (p.u.)	1.0113
P_{9055}^{G} (MW)	32.4047	V_{9055}^{G} (p.u.)	1.0069
Q_{117}^{C}(MVar)	253.8616	T_{87-94}(p.u)	0.99
Q_{120}^{C}(MVar)	18.2194	$T_{114-207}$(p.u)	1.01
Q_{154}^{C}(MVar)	17.4322	$T_{116-124}$(p.u)	0.94
Q_{164}^{C}(MVar)	-63.5705	$T_{121-115}$(p.u)	0.99
Q_{166}^{C}(MVar)	-29.1361	$T_{130-131}$(p.u)	1.05
Q_{173}^{C}(MVar)	39.1623	$T_{130-150}$(p.u)	1.06
Q_{179}^{C}(MVar)	44.8799	$T_{132-170}$(p.u)	1.02
Q_{190}^{C}(MVar)	-30.5400	$T_{141-174}$(p.u)	0.97
Q_{231}^{C}(MVar)	-58.3959	$T_{143-144}$(p.u)	0.97
Q_{238}^{C}(MVar)	-36.6317	$T_{143-148}$(p.u)	0.97
Q_{240}^{C} (MVar)	-40.4169	$T_{151-170}$(p.u)	0.99
Q_{248}^{C} (MVar)	18.9005	$T_{153-183}$(p.u)	1.03

Q^{C}_{9003} (MVar)	0.9558	$T_{155-156}$(p.u)	1.04
Q^{C}_{9034} (MVar)	0.9300	$T_{159-117}$(p.u)	1.01
$T_{37-9001}$(p.u)	1.00	$T_{160-124}$(p.u)	1.00
$T_{9001-9006}$(p.u)	0.95	$T_{163-137}$(p.u)	0.93
$T_{9001-9012}$(p.u)	0.99	$T_{164-155}$(p.u)	0.96
$T_{9005-9051}$(p.u)	1.09	$T_{182-139}$(p.u)	1.06
$T_{9005-9052}$(p.u)	0.92	$T_{189-210}$ (p.u)	1.01
$T_{9005-9053}$(p.u)	1.07	$T_{193-196}$ (p.u)	1.04
$T_{9005-9054}$(p.u)	1.06	$T_{195-212}$ (p.u)	0.98
$T_{9005-9055}$(p.u)	1.01	T_{201-69} (p.u)	1.04
$T_{9053-9533}$(p.u)	1.00	$T_{202-211}$ (p.u)	1.02
T_{3-1}(p.u)	1.00	$T_{204-2040}$ (p.u)	1.07
T_{3-2}(p.u)	0.96	$T_{209-198}$ (p.u)	1.03
T_{3-4}(p.u)	0.97	$T_{218-219}$ (p.u)	1.04
T_{7-5}(p.u)	0.94	$T_{229-230}$ (p.u)	0.98
T_{7-6}(p.u)	0.97	$T_{234-236}$ (p.u)	1.03
T_{10-11}(p.u)	1.03	$T_{238-239}$ (p.u)	1.02
T_{12-10}(p.u)	0.98	$T_{119-1190}$ (p.u)	1.07
T_{15-17}(p.u)	0.98	$T_{120-1200}$ (p.u)	0.92
T_{16-15}(p.u)	0.98	$T_{7062-62}$ (p.u)	0.94
T_{21-20}(p.u)	0.94	$T_{7017-17}$ (p.u)	0.98
T_{24-23}(p.u)	1.02	$T_{7039-39}$ (p.u)	0.95
T_{36-35}(p.u)	0.97	$T_{7057-57}$ (p.u)	0.97
T_{45-44}(p.u)	0.94	$T_{7044-44}$ (p.u)	0.96
T_{62-61}(p.u)	0.95	$T_{7055-55}$ (p.u)	0.94
T_{63-64}(p.u)	0.97	$T_{7071-71}$ (p.u)	0.96