



# GA Based Multi-Objective Optimization of Capacitor Sizing and Location in Distribution System with Electric Arc Furnaces

Le Minh Thang, Dinh Trung Hieu, Bach Quoc Khanh\*, Nguyen Khac Phong, Tran Manh Hung

**Abstract**— The problems of voltage quality and power loss in distribution system operation are always interested by utilities world-wide. With regard to the medium-voltage (MV) distribution system, using the fixed reactive power (Var) compensated capacitors is preferable for utilities such as the case of Vietnam because of its favorable advantage of economics. Although there were various methods developed for solving the problem of locating and sizing the capacitors in distribution system for loss reduction, new introductions about the problem modeling would have been still continuing, especially for various case studies of application. In this paper, in response to the rapid growth of industrial loads that causes poor power quality and the requirement of power loss reduction from utilities in Vietnam, a novel modeling of the problem of optimal capacitor placement in MV distribution system is introduced in the form of Genetic Algorithm (GA) based multi-objective optimization. The paper considers the installation of the fixed capacitors in a MV distribution system with the presence of electric arc furnaces that generates harmonics for loss reduction and THD improvement. The paper tests the optimization modeling on the IEEE's 33-bus distribution feeder which features to the real three-phase MV distribution system in Vietnam.

**Keywords**— Medium voltage, distribution system, power loss, voltage quality, fixed capacitor placement, multi-objective optimization, genetic algorithm.

## 1. INTRODUCTION

In operation of distribution systems in Vietnam, power loss is one of critical indices that the utilities have tried to improve for recent years. So far, the reactive power compensation by capacitors to reduce the power loss is one of effective solutions in distribution system. While automatic step-switching capacitor banks are popularly used in low-voltage networks, in the medium-voltage (MV) system, only fixed capacitor is used because of its much lower investment in comparison with the medium-voltage automatic Var compensating devices. However, the placement of the fixed capacitors in distribution system for loss reduction in Vietnam sometimes only bases on operating experience and that may cause issues of voltage quality as the system loading changes widely. Furthermore, the power quality issues increasingly affect adversely the operation of the grid, especially the problems relating with harmonics that result in overloading, damaging equipment as well as increasing power losses [1]. Using capacitors to reduce power losses due to the fundamental and harmonic currents is a larger problem than before.

It's been known that the problem of capacitor location and sizing for fundamental loss reduction is even a problem of nonlinear optimization. Various methods developed to solve this problem have been discussed [2, 3, 4, 5]. Generally, they are categorized in two groups of methods namely analytic and heuristics. [4] has analyzed

and selected genetic algorithms [6] as a method with many advantages such as fast convergence, various applications in the group of intelligent search methods to solve this problem effectively where a number of typical features of the industrial distribution systems in Vietnam are taken into account. When the harmonics is considered, there are also some studies [7, 8, 9, 10] that suggest either the optimal solution or the applied scenarios. [7] uses PSO to solve the optimal math problem, [8] using GA; however, the objective function only considers the fuel consumption of the power generation sources. [9] also uses GA and the penalty functions are included in the objective function to make the constraints possible. However, the model of harmonic calculation still has a number of assumptions. Harmonics simulation was performed by IEEE taskforce [10] and gathered at [11] as a reliable reference for power flow analysis at harmonics frequencies. This paper refers to the method of GA [6] in combination with the harmonic calculation model [10, 11] to select the position and size of the capacitor in an industrial distribution system where the harmonics is assumingly generated by the presence of electric arc furnace (EAF). This paper can also be considered as the development of the paper [9] with the application of GA based multi-objective optimization problem with regard to the industrial distribution network in Vietnam when considering harmonics generated by small-scale business steel production customers in industrial MV distribution systems and using MV fixed compensated capacitors.

## 2. PROBLEM DEFINITION

### 2.1 Objective function

In this paper, the problem of optimizing capacitor size and location is introduced in the form of multi-objective

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optimization [12] where objective function components includes total investment in compensation capacitors (or the total compensated VAR by capacitors in the system) and total annual resulting electric energy loss. The investment of compensation capacitors depends directly on total compensated VAR by capacitors. So the first objective function is defined as follows

$$f_1 = \sum_{i=1}^N Q_{ci} \Rightarrow \text{Min} \quad (1)$$

where,  $Q_{ci}$  is compensated VAR by the fixed capacitor at node  $i$ ,  $i:1 \div N$  and  $N$  is the number of nodes in the system.

The second objective function is the following

$$f_2 = \Delta A \Rightarrow \text{Min} \quad (2)$$

where,  $\Delta A$  is total annual electric energy loss (kWh) of the system;

$$\Delta A = 365 \times (\Delta A^1 + \Delta A^H) \quad (3)$$

where,  $\Delta A^1$  is daily electric energy loss (kWh) of the system for load's fundamental current;  $\Delta A^H$  is daily electric energy loss (kWh) of the system for load's harmonic currents.

The daily electric energy loss  $\Delta A^1$  is determined by the method of typical load profile. This method is normally applicable for small parts of a distribution system such as a distribution feeder outgoing from a distribution substation where the assumption that all loads supplied by the feeder have the similar load profiles, is accepted and the feeder's typical load profile is recorded by the metering device at the substation side of the feeder can be represented (e.g. in Vietnam, the medium voltage feeders outgoing from stepdown 110kV substation).

$$\begin{aligned} \Delta A_m^1 &= \sum_{k=1}^{24} \frac{S_{m,k}^2}{V_n^2} \times R_m = \frac{S_{m,max}^2}{V_n^2} \times R_m \times \sum_{k=1}^{24} \alpha_k^2 \\ &= \Delta P_{m,max}^1 \times \sum_{k=1}^{24} \alpha_k^2 \end{aligned} \quad (4)$$

where:

$S_{m,k}$ ,  $S_{m,max}$ : Power flow (VA) of the line segment  $m$  at the hour  $k$  and its daily peak hour.

$R_m$ : Resistance of the line segment  $m$ .

$V_n$ : Rated voltage.

$\Delta P_{m,max}^1$ : Maximum power loss of line segment  $m$  for load's fundamental current.

$\alpha_k$ : Per-unit power flow of the line segment  $m$  at the hour  $k$ . It's also assumed to be per-unit power of at the hour  $k$  in the daily typical load profile of the system (e.g. Fig.1).

$$\Delta A^1 = \sum_{m=1}^M \Delta A_m^1 \quad (5)$$

where,  $M$  is the number of line segments (branches) in the system. In distribution system with a single source,  $M=N-1$ .

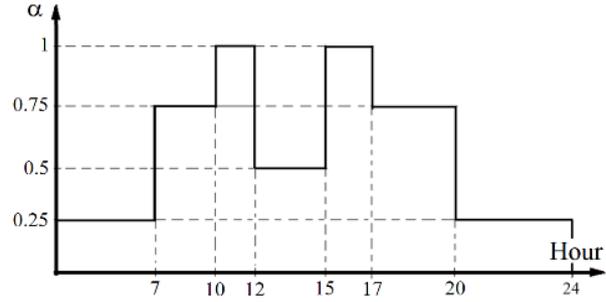


Fig. 1. An example of per-unit daily typical load profile.

For harmonic loss, we have

$$\Delta A^H = \sum_{m=1}^M \Delta A_m^H = \sum_{m=1}^M \left( \sum_{h=1}^{h,max} \Delta P_m^h \times \sum_{k=1}^{24} \alpha_k^2 \right) \quad (6)$$

where,  $\Delta P_{m,max}^h$  is maximum power loss of line segment  $m$  for load's harmonic current  $h$ ; Equ. (6) is subject to an assumption that harmonics load generated by non-linear load is in proportional to its fundamental load.

$$\begin{aligned} \Delta A &= 365 \times \sum_{m=1}^M \left( \Delta P_{m,max}^1 + \sum_{h=1}^{h,max} \Delta P_m^h \right) \times \sum_{k=1}^{24} \alpha_k^2 \\ &= 365 \times \Delta P_{max} \times \sum_{k=1}^{24} \alpha_k^2 = 365 \times \Delta P_{max} \times LsF \end{aligned} \quad (7)$$

where  $LsF$  is daily system loss factor.

Power loss of the test system is calculated from the power flow analysis (10) at the fundamental and (16) harmonic frequencies. Concretely, power loss of a branch of electric line segment  $m$  between node  $i$  and node  $j$  is also calculated as follows

$$\Delta P_m^1 = \Delta P_{ij}^1 = R_{ij} \times (|V_i^1 - V_j^1| \times |Y_{ij}^1|)^2 \quad (8)$$

$$\Delta P_m^h = \Delta P_{ij}^h = R_{ij} \times (|V_i^h - V_j^h| \times |Y_{ij}^h|)^2 \quad (9)$$

where:

$\Delta P_{ij}^1$ : Power loss of branch  $i-j$  at 50Hz

$\Delta P_{ij}^h$ : Power loss of branch  $i-j$  at the harmonic frequency  $h$  with  $R_{ij} = R_m$ .

$Y_{ij}^1$ : Admittance of branch  $i-j$  at 50Hz (15)

$Y_{ij}^h$ : Admittance of branch  $i-j$  at harmonic frequency  $h$  (21)

## 2.2 Constraints

a) Power flow calculation:

- Power flow equations for the fundamental frequency (50Hz)

Power flow equations at node  $i$  are as follows

$$\begin{aligned} P_i &= |V_i^1|^2 \times G_{ii}^1 \\ &+ \sum_{\substack{j=1 \\ j \neq i}}^n |V_i^1 \times V_j^1 \times Y_{ij}^1| \cos(\theta_{ij}^1 + \delta_j^1 - \delta_i^1) \end{aligned}$$

$$Q_i = -|V_i^1|^2 \times B_{ii}^1 + \sum_{\substack{j=1 \\ j \neq i}}^n |V_i^1 \times V_j^1 \times Y_{ij}^1| \sin(\theta_{ij}^1 + \delta_j^1 - \delta_i^1)$$

$$(i = 1 \div n) \tag{10}$$

where  $P_i, Q_i$ : Active and reactive power injected into the node  $i$  at  $f = 50\text{Hz}$ .

$$P_i = P_{i.l} + P_{i.n} \tag{11}$$

$$Q_i = Q_{i.l} + Q_{i.n} \tag{12}$$

with  $P_{i.l}$  and  $Q_{i.l}$ : Linear load (passive load) at node  $i$ .  $P_{i.n}$  and  $Q_{i.n}$ : (non-linear load) at node  $i$  that can generate harmonic.

Admittances at fundamental frequency:

$$Y_{ij}^1 = |Y_{ij}^1| \angle \theta_{ij}^1 = -y_{ij}^1 \tag{13}$$

$$Y_{ii}^1 = G_{ii}^1 + jB_{ii}^1 = y_{c.i}^1 + \sum_{j \neq i} y_{ij}^1 \tag{14}$$

$y_{ij}^1$ : Admittance of the branch  $i$ - $j$ :

$$y_{ij}^1 = \frac{1}{z_{ij}^1} \tag{15}$$

$y_{c.i}^1$ : Admittance of the capacitor connected with node  $i$ .

- Power flow calculation at the harmonic frequency:

System harmonic voltages are calculated by direct solution of linear equation [1]

$$[I_h] = [Y_h] \times [V_h] \tag{16}$$

where  $[I_h]$ : Matrix of harmonic currents injected in nodes of the system.

As the harmonic currents are only generated by non-linear load and calculated as follows based on the fundamental frequency non-linear load of the corresponding nodes  $i$

$$I_i^1 = \left[ \frac{P_{i.n} + jQ_{i.n}}{V_i^1} \right]^* \tag{17}$$

$$I_i^h = h\% \times I_i^1 \tag{18}$$

With  $h\%$  is percentage of harmonic  $h$  obtained by measurement or testing. In this paper,  $h\%$  is given in Table 2 for the EAF's typical harmonics.

Other test system parameters at the harmonic frequency  $h$ :

Passive load at node  $i$ :

$$Y_{t.i}^h = \frac{P_{i.l}}{|V_i^1|^2} - j \frac{1}{h} \times \frac{Q_{i.l}}{|V_i^1|^2} \tag{19}$$

Capacitor's admittance at node  $i$ :

$$Y_{c.i}^h = h \times Y_{c.i}^1 \tag{20}$$

Branch's admittance  $i$ - $j$ :

$$Y_{ij}^h = \frac{1}{R_{ij} + jh.X_{ij}} \tag{21}$$

Basing on nodal voltage, we calculate the power loss at the harmonic frequency as per (9).

b) Nodal voltage deviation constraint

Nodal voltage for all system nodes has to satisfy the following constraints for all operating conditions. In this paper, nodal voltage throughout the system of interest is verified for two critical operating conditions which are maximum and minimum system loading

$$V_{min} \leq V_i \leq V_{max} \tag{22}$$

where

$$V_i = \sqrt{\sum_{h=1}^{hmax} |V_i^h|^2} \tag{23}$$

In this paper,  $V_{min} = 0.95\text{pu}$ ;  $V_{max} = 1.05\text{pu}$ .

Variants of the optimization problem (1) are nodal voltage  $V_i$ , and compensated reactive power  $Q_{c.i}$  at node  $i$ .

c) Total harmonic distortion THD

Voltage THD ( $THD_V$ ) at all nodes in the system of interest is required not to be higher than the allowed limit ( $THD_{Vmax}$ )

$$THD_{V.i}(\%) = \frac{1}{|V_i^1|} \times \sqrt{\sum_{h=1}^{hmax} |V_i^h|^2} \leq THD_{Vmax} \tag{24}$$

where

$THD_{V.i}$ : Total voltage harmonic distortion at node  $i$ ;

$V_i^1$ : Fundamental voltage magnitude at node  $i$ ;

$V_i^h$ : Harmonic frequency  $h$  voltage magnitude at node  $i$ ;

$THD_{Vmax}$ : Total voltage harmonic distortion limit. In this paper,  $THD_{Vmax} = 5\%$ .

d) Capacitor rating

Capacitor size is limited to a maximum value as follows. We can use this constraint to change the compensated VAR distribution in the system

$$Q_{c.i} \leq Q_{c.max} \tag{25}$$

### 2.3 Generic Algorithm based multi-objective optimization solution

Multi-objective optimization is a suitable tool for managing various incommensurable objectives with agreeing/disagreeing relations or also not having any mathematical relation with each other. For the problem of optimizing capacitor placement, two objective functions mentioned in (1) and (2) are generally not converted into quantities in the same unit without assumptions. As discussed in the introduction part, to solve multi-objective optimization problems, methods can be separated into two groups: mathematical based and evolutionary based. In this paper, the problem of multi-objective optimization is solved by GA.

This research uses tool gamultiobj in Matlab to create a set of points on the Pareto front. gamultiobj uses a controlled, elitist genetic algorithm (a variant of NSGA-II). In applying this tool, the number of populations is set

in advance or took the default value (200). Each chromosome will be defined as a string of  $N$ -bit format, for example "1 0 0 0 5 0 0 0 6.5 0 ... 0 0", where "0" value means no capacitor placement and "non-zero" value is rated VAR of capacitor placement at the given node.  $N$  is the number of nodes in the distribution system of interest. In each generation, the non-dominated set of variables is identified and the fitness is ranked. The process of reproduction of new generation is continued until the relative change in best fitness function values is less or equals the function tolerance. The maximum generation is marked and Pareto optimal set is obtained. The step by step solution of GA based multi-objective optimization is shown in the flowchart in Fig. 2.

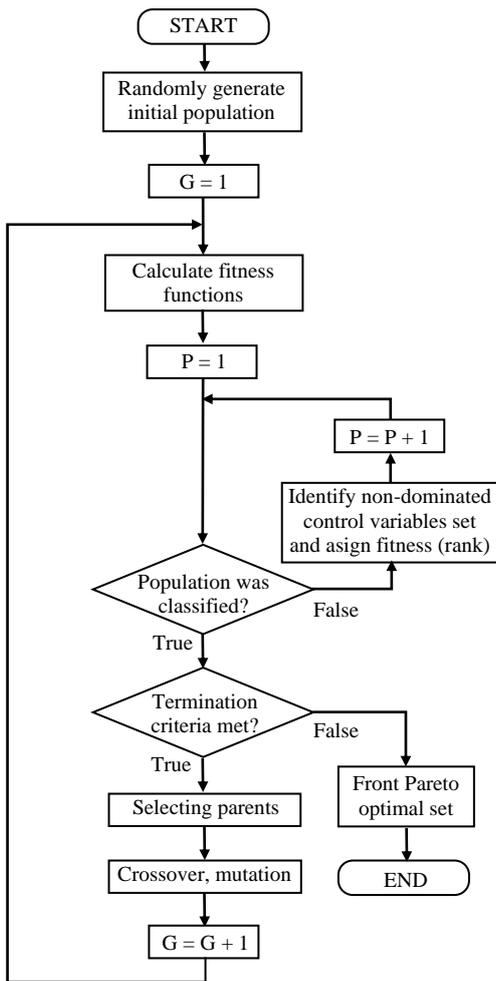


Fig. 2. Diagram of GA based multi-objective optimization

### 3. CASE STUDY

#### 3.1 Test system

With regard to the current MV three-phase distribution system in Vietnam, the IEEE 33-bus distribution feeder (Fig. 3) is used as the test system because it also features a balanced three-phase distribution system only, with three-phase loads and three-phase lines.

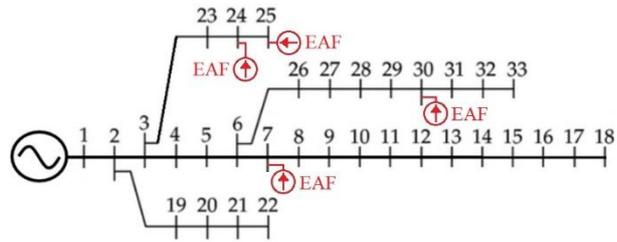


Fig. 3. IEEE 33-bus distribution feeder as the test system

This research assumes base power to be 100MVA. Base voltage is 11kV. The system voltage is 1pu. System impedance is assumed to be 0.1pu. In harmonic load flow calculation, the source of the distribution system is modeled as the system impedance at a certain harmonic  $h$ . Minimum system loading is assumed to be a half of the maximum system loading.

#### 3.2 Harmonic sources modeling

For considering the harmonics existence, four nodes assumed to be connected with EAF – non-linear loads are listed in Table 1. EAFs inject the largest harmonic in the network during the melting period.

Table 1. EAF – non-linear load connected nodes

Node	7	24	25	30
Non-linear load (%)	90	90	90	100

The paper considers a case study of EAFs where the typical harmonic spectrum generated by EAF is assumed in the Table 2 [14].

Table 2. EAF's typical spectrum of harmonics

h	1	2	3	4	5	6	7	9	11
h%	100	36	25	8	10	4	3	2	1

#### 3.3 Result and analysis

In this research, the constraints are assumed, basing on the current practice for power quality in Vietnam [13]

- Voltage tolerances:  $0.95 \div 1.05$ pu
- $THD_{Vmax}$ : 5%
- $Q_{c,max}$ : 230kVAr

Regarding GA parameters, the research set the number of populations as 400.

##### a) Test system operating performance without reactive power compensated capacitor placement

Firstly, considering system power loss with harmonics and without capacitor placement, the system power losses due to fundamental and harmonic currents are summarized in Table 3 as follows.

Table 3. The system power losses due to fundamental and harmonic currents

h	1	2	3	4	5
$\Delta P$ (kW)	281.891	5.351	2.577	0.263	0.407
h	6	7	9	11	
$\Delta P$ (kW)	0.064	0.036	0.015	0.004	

Total power loss in maximum loading operational condition is 290.608kW which is more than 3% higher than the total power loss without harmonic consideration. In reality, the increase of loss can be even greater for industrial distribution systems where more non-linear loads are existed like many locations producing steel in small businesses in Vietnam.

b) Test system operating performance with reactive power compensated capacitor placement

By solving the multi-objective optimization problem with assumed constraints and GA parameters, followings are remarked results.

The GA runs through 385 generations to get the Pareto optimal set with the process of non-dominated sorting and rank assigning for each generation and reproduction of new generations to solve the multi-objective optimization problem. The resulting Pareto front is plotted in Fig. 4 with the Pareto optimal set includes 138 solutions. It's noticed that the selection of population number is important for the algorithm to converge. A greater population number can help the algorithm converge easier, but the calculation time is longer. After quite a lot of trials of population number, the population of 400 is a trade-off.

Followings are the summarized outcomes from solving the multi-objective optimization problem by GA.

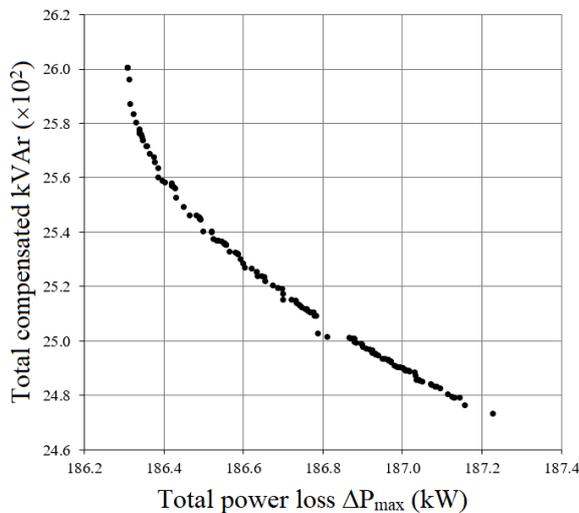


Fig. 4. Pareto front.

It's also noticed that the objective function of total electric energy loss indeed is represented by total power loss. Actually, this can be an important assumption that the VAR compensation has a little impact on the typical load profile. So, the daily system loss factor  $LsF(7)$  is assumed to be unchanged and the objective function of total electric energy loss is possibly replaced by that of total power loss.

The Pareto optimal set with optimal capacitor sizes and locations are shown in 3-D graphic in Fig. 5 for having a good vision of whole set of optimal solutions.

An example of an optimal solution (100) extracted from Pareto optimal set (138) that shows the optimal

compensated VAR capacitor distribution in the test system in Fig. 6.

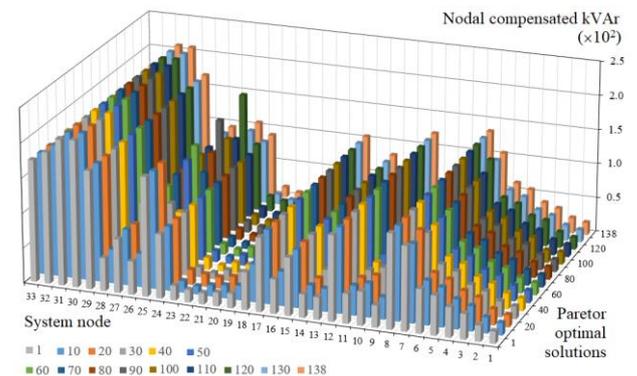


Fig. 5. Nodal compensated VAR for selected different Pareto optimal solutions in Pareto optimal set

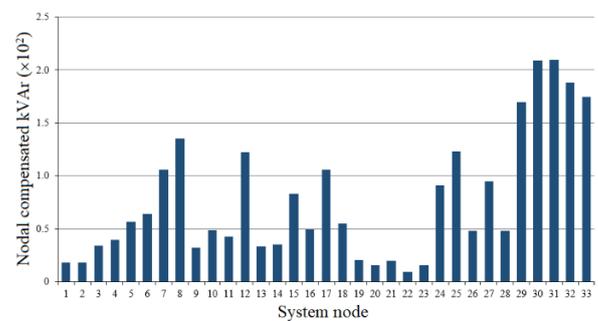


Fig. 6. An optimal solution from Pareto front of nodal compensated VAR distribution.

As the maximum compensated VAR is set as 230kVAR, we can see capacitor's nodal compensated VAR values are rather low but all nodes are compensated. The research tried other higher maximum compensated VAR values and the number of compensated nodes reduces but the nodal compensated VAR values are higher.

The paper depicts power quality indices (nodal voltage and  $THD_V$ ) for the maximum and minimum system loading operation for different case studies in Fig. 7 and Fig. 8. Red line remarks nodal voltage quality for the case without Var compensation which obviously violates power quality criteria (voltage deviation and  $THD_V$ ) at many nodes. Other lines plot nodal voltage and  $THD_V$  with optimal capacitor sizes and locations at some selected optimal solutions (Fig. 7 and Fig. 8). They are all satisfied voltage tolerance and allowable TDH limit.

In Fig. 7, nodal voltages in the maximum loading condition for different optimal solutions are almost the same. Similarly, they are also the same in the minimum loading condition. That's why it looks as if there're only two nodal voltage lines for the maximum and minimum loading conditions respectively. We can see that all the optimal results of capacitor placement satisfy the voltage tolerance and  $THD_V$  limit. Nodal  $THD_V$  in minimum loading operation is smaller because of the assumption that the load's harmonic current is in proportional to its fundamental current.

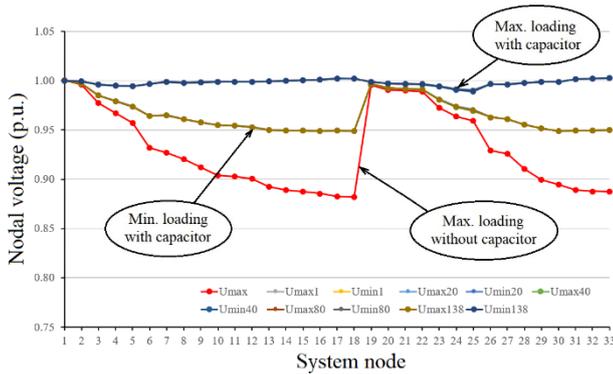


Fig. 7. Nodal voltage before and after optimally placing capacitor following some Pareto optimal solutions (1, 20, 40, 80 and 138).

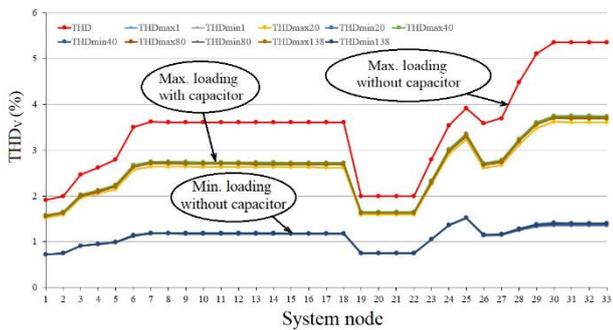


Fig. 8. Nodal THD<sub>v</sub> before and after optimally placing capacitor following some Pareto optimal solutions (1, 20, 40, 80 and 138).

4. CONCLUSION

This paper introduces the application of GA based optimization problem for optimally sizing and locating capacitors in a distribution system with the presence of harmonic sources like the electric arc furnaces. This research considers the real issues of power loss and poor power quality in industrial distribution systems in Vietnam, especially at steel producing small-scale business area. The problem considers the placement of fixed capacitors which either optimizes the system loss and investment or satisfies voltage quality in all operating conditions that reflects the real operation of distribution system in Vietnam. Therefore, in solving the problem, both maximum and minimum loading operations are verified to ensure voltage and THD constraints. The problem also considers the calculation of electric energy loss for easier conversion into cost and turns the multi-objective optimization problem in single-objective optimization problem. However, the calculation of electric energy loss is an approximation and thus this method is suitable for optimizing the capacitor placement for the distribution system where the compensate VAR is not much. In other words, the system loss rate is not high that does not need much kVAR compensation.

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