Evaluation of the Modified Tustin Friction Compensation in Simulation of the PID Controller System for Rotating-Shaft Dynamic

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Abstract— *Friction has been an important consideration for the analysis of dynamic movement between two or more contacting surfaces in mechanical devices such as automobile internal combustion engine, turbine, agitator, etc. This study aimed to improve the reliability of physical simulation by stabilization of the response from modified Tustin friction input using a proportional-integral-derivative (PID) controller embedded with an Extended Kalman Filter (EKF). Simulation results revealed an improvement in the performance of the PID controller with EKF function compared with the conventional system without friction compensation based on the Ziegler-Nichols tuning principles. The Root-Mean- Deviation (RMD) and Peak Error (P.E) were two important criteria used to evaluate the performance of the PID controller. A reduction in the RMD was documented for the sinusoidal (86.1%), triangular (30.2%) and square (20.0%) input signals. However, when considering P.E, it was observed that the P.E for the sinusoidal signal was reduced by 73.5%, for the square signal it remained the same and for the triangular signal is increased by almost 18.5%.*

Keywords— **Friction, modified tustin friction, PID controller, rotating-shaft.**

1. INTRODUCTION

Friction is usually study in terms of material's life-time, surface *interaction* and wear. It can also be useful in an automobile braking system. However, friction can cause a reduction in the performance of dynamic components which is referred to as frictional loss. For a mechanical system frictional loss is usually observed in the form of vibration, noise and heat. It is also the major cause of energy dissipation in automobile internal combustion engine resulting in an unwanted power loss of 18% [1]. For this reason, it is important for simulation in computer program to account for friction especially when simulating a control system. This procedure is usually referred as friction compensation. Many different models were employed to mathematically describe the friction that occur inside the specific system. The simplest models are the static models consisting of the Coulomb, the viscous and the Stribeck model. The more complicated algorithm for friction models included the Dahl model, the LuGre model and generalized Maxwell slip model (GMS). Another model which represent the speed dependent type of friction is the Tustin friction model developed in 1947 [2]. Most control systems in the industry relied on a proportional–integral–derivative (PID) controller because it is a relatively simple controller [3]. However, it is difficult for the PID controller to fully predict the magnitude of the effect of friction because the physical condition is usually in a form of non-linear mathematical models with various unknown variables, time delays and discontinuities. Many applications with non-linear characteristics suffer from these restrictions including micro-positioning of linear-piezoelectric motors [4], automobile fuel utilization [5] and peristaltic actuator for a linear-servo [6].

Over the years many modifications were made to increase the performance of the PID controller. Almeida et al. demonstrated the effectiveness of implementing a two-control loop system on motion control of a dynamic motor [7]. Barth et al. reported the efficient adaptive algorithm use to derive friction force of a servo system [8]. A feed-forward compensation technique was employed to increase the stability and tracking precision of friction compensation in an optoelectronic control system [9]. Another innovated advancement in the PID controller system is the application of the mind evolutionary algorithm (MEA) which can be used in many different industries [10]. Yu et al. developed a successful robust adaptive tracking system for compensation of friction considering stribeck effect, viscous friction and coulombic friction [11]. Many researches were devoted to enhance the PID controller system for robot [12]. Mitsantisuk et al. reported heat accumulation in sensorless robot system due to extensive friction. Friction compensation was therefore necessary to improve the stability of the robotic system [13].

The objective of this paper is to investigate the modification of Tustin friction compensation technique by using the Extended Kalman Filter (EKF) in comparison with the traditional Ziegler-Nichols methods. A close-loop system compensation controller was used to calculate the magnitude of friction acting on the shaft. Simulation of a PID controller were carried out based on the rotating shaft-pole including friction created on the bearing. Different types of input signal were observed

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including sine signal, triangular signal and square input. These simulating results were evaluated by calculating P.E and root-mean-square deviation (RMD).

2. ALGORITHM MODEL

2.1 Plant model

The performance of the rotating shaft system with tangible friction forces on the bearing was observed as shown in Figure 1.

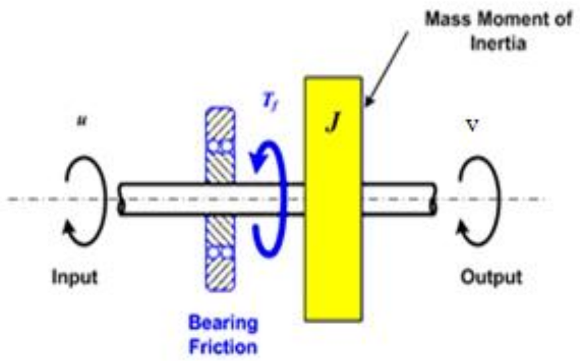

Fig. 1 Schematic diagram of the rotating shaft with friction on bearing.

Mathematic models of the rotating motion can be described as shown from equation 1 to equation 3 below.

$$\sum T = J\alpha \tag{1}$$

$$u - T_f = J\ddot{y} \tag{2}$$

$$v = \int \frac{1}{J}(u - T_f)dt \tag{3}$$

In equation 1-3, *u* represent the torque applied, *v* represents the angular velocity, *J* equaled the moment of inertia, is the angular acceleration and T_f is the friction torque from the bearing. A block diagram expressing the controller setup for the rotating shaft system is shown in Figure 2.

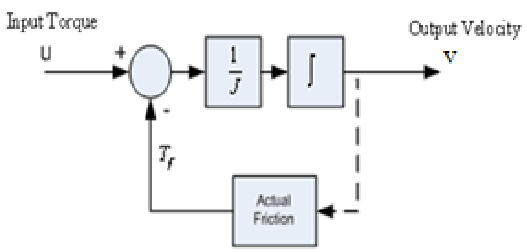

Fig. 2 PID controller diagram regarding the input torque and the output velocity.

The modified Tustin's friction model was employed to find the friction torque of the bearing, which can be given by the equation 4 [14].

$$T_f = \left(c_0 + c_1|v| + c_2^{-\left(\frac{|v|}{c_3}\right)^{c_4}} \right) \times \text{sign}(v) \tag{4}$$

where *v* is the angular velocity, and C_i are constants based on the location of the variables.

2.2 Fundamental of control

According to figure 1, it is observed that when friction was not considered the equation for the transfer function can be expressed as equation 5.

$$G_p(s) = \frac{V(s)}{U(s)} = \frac{1}{Js} \tag{5}$$

This equation represents a first order interaction considering the shaft of the system. A PID controller can be employed to improve the stability of the operating system. The equation that complied with the control law is shown in equation 6.

$$u(t) = K_p e(t) + K_I \int e(t)dt + K_d \frac{de(t)}{dt} \tag{6}$$

In equation 6, *u(t)* is the input from the plant as a function of time, *e* represents the error signal, is the integral gain, is the proportional gain, and is the derivative gain. For the traditional Ziegler-Nichols tuning method, K_p is equaled to 1.2 Nm/rad, K_I is equaled to 3.14 1/s and K_d is equaled to 0.0796 Nm/rad. The damping ratio for this equation is usually lower than 0.7 and the time constant is lower than 1 s considering J or the moment of inertia is equaled to 0.1 kg m^2.

3. FRICTION COMPENSATION FOR PID CONTROLLER

Figure 3 demonstrated a closed-loop system for friction estimation, which contained two sections including a PID controller and the EKF. The signal representing the estimated friction is referred to as $T_f(k)$ which was created by the EKF and then combined with the controlling signal derived in the PID controller.

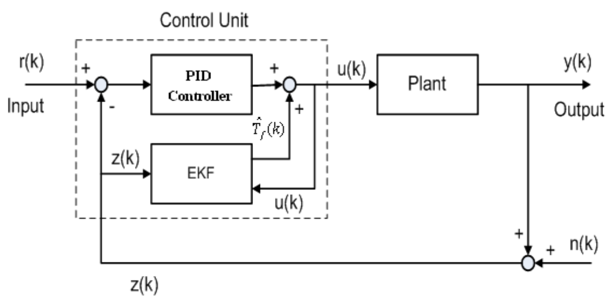

Fig. 3 Friction compensation via a close-loop feedback system.

Friction estimation can be achieved by the EKF through discrete mathematical formula of friction as shown in equation 7 and 8 [15].

$$T_f(k) = a(k)\text{sgn}(v(k)) \tag{7}$$

$$a(k+1) = a(k) + w(k) \tag{8}$$

According to equation 7 and 8, the discrete block diagram for plant is illustrated in figure 4. For this equation the $T_f(k)$ represent the period for sampling, *z(k)* is a measurement signal, *n(k)* is a measurement noise signal, and *w(k)* the system variable (state) are the $x_1(k)$ and $x_3(k)$.

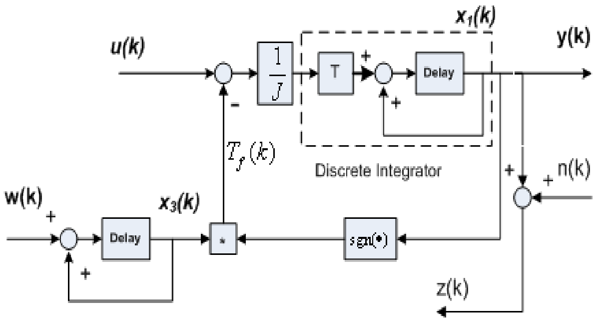

Fig. 4 PID controller for discrete friction compensation maneuver.

The algorithm described in figure 4 are embedded and illustrated in figure 4.

$$X(k+1) = f(X(k), u(k)) + G \times w(k) \quad (9)$$

$$z(k) = h(X(k)) + n(k) \quad (10)$$

Since the system compensation for friction is nonlinear, this research used the Extended Kalman Filter (EKF). The estimated state of the mentioned compensation system for friction is written in equation 11 and 12. These equations are in the form of error covariance matrix.

$$P^{-}(k+1) = A((\hat{X}(k), u(k)) \times P(k) \times A^{T}(\hat{X}(k), u(k)) + G \times Q \times G^{T} \quad (11)$$

$$\hat{X}^{-}(k+1) = f(\hat{X}(k), u(k)) \quad (12)$$

Other equations for the estimated state is described in the equations below.

$$P(k+1) = \{I - K(k+1) \cdot C(\hat{X}^{-}(k+1))\} \cdot P^{-}(k+1) \quad (13)$$

$$\hat{X}(k+1) = \hat{X}^{-}(k+1) + K(k+1) \cdot \{z(k+1) - h(\hat{X}^{-}(k+1))\} \quad (14)$$

$$K(k+1) = P^{-}(k+1) \cdot C^{T}(\hat{X}^{-}(k+1)) \cdot \{C(\hat{X}^{-}(k+1) \cdot P^{-}(k+1) \cdot C^{T}(\hat{X}^{-}(k+1)) + R\}^{-1} \quad (15)$$

The friction estimation equation is shown below.

$$\hat{T}_f(k) = \hat{a}(k)\,\mathrm{sgn}(\hat{v}(k)) \quad (16)$$

$$\hat{a}(k) = \hat{x}_3(k), \; \hat{v}(k) = \hat{x}_1(k) \quad (17)$$

4. COMPUTER SIMULATION RESULTS

The two different algorithms were employed in the computer simulation. These models included the no friction compensation (NFC) and the one with friction compensation (WFC). According to the NFC model, as shown in figure 3, the EKF was not presence. However, the WFC model contain EKF function to demonstrate the different between the two models. Each mathematical model is used to simulate input of different signals included the sinusoidal, triangular and square. The constants used in modified Tustin's friction algorithm used for the simulation included $C_0 = 0.1$, $C_1 = 0.5$, $C_2 = 0.1$, $C_3 = 0.1$, and $C_4 = 2$.

Figure 5 to 10 demonstrated computer simulation of the output response from the two different models (NCF and WFC) and the three different inputs (sinusoidal, triangular, and square).

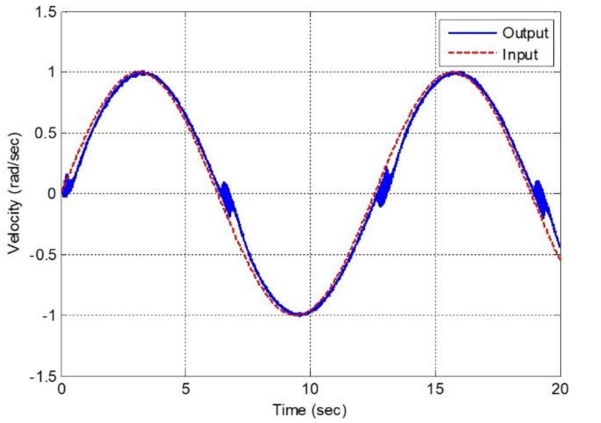

Fig. 5 Output response from a sine input signal when friction compensation was not included (RMD error is equaled to 0.081 and P.E = 0.254).

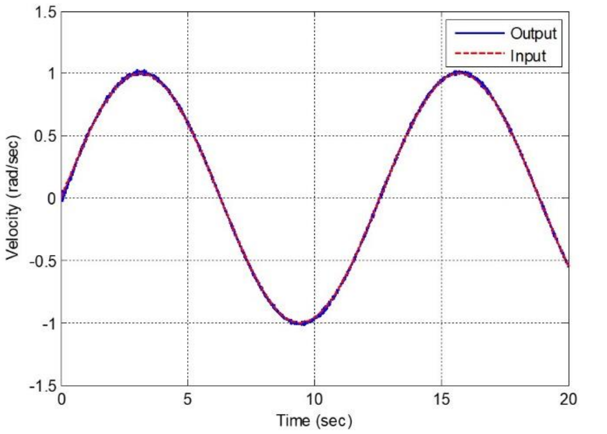

Fig. 6 Output response from a sine input signal when friction compensation was included (RMD error is equaled to 0.009 and P.E = 0.055).

The criteria used to evaluate each input signal for different type of models were the root-mean-square deviation (RMD) and the P.E described in equation 18 and 19.

$$\text{RMD Error} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - r_i)^2} \quad (18)$$

$$\text{Peak Error} = \text{Max.}\left|y_i - r_i\right| \quad (19)$$

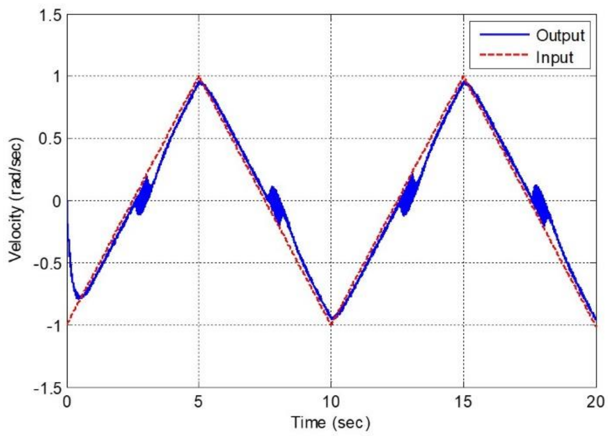

Fig. 7 Output response from a triangular input signal when friction compensation was not included (RMD error is equaled to 0.109 and P.E = 0.242).

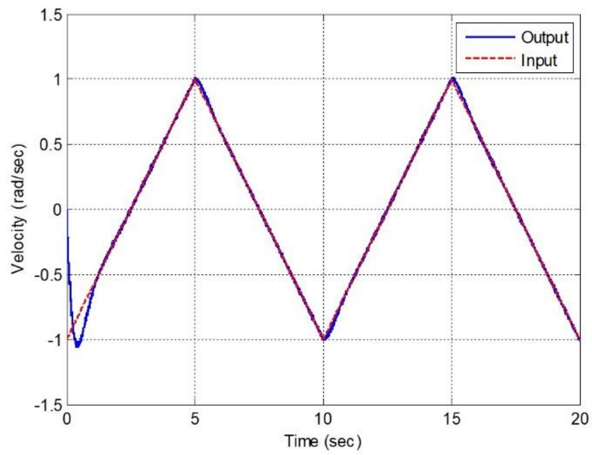

Fig. 8 Output response from a triangular input signal when friction compensation was included (RMD error is equaled to 0.066 and P.E = 0.236).

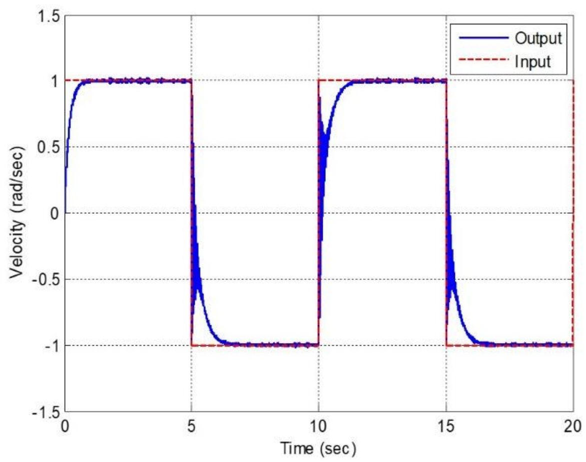

Fig. 9 Output response of a square input signal when friction compensation was not included (RMD error is equaled to 0.211 and P.E = 2.002).

In equation (18) and (19), y_i represent the output signal and r_i represent the input signal. Performance calculated from RMD and P.E for each type of models after simulation were demonstrated in Table 1 and Table 2. The computer simulation results for RMD and the P.E are showed in Table 2.

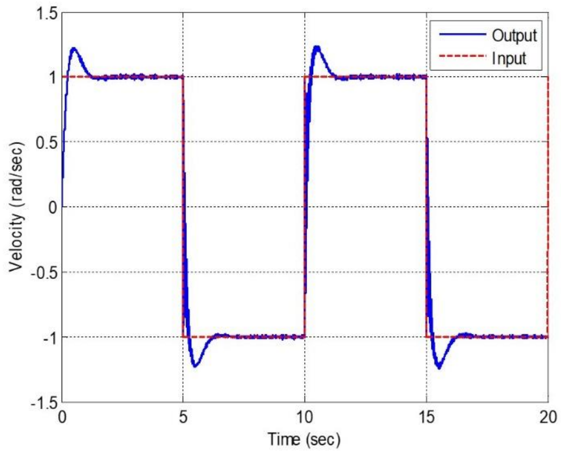

Fig. 10 Output response of a square input signal when friction compensation was included (RMD is equaled to 0.157 and P.E = 2.022).

Table 1 Performance in terms of RMD for NFC and WFC model via the three types of input signal

Input signals	NFC	WFC	Improvement (%)
Sinusoidal	0.065	0.009	86.05%
Triangular	0.094	0.066	30.20%
Square	0.196	0.157	19.88%

Table 2 Performance in terms of P.E for NFC and WFC model via the three types of input signal.

Input signals	NFC	WFC	Improvement (%)
Sinusoidal	0.197	0.052	73.51%
Triangular	0.198	0.235	-18.53%
Square	2.002	2.002	0.00%

5. CONCLUSION

Computer simulated results for the usage of modified Tustin Friction compensation applied in PID controller were investigated in this research. The stability and performance of the rotating shaft system by the EKF was evaluated through three different types of signals including sine, triangular and square input. The criteria for evaluation included root-mean-square error (RMD error) and peak error. Results have shown that the RMD errors declined by 86.1% (sinusoidal input signal), 30.2% (triangular input signal) and 20.0% (square input signal). Peak error was reduced by 73.5% when simulating sinusoidal input signal response. It was also illustrated that peak error did not change significantly for the square input signal after applying the modified Tustin Friction compensation method. However, it was shown that peak error increased by 18.5% when the triangular input signal was simulated.

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REFERENCES

[1] Ali M. K. A. ;Xianjun H. ;Abdelkareem M. A. A. ;Gulzar M.; and Elsheikh A. H. 2018.Novel approach of the graphene nanolubricant for energy saving via anti-friction/wear in automobile engines. *Tribology International* 124:209-29.

[2] Tustin A. 1947.The effects of backlash and of speed-dependent friction on the stability of closed-cycle control systems. *Journal of the Institution of Electrical Engineers - Part IIA: Automatic Regulators and Servo Mechanisms* 94(1):143-51.

[3] Aidan O. Handbook Of Pi And Pid Controller Tuning Rules (3rd Edition): World Scientific Publishing Company; 2009.

[4] Tan K. K. ;Tong Heng L.; and Zhou H. X. 2001.Micro-positioning of linear-piezoelectric motors based on a learning nonlinear PID controller. *IEEE/ASME Transactions on Mechatronics* 6(4):428-36.

[5] Liying Z. ;Ting X. ;Xuejun L. ;Liyan L. ;Pan G.; and Yubao Z., editors. Study on Nonlinear Compensation Control Method for Electronic Throttle Valve. 2018 Chinese Automation Congress (CAC); 2018 30 Nov.-2 Dec. 2018.

[6] Carneiro J. F.; and de Almeida F. G. 2018.Friction characteristics and servo control of a linear peristaltic actuator. *The International Journal of Advanced Manufacturing Technology* 96(5):2117-26.

[7] Carneiro J. F.; and de Almeida F. G. 2012.A high-accuracy trajectory following controller for pneumatic devices. *The International Journal of Advanced Manufacturing Technology* 61(1):253-67.

[8] Zhu Y.; and Barth E. J. 2010.Accurate Sub-Millimeter Servo-Pneumatic Tracking using Model Reference Adaptive Control (MRAC). *International Journal of Fluid Power* 11(2):43-55.

[9] Yang P. ;Zhang Z. ;Zhao J.; and Zhou D. 2014.Improved PID Friction Feed-forward Compensation Control Based on Segment Friction Model *The Open Automation and Control Systems Journal* 6:1620-8

[10] Zhang Y. ;Zhang L.; and Dong Z. 2019.An MEA-Tuning Method for Design of the PID Controller. *Mathematical Problems in Engineering*:1-11.

[11] Wang S. ;Yu H.; and Yu J. 2019.Robust adaptive tracking control for servo mechanisms with continuous friction compensation. *Control Engineering Practice* 87:76-82.

[12] Gaidhane P. J. ;Nigam M. J. ;Kumar A.; and Pradhan P. M. 2018.Design of interval type-2 fuzzy precompensated PID controller applied to two-DOF robotic manipulator with variable payload. *ISA Transactions*.

[13] Prajumkhaiy N.; and Mitsantisuk C. 2016.Sensorless Force Estimation of SCARA Robot System with Friction Compensation. *Procedia Computer Science* 86:120-3.

[14] Armstrong-Hélouvry B. ;Dupont P.; and De Wit C. C. 1994.A survey of models, analysis tools and compensation methods for the control of machines with friction. *Automatica* 30(7):1083-138.

[15] Gomonwattanapanich O. Identification and Compensation of Friction in Mechanical Systems with Stochastic Inputs: University of Texas at Arlington; 2003.