Improved Methodology for Determining Seasonal and Fixed Optimum Tilt Angles for Solar Collectors

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ABSTRACT

The world faces a series of severe energy and environmental challenges including depletion of fossil fuel reserves, and climate change. These problems are affecting societies directly or indirectly across the world. Solar energy is one of the promising renewable energy resources. However, the availability of sun energy is greatly influenced by the weather and climate changes. The work proposes an improved methodology for determining seasonally adjusted and yearly fixed optimum tilt angles for solar collectors. The optimum angle, in contrast with conventional method, has been found by firstly estimating cumulative radiations for selected span of months, on surface at tilt angles varying from -90° to 90°, and then finding angle against which maximum radiations are obtained. As a case study, total solar radiations on tilted surface and loss of solar radiations have also been compared and discussed with results obtained from literature. Improvement in energy collection has been found out to be as high as 8.69%.

1. INTRODUCTION

Energy is the backbone of modern societies and is crucial for sustainable development. The existing energy scenario faces a number of challenges including depletion of fossil fuels, fluctuation in energy prices, global warming and growing concerns about energy security [1].Renewable energy is an appropriate solution to help address these problems. Renewable energy is amongst the fastest growing energy resources in the world and projections indicate that these resources will have huge contribution in the future [2].

Solar energy is one of the most promising renewable energy resources, and is free, clean and abundant. The direct use of solar energy offers good potential for residential space heating and domestic hot water production. Solar energy offers significant potential for industrial applications as well [3]. Among various solar energy technologies, the most popular are flat-plate collectors for heating purposes and Photovoltaic (PV) panels for direct electricity generation. Solar concentrating technologies e.g. troughs, dishes, towers etc. are generally used for steam generation in power plants. Also, the use of renewable energy is showing an increasing trend in hybrid automobile industry [4] and in desalination systems [5]. However, Solar energy; like other renewable resources, faces the issue of intermittency.

The performance of solar photovoltaic (PV) panels and thermal collectors (hereafter called "collectors"), is mostly affected by the orientation and angle of tilt. Orientation is measured in terms of collectors' surface azimuth angle (γ), with zero due south, east negative and west positive. In the northern hemisphere, the optimum orientation is regarded as south facing (i.e. $\gamma_{opt}=0$). Slope (β) is the tilt angle of collector measured with horizontal. The optimum value of tilt angle (β_{opt}) depends upon latitude (∅) and solar declination (δ) and hence it varies with location and day of the year. In solar systems, tracking mechanism can be employed to maximize the collection of energy from the sun [6]. However, trackers come at a cost resulting into more investment besides consuming energy to operate. Also due to moving components, there are issues with regards to maintenance. Tracking systems are therefore not always a preferred choice. For non-tracking and single-axis tracking collectors, manually changing the tilt angle daily is practically very difficult. As a "rule of thumb", as also suggested by many researchers [7-10], β_{opt} should be equal to the latitude for collecting maximum solar energy annually using fixed-type collectors. In literature, single values of β_{opt} for locations in northern hemisphere have been reported, for example, ∅+20° [11], ∅-10° [12] and ∅+10° [13]. Several researchers have recommended changing the tilt angle monthly, seasonally or twice a year for collecting maximum energy for specific period of time. For example, ∅±20° [14], ∅±15° [15, 16] and ∅±8 [17], where '+' is used for winter and '-' for summer.

In recent decades, significant work has been undertaken

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towards estimation of optimum tilt angle, for particular regions, countries or cities, for specific time periods. Ulgen [18], for example, computed monthly, seasonal and yearly optimum tilt angles for a solar collector and total solar radiation on a tilted surface in Izmir (Turkey). In this work, solar radiations data were taken from the Solar-Wind Metrological Station, located on the roof of Solar Energy Institute Building in Ege University. Isotropic sky model was used for calculating monthly radiations on tilted surfaces that were varied from 0° to 90° from horizontal. Optimum tilt angle for a month was the one against which maximum radiations were received. The seasonal and yearly tilt angles were calculated by finding the average value of the tilt angle for each season and whole year, respectively. Ibrahim [19] calculated monthly, seasonal and yearly optimum tilt angles for Cyprus. Solar radiations data were obtained from meteorology office, located in the northern part of Cyprus. For seasonal and yearly optimum tilt angles, average values of monthly optimum tilt angles were used. Benghanem [20], in the case study for optimizing tilt angle of solar panels for Madinah (Saudi Arabia), found that the annual optimum tilt angle is approximately equal to the latitude and the loss in the amount of collected energy when using the yearly average fixed angle is around 8% compared with the monthly optimum tilt angle. Bari [21] determined optimum tilt angles between any specific numbers of days for Malaysian territory. The isotropic diffuse sky model was assumed to calculate the diffused-scattered component utilizing the data from ASHRAE handbook. Yakup and Malik [22] calculated average optimum tilt angles at different hours of the day, monthly, seasonal and yearly averaged tilt angles in Brunei Darussalam. Fixed tilt angle for each season was evaluated as an average of monthly optimum tilt angles for a period of three months. Also, there is lot of research available discussing about the solar energy potential using different radiation models [23-26].

This paper proposes an improved methodology for finding optimum tilt angles for seasonal and fixed optimum tilt angles for a collector. The proposed approach has demonstrated to yield better results in terms of energy collection as compared to existing ones. As a case study, total solar radiations on tilted surface and the loss of solar radiations have been compared and discussed with the results obtained from literature for Cyprus and Izmir. Radiations on tilted surface will be estimated assuming isotropic sky model.

2. METHODOLOGY

The optimum tilt angle (β_{opt}) for a particular span of time at any location is the one at which the collector will receive maximum solar energy when compared to the energies received at any other angles.

The total radiation on a tilted surface for a span of months, which is to be maximized, can be expressed by:

$$\sum_{m_a,m_b,\dots m_l} H_T = H_{T_{m_a}} + H_{T_{m_b}} + \cdots + H_{T_{m_l}} \tag{1}$$

where, $H_{T_{m_a}}$, $H_{T_{m_b}}$... $H_{T_{m_l}}$ are the total radiation on tilted surface of specific months and can be calculated as:

$$H_{T_m} = n_m \bar{H}_{T_m} \tag{2}$$

where, subscript $m$refers to a specific month,$n_m$is the total number of days of month (see Table I) and $\bar{H}_{T_m}$ is the monthly average daily solar radiation on tilted surface. It can be determined using:

$$\bar{H}_{T_m} = \bar{R}_m \bar{H}_m \tag{3}$$

where, $\bar{H}_m$ is monthly average daily global radiation and $\bar{R}$ is the ratio of monthly average total radiation on a tilted surface to that on a horizontal surface which can be calculated as [27]:

$$\bar{R}_m = \left(1 - \frac{\bar{H}_{d_m}}{\bar{H}_m}\right)\bar{R}_{b_m} + \frac{\bar{\bar{H}}_{d_m}}{\bar{H}_m}\left(\frac{1+\cos\beta}{2}\right) + \rho\left(\frac{1-\cos\beta}{2}\right) \tag{4}$$

where, $\bar{H}_{d_m}$ is the monthly averaged daily diffuse radiation on horizontal, ρ is the ground reflectivity and $\bar{R}_{b_m}$ is the ratio of monthly average daily beam radiation on tilted surface to that on horizontal surface. Using isotropic sky model,$\bar{R}_{b_m}$can be calculated as:

$$\bar{R}_{b_m} = \frac{\cos(\emptyset - \beta)\cos\delta_m \sin\acute{\omega}_{s_m} + \left(\frac{\pi}{180}\right)\acute{\omega}_{s_m}\sin(\emptyset - \beta)\sin\delta_m}{\cos\emptyset\cos\delta_m\sin\omega_{s_m} + \left(\frac{\pi}{180}\right)\omega_{s_m}\sin\emptyset\sin\delta_m} \tag{5}$$

where, $\emptyset$ is latitude of location, β is tilt angle and δ_m is declination angle on average day of the month. Sun set-hour angle for horizontal surface (ω_{s_m}) and the monthly average daily mean sunset hour angle for the tilted surface ($\acute{\omega}_{s_m}$) are given by:

$$\omega_{s_m} = \cos^{-1}(-\tan\emptyset\tan\delta_m) \tag{6}$$

$$\acute{\omega}_{s_m} = \min\left[\omega_{s_m} = \cos^{-1}(-\tan\emptyset\tan\delta_m), \cos^{-1}(-\tan(\emptyset - \beta)\tan\delta_m)\right] \tag{7}$$

The monthly average daily extra-terrestrial radiation on horizontal surface can be determined using:

$$\bar{H}_{o_m} = \frac{3600 \times 24}{\pi} G_{sc}\left(1 + 0.033\cos\frac{360\bar{n}_m}{365}\right)\left(\cos\emptyset\cos\delta_m\sin\omega_{s_m} + \frac{\pi}{180}\omega_{s_m}\sin\emptyset\sin\delta_m\right) \tag{8}$$

where, G_{sc} is solar constant ($= 1367\ W/m^2$) and $\bar{n}_m$is the average day of the month [28] which can be found from Table 1.

Declination angle can be calculated as:

$$\delta_m = 23.45 \sin\left(360 \frac{284 + \bar{n}_m}{365}\right) \quad (9)$$

Table 1. Total number of days and recommended average day for the months

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
m	1	2	3	4	5	6	7	8	9	10	11	12
n_m	31	28	31	30	31	30	31	31	30	31	30	31
$\bar{n}_m$	17	47	75	105	135	162	198	228	258	288	318	344

The methodology as outlined by Ulgen [18] and Ibrahim [19] finds out the optimum tilt angle of a particular season, say, comprising of December January and February, by calculating β_{opt} of each month against the maximum amount of radiation received in that particular month and then taking the average of β_{opt} of individual months to find out β_{opt} for the season. However this may not be the β_{opt} for these months cumulatively. The proposed methodology calculates β_{opt} by executing a greater number of iterations since it varies β through several steps to calculate amount of energy received and hence results in better accuracy. For different values of β, total radiations on the tilted surface for a particular month or span of months can be evaluated using Eq. (1) through Eq. (9). To find the optimum value of β at which $\sum H_T$ is maximum, β is varied from -90° to 90° in steps of 1° (or 0.1° for more accuracy) and $\sum H_T$ is calculated against it. The optimum tilt angle will be the β against the maximum value of $\sum H_T$ for particular span of months.

3. RESULTS AND DISCUSSION

A simulation program based upon the proposed methodology to calculate optimum tilt angle and total energy on that tilted surface, facing south, for a span of months has been developed. It is shown below in the form of Program Description Language (PDL). The program based upon the simulation model has been coded in PHP and is listed at "Building Energy Software Tools Directory" which is sponsored and maintained by U.S. Department of Energy's (DOE) office of Energy Efficiency and Renewable Energy (EERE), hosted at National Renewable Energy Laboratory (NREL) official website [29].

START
INITIALISE $\bar{n}(12)$ AS ARRAY = 17,47,75,105,135,162,198,228,258,288,318,344
INITIALISE n(12) AS ARRAY = 31,28,31,30,31,30,31,31,30,31,30,31
INITIALISE $\emptyset$ AS CONSTANT = *latitude*, G_{sc} AS CONSTANT = 1367, ρ AS CONSTANT = 0.2
INITIALISE $\sum H_{T_m}$, $\sum H_{T_{max}}$ and β_{opt} AS VARIABLE
INITIALISE $H_{T,m}(12)$ and $H_{T_{m,max}}(12)$ AS ARRAY
FOR β = -90 **TO** 90 **STEP** 1
    SET $\sum H_{T_m} = 0$
    FOREACH month for which results are required
        READ $\bar{H}_m$ for month from solar radiations database
        SET $\bar{n}_m = \bar{n}$ for month
        CALCULATE δ_m (using $\bar{n}_m$)
        CALCULATE ω_{s_m} (using $\emptyset$ and δ_m)
        CALCULATE $\acute{\omega}_{s_m}$ (using $\emptyset$, δ_m and β)
        CALCULATE $\bar{H}_{o_m}$ (using G_{sc}, $\bar{n}_m$, $\emptyset$, δ_m and ω_{s_m})
        CALCULATE $\bar{R}_{b_m}$ (using $\emptyset$, β, δ_m, $\acute{\omega}_{s_m}$ and ω_{s_m})
        CALCULATE $\bar{R}_m$ (using $\frac{\bar{H}_{d_m}}{\bar{H}_m}$, $\bar{R}_{b_m}$, ρ and β)
        CALCULATE $\bar{H}_{T_m}$ (using $\bar{R}_m$ and $\bar{H}_m$)
        SET H_{T_m} (associated with month) = $\bar{H}_{T_m} \times$ days in month
        SET $\sum H_T$ **ASSIGN SUM** H_{T_m}
    ENDFOR
        IF $\sum H_T$ **IS GREATER THAN** $\sum H_{T_{max}}$
        THEN
            SET $\sum H_{T_{max}} = \sum H_{T_m}$
            SET $\beta_{opt} = \beta$
            SET $H_{T_{m,max}} = H_{T,m}$
        ENDIF
    ENDFOR
PRINT β_{opt}, $\sum H_{T_{max}}$ and $H_{T_{m,max}}$ (for each month)
END

Using the solar radiations data obtained from published researches for Cyprus [19] and Izmir [18], seasonal and yearly fixed optimum tilt angles using the proposed methodology were calculated along with the energy collection on tilted surfaces, facing south. Tables 2 and 3 provide comparison between proposed and published methodologies for Cyprus and Izmir respectively. Positive differences in energy collection are evident in each of the cases, exceeding 3%for autumn and winter, and reaching up to even 8% as shown.

Table 2. Comparisons of results evaluated from proposed and published methodologies for Cyprus

Adjustment		Published methodology		Proposed methodology		Comparison
		β_{opt}	Energy	β_{opt}	Energy	Energy difference
			MJ/m^2/span	Deg.	MJ/m^2/span	MJ/m^2/span
Seasonal	Summer	14°	2222	2.1	2238.99	16.99 (0.76%)
	Autumn	40°	1583	43.8	1634.76	51.76 (3.27%)
	Winter	48°	1068	54.8	1160.85	92.85 (8.69%)
	Spring	22°	1837	18.06	1845.63	8.63 (0.47%)
Fixed	Year	31°	1822	27	1831.55	9.55 (0.52%)

Table 3. Comparisons of results evaluated from proposed and published methodologies for Izmir

Adjustment		Published methodology		Proposed methodology		Comparison
		β_{opt}	Energy	β_{opt}	Energy	Energy difference
			MJ/m^2/span	Deg.	MJ/m^2/span	MJ/m^2/span
Seasonal	Summer	4.3°	2364.07	5.1	2368.75	4.68 (0.20%)
	Autumn	43°	1568.36	46.4	1628.55	60.19 (3.84%)
	Winter	55.7°	1116.63	58.5	1154.24	37.61 (3.37%)
	Spring	18.3°	1799.63	16.74	1833.56	33.93 (1.89%)
Fixed	Year	30.3°	1761.59	29.5	1819.76	58.17 (3.30%)

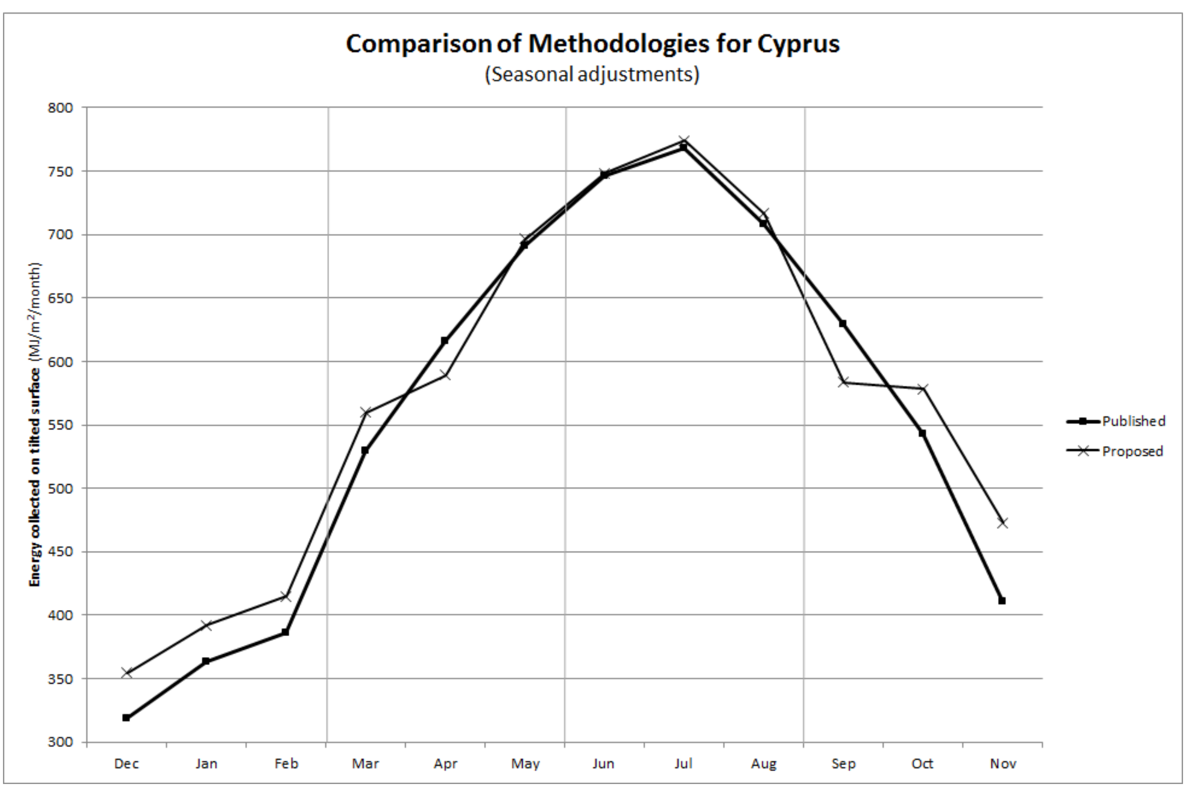

Fig. 1. Comparison of Monthly Energy Collection at Tilted Surfaces for Cyprus, using Proposed and Published Methodologies (Adjustments Made Seasonally).

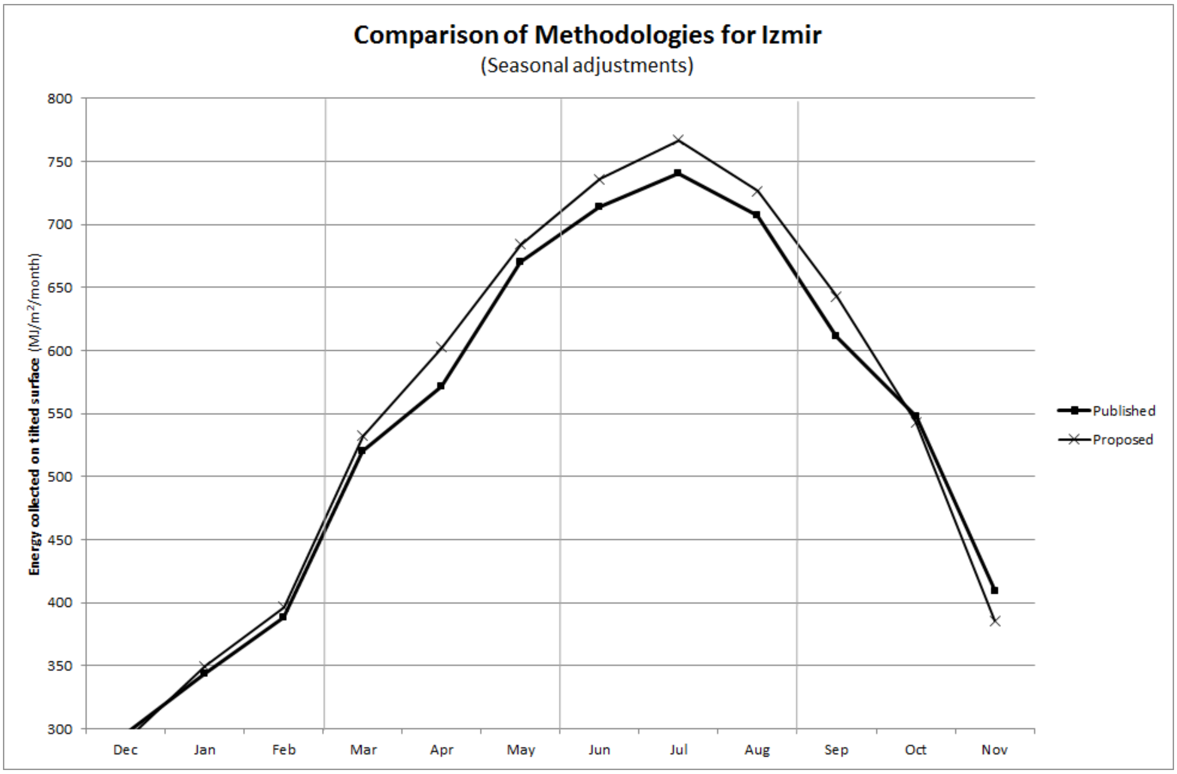

Fig. 2. Comparison of Monthly Energy Collection at Tilted Surfaces for Izmir, Turkey, using Proposed and Published Methodologies (Adjustments Made Seasonally).

Figures 1 and 2 show comparison of monthly energy collection at tilted surfaces calculated using proposed and published methodologies for Cyprus and Izmir, respectively. Adjustments were made seasonally. It is obvious that the energy collection by tilting the collector surface at optimum angle calculated using proposed methodology exceeds energy collection using existing methodologies. It may also be noted that the graphs show very few months for which the energy collected using proposed methodology for the particular month yields lower energy collection; however, the energy collected for the whole season will still remain higher as calculated in Tables 2 and 3. These increments should not be considered as trivial, since those incremental values are for a unit area. The larger the collector area, greater would be the gain in energy collection. For example, the gain of 58.17MJ/m^2, as indicated in last row of Table 2, would become 290.85 MJ for a surface area of 5 m^2. Thus larger benefits can be obtained for large sized collector systems.

4. CONCLUSIONS

An improved methodology for determining optimum seasonal and fixed tilt angles for solar collectors has been proposed which calculates optimum tilt angle as the one which returns maximum sum of monthly radiations on tilted surface for any season or year by varying tilt angles from -90° to 90°. This is in contrast with various existing methodologies which calculate optimum tilt angle by taking simple average of monthly optimum tilt angles. Comparison of results with already published case studies for Cyprus and Izmir show better results. Energy collection has been observed to increase by as much as 8.69%. It has also been emphasized that the gain in energy would increase with an increase in collector area.

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