

# **Output Limitation Control for 4DOF Magnetic Bearings**

Ngoc Hoi Le, Tung Lam Nguyen<sup>\*</sup>, Minh Duc Duong, and Quang Dich Nguyen

Abstract— In the currently-used electrical drive systems, bearings are increasingly widely applied owing to its capability of minimizing friction during operation, allowing rotating shafts to smoothly and easily rotate, as well as suffer from almost no mechanical wear. Recently with the development of control techniques and semiconductor components, magnetic bearings, mainly Active Magnetic Bearing (AMB), gradually substitute original bearings in numerous applications, typical high-speed rotation systems, and high-accuracy-demanding systems. In addition, magnetic bearings result in long life without the use of any lubrication system, thereby eliminating the complexity of lubricant systems and encouraging green operations of rotating systems. Therefore, an appropriate control system is essential for an AMB system to utilize its positive characteristics and improve working efficiency. In this study, based on the Lyapunov Stability, a nonlinear Adaptive Backstepping Controller is applied to the AMB system, so that it is possible to maintain the rotor - a rotating shaft - at equilibrium point during operation. Besides, an exponentially convergent nonlinear observer is also introduced into the system to estimate the unmeasured variables - displacement velocity of the rotating shaft since this is unavailable for measurement and feedback as well. The reliability of all methods applied is verified and confirmed by simulation results.

Keywords- Active magnetic bearing, barrier Lyapunov function, nonlinear observer, backstepping.

## 1. INTRODUCTION

The requirement of a supporting mechanism replacing conventional mechanical bearings is essential in modern manufacturing industry. Possessing non-contact and nonlubrication properties, an active magnetic bearing (AMB) has been a potential candidate recently [1][2][3]. The pivot problem of magnetic bearing system is to maintain rotor shaft in the face of the nonlinearity in the system. In recent years, various control approaches have been proposed to AMB systems. In [4] and [5], various linear controller structures for AMBs such as PI/PD, PID, LQ, LQ/LTR have been considers. Genetic algorithm is also used for parameter optimization. In order to improve the operational performance of AMB with linear controller structures, Linear Quadratic Gaussian Controller with Extended Kalman Filter is also applied to AMB in [6]. In [7], four degrees of freedom AMB with gyroscopic impacts is considered, and due to the lack of velocity feedback information, the authors employ robust control with rotor position output feedback. In [8], the state feedback controller is proposed to control the AMB. Five explicit sets of stability constraints had been proposed to guarantee the state feedback loop is not over-designed. Adaptive control with nonlinear observer is also used in [9] to overcome the nonlinear and parametric

uncertainties in AMB. In addition, to attenuate the disturbance and increase the robustness to model uncertainties, sliding mode control is applied to control the AMBs in [10] and [11]. In [12], a robust Takgi-Sugeno model based fuzzy control has been proposed to stabilize the AMB system with parameter uncertainties and voltage saturation. To increase the robustness and improve the performance of the AMB system subject to disturbances, a hybrid control scheme including a feedback Ho controller and disturbance observer-based control is proposed in [13]. In addition, to reject the influence of moving gimbal effect and parameter variation, feedback linearization and extend state observer are used in [14]. Although various problems have been considered in the aforementioned works, there still some limitations such as the motor shaft is considered as a mass point, or the output limitation has not taken into account.

To overcome the above-mentioned limitation, in this paper, we consider the 4DOF Active Magnetic Bearing in which the motor shaft is not a mass point but a cylindrical shaft. Due to only output feedback is available, an exponentially convergent nonlinear observer is used to estimated unmeasured rotor speed. To guarantee the system stability and keep the rotor shaft of AMBs at the equilibrium point during operation, a nonlinear adaptive controller with backstepping design strategy is introduced. In order to avoid multi-input situation, a current switching scheme is utilized. Moreover, the paper introduces barrier term in Lyapunov candidate function to control system output in a limit which bear a practical meaning since it prevents the rotor from coming into contact with the stator.

This paper is organized as follows. First, in the section 2, the model of 4DOF Active Magnetic Bearing is presented. Then, the nonlinear adaptive controller is

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designed using backstepping strategy in Section 3. Simulations is done in Section 4. Finally, Section 5 concludes this paper.

#### 2. SYSTEM MODELLING

Considering a 4DOF Active Magnetic Bearing as in Figure 1, this model can be divided into 2 parts - upper part composed of magnet 1 and 2 while lower counterpart comprising magnet 3 and 4. The upper part includes two identical magnets that are placed opposite to each other, creating two forces  $(F_1, F_2)$  of opposite direction. These two forces keep the rotor remained at equilibrium point between two magnets and can be adjusted by adjusting voltage  $u_1$  and  $u_2$  applied to the coils so as to adjust current  $i_1$  and  $i_2$ . Similarly, the lower part includes two identical magnets that are placed opposite to each other, creating two forces  $(F_3, F_4)$  of opposite direction These two forces keep the rotor remained at equilibrium point between two magnets and can be adjusted by adjusting voltage  $u_3$  and  $u_4$  applied to the coils so as to adjust current  $i_3$  and  $i_4$ . It is supposed that the effect rotational motion is negligible, i.e, it is totally decoupled from transverse motions.



Fig.1. Vertical model of 4-degree-of-freedom AMB.

Now, let's consider the upper part. According to Newton's law:

$$m\ddot{x_u} = F_1 - F_2 , \qquad (1)$$

where  $x_u$  is the displacement of upper part from equilibrium point. Two electromagnetic force  $F_1$  and  $F_2$  can be calculated as [15]:

$$F_1 = \frac{\mu_g N^2 i_1^2 A_g}{4(x_0 - x_u)^2} = \frac{\kappa}{4} \left(\frac{i_1}{x_0 - x_u}\right)^2,\tag{2}$$

$$F_2 = \frac{\mu_g N^2 i_2^2 A_g}{4(x_0 + x_u)^2} = \frac{\kappa}{4} \left(\frac{i_2}{x_0 + x_u}\right)^2,$$
(3)

with  $x_0$  is the nominal position of the rotor at equilibrium point,  $K = \mu_g N^2 A_g$ ,  $\mu_g$  is flux factor of air gap, N is number of coil round,  $A_g$  is the cross-section area. It is noted that, the force and current/displacement equation exhibits a nonlinear relationship which causes difficulites in control design. According to Kirchoff's Voltage Law:

$$u_1 = Ri_1 + L_s \frac{di_1}{dt} + N \frac{d\phi_1}{dt}, \qquad (4)$$

$$u_2 = Ri_2 + L_s \frac{di_2}{dt} + N \frac{d\phi_2}{dt},\tag{5}$$

where R is coil resistor and  $L_s$  is coil inductance.

From (1) - (5) we have the equations representing the AMB dynsmics for upper part is:

$$\begin{pmatrix}
\dot{x}_{u} = v_{u} \\
\dot{v}_{u} = a_{u} \cdot \left(\frac{i_{1}}{x_{0} - x_{u}}\right)^{2} - a_{u} \cdot \left(\frac{i_{2}}{x_{0} + x_{u}}\right)^{2} \\
\frac{di_{1}}{dt} = \frac{2 \cdot (x_{0} - x_{u})}{2L_{s} \cdot (x_{0} - x_{u}) + K} \cdot (u_{1} - R \cdot i_{1} - \frac{K \cdot v_{u} \cdot i_{1}}{2 \cdot (x_{0} - x_{u})^{2}}) \\
\frac{di_{2}}{dt} = \frac{2 \cdot (x_{0} + x_{u})}{2L_{s} \cdot (x_{0} + x_{u}) + K} \cdot (u_{2} - R \cdot i_{2} + \frac{K \cdot v_{u} \cdot i_{2}}{2 \cdot (x_{0} + x_{u})^{2}})$$
(6)

in which  $a_u = \frac{\kappa}{4m}$ . Similarly, the equations representing the AMB motions for lower part is:

$$\begin{cases} \dot{x}_{l} = v_{l} \\ \dot{v}_{l} = a_{l} \cdot \left(\frac{i_{3}}{x_{0} - x_{l}}\right)^{2} - a_{l} \cdot \left(\frac{i_{4}}{x_{0} + x_{l}}\right)^{2} \\ \frac{di_{3}}{dt} = \frac{2 \cdot (x_{0} - x_{l})}{2L_{s} \cdot (x_{0} - x_{l}) + K} \cdot (u_{3} - R \cdot i_{3} - \frac{K \cdot v_{l} \cdot i_{3}}{2 \cdot (x_{0} - x_{l})^{2}}) \\ \frac{di_{4}}{dt} = \frac{2 \cdot (x_{0} + x_{l})}{2L_{s} \cdot (x_{0} + x_{l}) + K} \cdot (u_{4} - R \cdot i_{4} + \frac{K \cdot v_{l} \cdot i_{4}}{2 \cdot (x_{0} + x_{l})^{2}}) \end{cases}$$
(7)

where  $x_l$  is the displacement of lower part from equilibrium point,  $a_l = \frac{K}{4m}$ .

### 3. CONTROLLER DESIGN

Practically, almost every parameter of AMB system can be measured, and the only exception is velocity. Since the feedback of velocity is needed for controller design, an exponentially convergent nonlinear speed observer [9] is introduced into the system as:

$$\hat{\nu}_u = \zeta_1 + A_u \xi_1 + k_u x_u,\tag{8}$$

$$\hat{v}_l = \zeta_2 + A_l \xi_2 + k_l x_l, \tag{9}$$

in which  $\dot{\zeta_1} = -k_u \zeta_1 - k_u^2 x_u$ ,  $\dot{\zeta_2} = -k_l \zeta_2 - k_l^2 x_l$ , and

$$\dot{\xi_1} = -k_u \xi_1 + \frac{i_1^2}{(x_0 - x_u)^2} - \frac{i_2^2}{(x_0 + x_u)^{2\prime}}$$
$$\dot{\xi_2} = -k_l \xi_2 + \frac{i_3^2}{(x_0 - x_l)^2} - \frac{i_4^2}{(x_0 + x_l)^{2\prime}}$$

the initial states  $\zeta_u(0) = \zeta_l(0) = 0$ ,  $\xi_u(0) = \xi_l(0) = 0$ , and  $k_u$ ,  $k_l$  are positive factors.

Substituting (8) into (6) and (9) into (7), the equations representing the system for the upper part can be rewritten as:

$$\begin{pmatrix}
\dot{x}_{u} = v_{u} \\
\dot{v}_{u} = \zeta_{1} + A_{u}\xi_{1} + k_{u}x_{u} \\
\dot{x}_{u} = a_{u} \left[ \frac{i^{2}_{1}}{(x_{0} - x_{u})^{2}} - \frac{i^{2}_{2}}{(x_{0} + x_{u})^{2}} \right] \\
\frac{di_{1}}{dt} = \frac{2.(x_{0} - x_{u})}{2L_{s} \cdot (x_{0} - x_{u}) + K} \cdot (u_{1} - R \cdot i_{1} - \frac{K \cdot v_{u} \cdot i_{1}}{2.(x_{0} - x_{u})^{2}}) \\
\frac{di_{2}}{dt} = \frac{2.(x_{0} + x_{u})}{2L_{s} \cdot (x_{0} + x_{u}) + K} \cdot (u_{2} - R \cdot i_{2} - \frac{K \cdot v_{u} \cdot i_{2}}{2.(x_{0} + x_{u})^{2}})$$
(10)

And for the lower part

$$\begin{pmatrix} \dot{x}_{l} = v_{l} \\ \hat{v}_{l} = \zeta_{2} + A_{l}\xi_{2} + k_{l}x_{l} \\ \ddot{x}_{l} = a_{l} \left[ \frac{i^{2}_{3}}{(x_{0} - x_{l})^{2}} - \frac{i^{2}_{4}}{(x_{0} + x_{l})^{2}} \right]$$

$$\begin{pmatrix} \frac{di_{3}}{dt} = \frac{2.(x_{0} - x_{l})}{2L_{s}.(x_{0} - x_{l}) + K}. (u_{3} - R.i_{3} - \frac{K.v_{l}i_{3}}{2.(x_{0} - x_{l})^{2}}) \\ \frac{di_{4}}{dt} = \frac{2.(x_{0} + x_{l})}{2L_{s}.(x_{0} + x_{l}) + K}. (u_{4} - R.i_{4} - \frac{K.v_{l}i_{4}}{2.(x_{0} + x_{l})^{2}})$$

$$(11)$$

Since in this model, two parts of the system are identical to each other, design controller will be designed for one side, and applied similarly to the other counterpart. The system is in of strict feedback form, the control design is based on backstepping strategy as follows.

Considering  $z_1$  the displacement between rotor and equilibrium point:  $z_1 = x_u$ . The derivative of  $z_1$  can be written:

$$\dot{z}_{1} = \dot{x}_{u} = \zeta_{1} + A_{u}\xi_{1} + k_{u}x_{u} + \tilde{v}_{1}$$
(12)

Practically, the displacement needs being limited to a certain extent, or collision between hardware components can make the system collapsed. Therefore, in this paper, Lyapunov stability is used to design the a controller and the Barrier Lyapunov function is applied in order to avoid this incident.

Here, Barrier Lyapunov function V<sub>1</sub> was used:

$$V_1 = \frac{1}{2} ln \frac{k_b^2}{k_b^2 - z_1^2} + \frac{1}{2k_u d_1} \tilde{v}_1^2$$
<sup>(13)</sup>

in which  $k_b$  is the limitation of displacement  $(-k_b \le z_l \le k_b)$ ,  $d_l$  is a positive factor, and  $\tilde{v}_1 = v_u - \hat{v}_u$ . Different from conventional Lyapunov candidate functions taking a form of quadratic of system errors, the logarithm term in  $V_1$  is used to limit  $z_l$  in a predefined value  $k_b$  [16] and [17].

Taking the derivative of  $V_1$ :

$$\dot{V}_1 = \frac{z_1(\zeta_1 + A_u\xi_1 + k_ux_u + \tilde{v}_1)}{k_b^2 - z_1^2} - \frac{1}{d_1}\tilde{v}_1^2$$
(14)

It is noted that  $\dot{\tilde{v}}_1 = \dot{v}_u - \dot{\tilde{v}}_u = k_u \tilde{v}_1$ .

To achieve  $\dot{V}_1 \leq 0$ , a virtual controller  $\xi_{1\text{ctrl}}$  can be chosen as:

$$\xi_{1_{ctrl}} = \frac{1}{A_u} \left[ -(k_b^2 - z_1^2) k_1 z_1 - d_1 \frac{z_1}{k_b^2 - z_1^2} - \right]$$
(15)

$$k_u x_u - \zeta_1$$

in which k1 and d1 are positive factors. Substituting this into (14), the derivative  $\dot{V}_1$  is obtained as:

$$\dot{V}_1 = -k_1 z_1^2 - d_1 \left[ \frac{z_1}{k_b^2 - z_1^2} - \frac{1}{2d_1} \tilde{v}_1 \right]^2 - \frac{3}{4d_1} \tilde{v}_1^2 \le (16)$$

Here, it is proved that  $V_I$  exponentially converges to zero if  $\xi_I = \xi_{Ictrl}$ . Since the global stability condition is not achieved yet, the control design needs expanding to include the error variable  $z_2$ :

$$z_2 = \xi_1 - \xi_{1ctrl} = \xi_1 - \alpha_1. \tag{17}$$

Then,

$$\dot{z}_2 = \dot{\xi}_1 - \frac{\partial \alpha_1}{\partial z_1} (\hat{v}_1 + \tilde{v}_1) - \frac{\partial \alpha_1}{\partial \zeta_1} \zeta_1$$
(18)

Substituting (17) into (12), the derivative of  $z_1$  can be rewritten as:

$$\dot{z}_1 = A_u z_2 - (k_b^2 - z_1^2) k_1 z_1 - d_1 \cdot \frac{z_1}{k_b^2 - z_1^2} + \tilde{v}_1 \qquad (19)$$

Considering the Barrier Lyapunov function  $V_2$ :

$$V_2 = V_{1nw} + \frac{1}{2}z_2^2 + \frac{1}{2k_u d_2}\tilde{v}^2$$
(20)

in which  $d_2$  is a positive factor and  $\dot{V}_{1nw} = A_u \frac{z_1 z_2}{k_b^2 - z_1^2} + \dot{V}_1$ . Taking the derivative of  $V_2$ , it is obtained that:

$$\dot{V}_{2} = \dot{V}_{1} + A_{u} \frac{z_{1} z_{2}}{k_{b}^{2} - z_{1}^{2}} + z_{2} (\dot{\xi}_{1} - \frac{\partial \alpha_{1}}{\partial z_{1}} (\hat{v}_{1} + \tilde{v}_{1}) - \frac{\partial \alpha_{1}}{\partial \zeta_{1}} \dot{\zeta}_{1}) - \frac{1}{d_{2}} \tilde{v}_{1}^{2}$$
(21)

To render  $\dot{V}_2 \leq 0$ ,  $\dot{\xi}_1$  is defined as:

$$\dot{\xi}_1 = \alpha_2 = -k_2 z_2 - A_u \frac{z_1}{k_b^2 - z_1^2} - d_2 z_2 \left(\frac{\partial \alpha_1}{\partial z_1}\right)^2 + \frac{\partial \alpha_1}{\partial z_1} \hat{\nu}_1 + \frac{\partial \alpha_1}{\partial \zeta_1} \dot{\zeta}_1$$
(22)

in which  $k_2$  and  $d_2$  are positive factors. With this control variable, we have

$$\dot{V}_{2} = \dot{V}_{1} - k_{2} z_{2}^{2} - d_{2} \left[ z_{2} \frac{\partial \alpha_{1}}{\partial z_{1}} + \frac{1}{2d_{2}} \tilde{v}_{1} \right]^{2} - \frac{3}{4d_{2}} \tilde{v}_{1}^{2} \leq 0$$
(23)

From (8), it is clear that  $\dot{\xi}_1$  can be demonstrated as a function of  $i_1$  and  $i_2$ , then  $\alpha_2 = \dot{\xi}_1$  can also be demonstrated as:

$$\alpha_2 = \dot{\xi}_1 = -k_u \xi_1 + \frac{i_1^2}{(x_0 - x_u)^2} - \frac{i_2^2}{(x_0 + x_u)^2}$$
(24)

Setting  $\alpha_u = k_u \xi_1 + \alpha_2$ , then,

$$\alpha_u = \frac{i_1^2}{(x_0 - x_u)^2} - \frac{i_2^2}{(x_0 + x_u)^2} \,. \tag{25}$$

Ultimately, the controlling purpose becomes adjusting the actual current value *i* stick to the desired current value  $i_d$ . As it can be seen, virtual control function  $\xi_1$  is defined based on  $i_1$  and  $i_2$  - current generated by magnet 1 and magnet 2, respectively. However, letting both currents flowing simultaneously leads to high consumption of electricity, as well as make it difficult to design control function for  $u_1$  and  $u_2$  to control the current flowing in both coils. Therefore, in this research, the currents are alternatively switched on and off. To be more specific: when the rotor part is displaced towards magnet 2,  $i_2$  is switched off. Then:  $\alpha_u = \frac{i^2_1}{(x_0 - x_u)^2}$ , and

$$i_{1d} = (x_0 - x_u)\sqrt{\alpha_u}$$
 (26)

Considering  $z_3$  the error variable of  $i_1$  and  $i_{1d}$ :

$$z_3 = i_1 - i_{1d} (27)$$

Taking the derivative:

$$\dot{z}_{3} = \frac{2(x_{0} - x_{u})}{2L_{S}(x_{0} - x_{u}) + K} \left( -Ri_{1} - \frac{ki_{1}}{2(x_{0} - x_{u})^{2}} \dot{x}_{u} + u_{1} \right) - \frac{di_{1d}}{dt}$$
(28)

Setting  $A_1 = \frac{2(x_0 - x_u)}{2L_S(x_0 - x_u) + K}$  and  $B_1 = -\frac{A_1ki_1}{2(x_0 - x_u)^2} - \frac{\partial i_{1d}}{\partial z_1}$ , equation (28) can be rewritten:

$$\dot{z}_{3} = A_{1}Ri_{1} + B_{1}(\hat{v}_{1} + \tilde{v}_{1}) + A_{1}u_{1} - \frac{\partial i_{1d}}{\partial \xi_{1}}\dot{\xi}_{1} - \frac{\partial i_{1d}}{\partial \zeta_{1}}\dot{\zeta}_{1}$$
(29)

Considering the Lyapunov candidate function:

$$V_3 = V_2 + \frac{1}{2}z_3^2 + \frac{1}{2k_u d_3}\tilde{v}_1^2 , \qquad (30)$$

in which  $d_3$  is a positive factor. Derivative of  $V_3$  is obtained as:

$$\dot{V}_{3} = \dot{V}_{2} + z_{3} \left[ -A_{1}Ri_{1} + B_{1}(\hat{v}_{1} + \tilde{v}_{1}) + A_{1}u_{1} - \frac{\partial i_{1d}}{\partial \xi_{1}}\dot{\zeta}_{1} - \frac{\partial i_{1d}}{\partial \zeta_{1}}\dot{\zeta}_{1} \right] - \frac{1}{d_{3}}\tilde{v}_{1}^{2}$$
(31)

At this point, to make  $V_3 \le 0$ , the control function for  $u_1$  can be defined as:

$$u_{1} = \frac{1}{A_{1}} \Big[ A_{1} R i_{1} - B_{1} \hat{v}_{1} - k_{3} z_{3} - d_{3} z_{3} B_{1}^{2} + \frac{\partial i_{1d}}{\partial \xi_{1}} \dot{\xi}_{1} + \frac{\partial i_{1d}}{\partial \zeta_{1}} \dot{\zeta}_{1} \Big]$$
(32)

in which  $k_3$  and  $d_3$  are positive constants. Substituting (32) into (31), we can achieve:

$$\dot{V}_3 = \dot{V}_2 - k_3 z_3^2 - d_3 \left[ z_3 B_1 - \frac{1}{2d_3} \tilde{v}_1 \right]^2 - \frac{3}{4d_3} \tilde{v}_1^2 \le 0$$
(33)

When the rotor part is displaced towards magnet 1,  $i_1$  is switched off, then  $\alpha_u = -\frac{l_2^2}{(x_0+x_u)^2}$ 

Control function for  $u_2$  is obtained as:

$$u_{2} = \frac{1}{A_{2}} \Big[ A_{2}Ri_{2} - B_{2}\hat{v}_{1} - k_{4}z_{4} - d_{4}z_{4}B_{2}^{2} + \frac{\partial i_{2d}}{\partial \xi_{1}}\dot{\xi}_{1} + \frac{\partial i_{2d}}{\partial \zeta_{1}}\dot{\zeta}_{1} \Big]$$
(34)

in which  $k_4$  and  $d_4$  are positive factors, and

$$z_4 = i_2 - i_{2d} = i_2 - (x_0 + x_u)\sqrt{-\alpha_u} .$$
(35)

Here the design procedure for upper part of rotor is completed. To design controller for the lower one, it is possible to identically follow the steps for the upper, and finally obtain:

$$u_{3} = \frac{1}{A_{3}} \Big[ A_{3} R i_{3} - B_{3} \hat{v}_{2} - k_{7} z_{7} - d_{7} z_{7} B_{3}^{2} + \frac{\partial i_{3d}}{\partial \xi_{2}} \dot{\xi}_{2} + \frac{\partial i_{3d}}{\partial \zeta_{2}} \dot{\zeta}_{2} \Big]$$
(36)

$$u_{4} = \frac{1}{A_{4}} \Big[ A_{4}Ri_{4} - B_{4}\hat{v}_{2} - k_{8}z_{8} - d_{8}z_{8}B_{4}^{2} + \frac{\partial i_{4d}}{\partial \xi_{2}} \dot{\xi}_{2} + \frac{\partial i_{4d}}{\partial \zeta_{2}} \dot{\zeta}_{2} \Big]$$
(37)

### 4. SIMULATIONS

With the acquired the AMB equations of motion and control, a simulation model is constructed to assess the stability and reliability of the controller for a 4-DOF AMB system. Parameters of the system as below:

Table 1. System's parameters for simulations

Parameter	Symbol	Value
Rotor weight	m	5kg
Coil rounds	N	400 rounds
Nominal air gap	X <sub>0</sub>	0.004m
Displacement limit	k <sub>b</sub>	0.001m
Initial position of rotor upper part	Xu	0.0004m
Initial position of rotor lower part	x <sub>1</sub>	0.0004m
Coil inductance	Ls	0.001H
Cross-section area	Ag	0.001m
Air gap factor	$\mu_{g}$	1.256 x 10 <sup>-6</sup>
Inertia	Ir	2.90 x 10 <sup>-2</sup> kgm <sup>2</sup>
Distance from shaft center to upper magnet	D <sub>u</sub>	4.166 x 10 <sup>-2</sup> m
Distance from shaft center to lower magnet	D <sub>1</sub>	7.602 x 10 <sup>-2</sup> m

Control gains are selected as below:

- For the upper part:  $k_1=75$ ;  $k_2=250$ ;  $k_3=160$ ;  $k_4=160$ ;  $d_1=9.5 \times 10^{-6}$ ;  $d_2=10^{-10}$ ;  $d_3=3 \times 10^{-15}$ ;  $d_4=3 \times 10^{-15}$ ;  $k_u=7.75$ .

- For the lower part:  $k_5=65$ ;  $k_6=300$ ;  $k_7=180$ ;  $k_8=180$ ;  $d_5=9x10^{-6}$ ;  $d_6=5x10^{-10}$ ;  $d_7=4x10^{-15}$ ;  $d_8=4x10^{-15}$ ;  $k_1=5$ .



Fig. 4. Upper rotor displacement velocity.

At the initial condition, the AMB rotor is attached to one side of the stator and then the control is activated to drive the rotor to the center of the two stators. Fig. 2 shows that after 1s, the control successfully steers the rotor to zero position, consequently the rotor angle also reaches zero degree. The effectiveness of introducing barrier term in selection of the Lyapunov is presented in Fig. 1 where the displacement overshoot is limited below the nominal airgap ( $k_b$ =0.004). Fig. 4 and 5 indicatives that the estimated speed closely track the actual values. Electrical signals in the AMB in Fig. 6, 7, 8, and 9 clearly demonstrate complementarily switching nature of two opposite magnets. In addition, the control input voltage is practical for the AMB under consideration.



Fig. 7. Lower rotor currents in magnet coils.

1

Time (s)

1.5

0.5

ſ

Ó

-0.05

2



Fig. 8. Upper rotor voltages applied to magnet coils.



Fig. 9. Lower rotor voltage applied to magnet coils.

## 5. CONCLUSIONS AND FUTURE WORKS

This study introduces the procedures in designing a nonlinear controller based on the Backstepping method and with the use of Barrier Lyapunov function for Active Magnetic Bearings (AMB). With the feedback of rotor displacement, current and voltage of the magnets, as well as estimated velocity of displacement, the controller succeeds in not only maintaining the shaft at equilibrium point but also restricting movement range during initiation. As can be seen in the simulation results, the usage of the Lyapunov method leads to the effectiveness and robustness of the control system, hence the stability of the AMB system. Besides, the convergent nonlinear observer introduced is proved to bring high-accuracy estimation of velocity, allowing us to overcome the difficulties of unknown parameters in the system. Experimental results are found satisfactory and promising for the active magnetic bearing system.

Nevertheless, there still stand some shortcomings within this study. One is, the controller factors significantly affect the outcomes of the controller, and even minor changes in these factors can lead to completely different results. Therefore, a clear study to discover and tune these coefficients should be found, or else this process is conducted without basing on any basis, hence taking a considerable amount of time. Certain methods such as genetic algorithm or particle swarm might be the potential to handle this difficulty. Another point is that AMB system, as well as the backstepping method, stands a great chance of being applied widely in the real world, meaning that its reliability should be ensured as much as possible. Therefore, some ignored conditions, such as power loss or environmental affection, can be taken into consideration to make the entire system more practical and applicable.

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