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A New Technique of Genetic Algorithm for Optimal Placement of Phasor Measurement Units in Power System Observability

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ABSTRACT

This paper proposes a new technique of genetic algorithm to optimize the placements of phasor measurement units in power system observability. This technique bases on three rules: Placing PMU at a bus: that bus and its adjacent buses will be observed; Consider a group of N buses consisting of (N-1) buses connected to a ZIB: if (N-1) buses are observed, the remaining buses will also be observed; consider a bus connected to several ZIBs: if one of that ZIB observability conditions is satisfied, it will be observed. Hence, the technique consists of three simple steps: the first one is to calculate the bus observability index with no ZIBs effect; and then to consider the effect of ZIBs; the last one is considering effect of ZIBs observability to another ZIB. The proposed technique is validated on IEEE 14, 30, and 118 buses. Comparison of its results with the litterature shows the efficiceny of the technique.

1. INTRODUCTION

Power systems become more and more extensive and complex. To remain the systems in steady state, it is very important to estimate the state of the systems. The Supervisory Control and Data Acquisition (SCADA) system and the Energy Management System (EMS) are equiped to measure signals of the systems. From these signals, one can use different methods to estimate the state of the systems. The measurements from the SCADA system are analog signals, signal magnitude, and asynchronous signals [1]. These make the realtime system evaluation has low reliability. The Wide Area Measurement System (WAMS) is used and developed to overcome the disadvantages of the EMS/SCADA system through the information collected from Phasor Measurement Units (PMUs) which measure phasor voltage and phasor current signals at the same time. Informations from PMUs are more frequent than SCADA. PMUs take 30-60 times of data sample per second while SCADA takes measurement in a few seconds a time [2],[3]. Due to expensive cost, installing PMUs at all buses on the grid will meet the economic problem [4]. Consequently, a lot of research studies focus on the problem of Optimal Placement of PMUs (OPP). Their objective is to minimize number of PMUs and assure the system is fully observable. This problem can be solved using numerical methods or topological methods [5]. The numerical ones using the Jacobi's matrix are complex. Moreover, when solving large power systems, the Jacobi's matrix will be huge. These reasons may lead to an increase in computation time [6]. The second ones use graph theory which only requires a system topology, measurement types, and the device locations [5]. It does not require line and transformer parameters, and complex calculations. Hence, most of current applications use the topological methods such as integer programming, heuristic algorithms, . . .

In power systems, there are intermediate buses whose injection power is zero. This kind of bus are called Zero Injection Bus (ZIB). Taking into account ZIB, the constraints become non-linear. In this case, heuristic search algorithms are usually applied, such as greedy algorithm [11], particle swarm optimization algorithm [12], genetic algorithm [3], [4], [6], [10], [13]-[15], and Tabu search algorithm [16]. The consideration of the ZIB in OPP reduces the number of PMUs [6], [10], [16]. These works use different techniques when considering ZIB, such as simplifying constraints by using properties of logical AND and OR [7], topology reconfiguration rule and PMU configuration rule [18], and adding auxiliary binary variables to the constraints [19], [24], [26].

Applying the GA, the paper [3] uses a set of mathematic equation to transform the constraints when considering the ZIBs. In [4], the authors use a hybrid genetic algorithm - the Minimum Spanning Tree method. The latter method handle solutions that do not guarantee the observability. In [6], the connectivity matrix is transformed by merging the ZIB with one of its connections. The authors in [10] use four observability rules corresponding to extra constraints when considering the ZIBs. The connection matrix is

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changed for constraint functions in paper [13] by adding matrices containing ZIB information. In [14], the author used CPI index, which is calculated based on information form measurements and state matrix H, for comparison between alternatives. The authors in the article [15] use the immunity GA with the addition of vaccines corresponding to the observability rules to affect individuals.

With these above analysis, this paper proposes to apply the binary genetic algorithm and a new technique to determine Bus Observability Index (BOI) when taking into account ZIB. The algorithm is implemented and applied for three cases: OPP problem in a normal condition, OPP problem considering ZIB and RB, and OPP problem when one PMU is lost.

The following content is presented in four parts. Section 2 illustrates the objective function and the analysis of calculating the bus observability index with or without considering the influence of the ZIB. The section 3 proposes a new process to calculate the BOI that is simpler than previous studies. The binary genetic algorithm uses this calculation to evaluate the placing options and choose the optimal one. In section 4, the simulations results of the IEEE 14, 30, and 118 buses system are compared with other studies. Finally, the conclusion is presented in section 5. The comparison of results with the previously published papers shows the distinction and suitability of this method for finding the allocation of PMUs.

2. OPTIMAL PMUs PLACEMENT FORMULATION

Finding the optimal PMU placement problem can be reduced to the issue of finding the minimum value of the PMU installation cost function on the grid so that the system is fully observed. The problem is formulated as an Integer Programming problem for a system of N buses as follows [7]:

Minimizing the objective function:

$$F(x) = \sum_{i=1}^{N} w_i x_i \qquad (1)$$

where, x_i is a binary variable that indicates where PMU is located.

$$x_i = \begin{cases} 1 \text{ if PMU is placed at bus i} \\ 0 \text{ if PMU is not placed at bus i} \end{cases} \qquad (2)$$

and w_i is the PMU's installation cost at bus i-th. It is usually assumed the same for all bus systems and its value is set as one [10], [17].

2.1. The observability without the effect of zero injection bus

The objective of the OPP problem is to find the minimum number of PMUs so as to satisfy the observability conditions. Besides, this problem may have more than one suitable output with the same value of the objective function. In that case, previous research uses the BOI and System Observability Redundancy Index (SORI) to evaluate the system observability, then choose the optimal solution [10]. The BOI indicates how many PMUs observe a bus, and SORI is the total BOI of the studied system.

Using the topology method, some system observability rules are considered [6]:

- At the locating PMU bus, its phasor voltage and phasor current of all branches connected to it are measured.
- If the value of phasor voltage and phasor current at one end of a branch is known, the complex bus voltage at the other end will be calculated.
- If the phasor voltages of the two buses are known, the phasor current of a connecting branch will be obtained.

The observability condition of the OPP problem is ensured using the following constraint:

$$f(x) = A.X \geq b \qquad (3)$$

where, A is a binary bus connectivity matrix in which each element is defined as:

$$A_{ij} = \begin{cases} 1 \text{ if i = j or i connect to j} \\ 0 \text{ if i and j do not connect} \end{cases} \qquad (4)$$

and b is a vector matrix whose elements are all equal to 1 or 2 depending on the condition of the considered problem.

Considering i-th bus, BOI(i) is the value of the function fi(x) on the left side of the constraint function (3).

If the elements of b are all equal to 1, it means that the PMU placement ensures to see each bus in the system at least once [7]. If all b's elements are equal to 2, it means that every bus in the system is observed by at least two PMUs. This case is applied to make sure that the system is still observed in the case of any single PMU loss or line power failure [9], [10], [17].

2.2. The observability with the effect of zero injection bus

When considering the ZIBs, one more observability rule is as follows [6], [7].

- If there is one unobservable bus in a group of N buses connecting to one ZIB, it will be observable through Kirchhoff's current law applied at ZIBs.

When considering the ZIBs, there are many studies with different constraint models to achieve the observability condition. In [7], the authors describe the constraint functions in three cases through an implementation example for a 7-bus grid. When considering a group of buses connected to one ZIB, the constraint functions of these buses are reestablished, having components depending on the observability of the others, and then is simplified based on the logical operations.

The paper [18] presents a method to reconfigure the system when considering the ZIBs and then apply GA based on serial number coding. This one gives many sets of PMU. However, within these options, some do not meet the observability conditions for the IEEE 118 bus system.

Particularly, with the first set (3, 8, 11, 12, 17, 20, 23, 28, 34, 37, 41, 45, 49, 53, 56, 59, 67, 73, 75, 77, 80 , 85, 86, 89, 92, 94, 105, 110, 115) in Table 6 [18], 29 PMUs cannot observe bus 101. Looking at Fig. 1.a, bus 101 connects to bus 100 and 102, which neither of these is ZIB, and also no PMU at these buses. Therefore, the observability of bus 101 is not guaranteed.

The article [19] adds binary variables to equation (3) for the ZIBs and the buses connected to the ZIBs. This one gives the results of 28 PMU locations (3 9 11 12 17 21 25 28 34 37 40 45 49 53 56 62 72 75 77 80 85 86 90 94 102 105 110 114). The authors in [20] mention that this PMU set cannot reach buses 63 and 64. However, this conclusion is not entirely correct. More specific, considering a part of the IEEE 118-bus system as shown in Fig. 1.b, applying Kirchhoff's current law at bus 63 and bus 64 we have two equations as in (5):

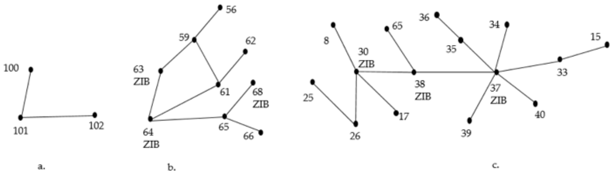

Fig. 1. Partial connection diagrams of IEEE 118-bus

$$\begin{gathered}\frac{\mathrm{V}_{59}-\mathrm{V}_{63}}{\mathrm{Z}_{59-63}}+\frac{\mathrm{V}_{64}-\mathrm{V}_{63}}{\mathrm{Z}_{63-64}}=0\\ \frac{\mathrm{V}_{61}-\mathrm{V}_{64}}{\mathrm{Z}_{61-64}}+\frac{\mathrm{V}_{65}-\mathrm{V}_{64}}{\mathrm{Z}_{64-65}}-\frac{\mathrm{V}_{64}-\mathrm{V}_{63}}{\mathrm{Z}_{63-64}}=0\end{gathered} \tag{5}$$

where, V_{59}, V_{61}, V_{63}, V_{64}, V_{65} are voltage at bus 59, 61, 63, 64, and 69 respectively; Z_{59-63}, Z_{63-64}, Z_{61-64}, Z_{64-65} are impedance between bus 59-63, 63-64, 61-64, and 64-65, respectively.

In [19], the placement of PMUs at buses 56 and 62 makes buses 59 and 61 observable. Bus 65 connects to two ZIB, 38 and 68, so its voltage can be calculated when the condition at bus 38 or bus 68 is satisfied. Then, through equation (5) we can obtain the voltage at buses 63 and 64. A similar observability problem of buses 63 and 64 is also found in the papers [15], [16], [21], and [22].

The article [23] indicates that with the given positions in [20], buses 33 and 35 in the IEEE 118-bus system are not observed. Considering the diagram of two buses 33 and 35 in Fig. 1.c, only bus 37 is ZIB, connects to 33, 34, 39, 38, 40, and 35. Bus 33 links to 15 and 37. Bus 35 connects to 36 and 37. Besides, PMUs are not located at buses 15, 36, and 37. Consequently, even though buses 34, 39, 38, 40, and 37 are all observed, buses 33 and 35 cannot be observed through Kirchhoff's current law applied at bus ZIB 37.

The paper [24] shows how to set up the constraint function when taking into account the influence of the group of interconnected ZIBs. Though, with the given PMU placement set (3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 40, 45, 49,52, 56, 62, 65, 72, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110) for the IEEE 118-bus, it also does not guarantee the observabilitys at buses 33 and 35.

The authors in [14] indicate that several PMU allocations in the previous literature for the IEEE 118-bus are not fully cover the system. They suggest putting PMUs on 29 buses include 3, 9, 11, 12, 15, 17, 21, 27, 31, 32, 34, 40, 45, 49, 52, 56, 59, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, and 110. Looking at Fig. 1.b and Fig. 1.c, bus 65 could be observed through the ZIB effects of bus 64 or 68. Then apply Kirchhoff’s current law at bus 38, we can calculate its phasor voltage. This voltage value is considered in the equation of ZIB 37 and ZIB 38 to obtain bus 35 and 26, respectively.

In literature [25], the authors point out that observing the system through the ZIB bus can transmit and “The reliability of ZI observability becomes weaker, as ZI observability transmits ‘‘deeper.’’”. It suggests a constraint model that considers the zero injection observability depth, which is set less than or equal to one.

Article [21] mentions that the constraints when considering the ZIB in [9] are not clear in the case of a bus connecting with more than two ZIBs. This article uses a ZIB set and buses connected to the ZIB set to guarantee that if the ith bus connects to two or more zero-injection buses, its constrain is only considered in the function of one of these zero injection buses.

In [26], the authors point out that in the case of two or more ZIBs connected to one bus, the linear constraint models used before may not fully guarantee the system observability. This paper presents a new linear constraint function model to solve this problem.

The authors in [27] use Cellular Learning Automata (CLA) with new CLA local rules. In this paper, the constraint function model is unchanged when considering the ZIBs or injection measurements.

In [22], the OPP problem solves using mixed-integer linear programming with the ZIBs property model based on the fixed and floating concept.

Through the above analysis, solving the OPP problem considering the ZIB effect may need constraint functions with more components than normal. Several papers implement a new model function but do not fully solve the problem of observability as in [18] and [20]. Some other articles give better solutions as in [23] and [25]-[27]. However, even with these options, with the PMU placement set shown in Table 1, we found that all the options are not guaranteed the observability at bus 47. Moreover, it is noticeable that this issue exists in almost prior published studies. Specifically, consider the graph connection of bus 47 in the IEEE 118-bus system in Fig. 2, it shows that bus 47 connected to three buses (46, 69, and 79) which all are not ZIBs. Besides, there are no PMU given on these buses. Therefore, this results in bus 47 not being observed directly by the PMU or via the effect of ZIBs.

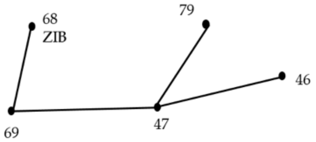

Fig. 2. Connection diagram of bus 47 in the IEEE 118-bus system.

Table 1. PMU location in several references

Ref.	No. of PMU	PMU location
[25]	29	3, 9, 11, 12, 17, 21, 23, 29, 34, 37, 40, 45, 49, 52, 56, 62, 65, 71,75, 77, 80, 85, 87, 91, 94, 101, 105, 110, 115
[21]	28	3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110
[26]	29	3, 5, 8, 12, 15, 17, 21, 27, 31,32, 34, 40, 45, 49, 53, 56, 59, 66, 72, 75, 77, 80, 85, 86, 91, 94, 102, 105, 110
[23]	29	2, 8, 11, 12, 15, 19, 21, 27, 31, 32, 34, 40, 45, 49, 52, 56, 62, 65, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110
[27]	29	2, 8, 11, 12, 15, 19, 21, 27, 31, 32, 34, 40, 45, 49, 52, 56, 62, 65, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110
[22]	28	3, 9, 11, 12, 17, 21, 23, 29, 34, 37, 40, 45, 49, 53, 56, 62, 71, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 115
[16]	28	3, 8, 11, 12, 17, 21,27, 31, 32, 34, 37, 40,45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90,94, 102, 105, 110
[15]	28	3, 8, 11, 12, 17, 21, 25, 28, 34, 35, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110, 114

To overcome the complexity of establishing constraint functions when considering the ZIB and ensuring the system observability, the GA with the simple bus observability index calculation process is proposed in Section 3.

3. PROPOSED TECHNIQUE OF GENETIC ALGORITHM FOR OPP

The Genetic Algorithm is that adapted to the evolutionary process of biological populations based on Darwin's theory. GA was first introduced by Holland and developed by Goldberg. In GA, the optimal individual search is started with a set of individuals. The chromosomes of the current population are the source for the next generation through three main operators: selection, crossover, and mutation. At each step, the individual is evaluated through its fitness value. Chromosomes that have good fitness value will survive, and the others will be discarded. The GA can implement with binary variables or decimal variables. In binary GA, each gene in an individual corresponding to one decimal variable will be represented by a binary value set. Performing decimal variables in binary variables form with high precision requirements can cause an increase in the individual length and then computation time. Therefore, depending on the problem, it is possible to choose which appropriate GA algorithm is.

The binary GA is appropriate for the OPP problem because each variable that referred to the choice of whether or not to place PMU at a bus only needs to represent by a value of 0 or 1. For this, the GA works on a set of PMU placement options. Each option is evaluated through the total number of PMUs make the system fully observable. In this paper, the observability taking into account the influence of the ZIB according to the observed laws mentioned in Section 2.2, is as follows:

- Placing PMU at a bus: that bus and its adjacent buses will be observed.

- Consider a group of N buses consisting of (N-1) buses connected to a ZIB: if (N-1) buses are observed, the remaining buses will also be observed.

- Consider a bus connected to several ZIBs: if one of that ZIB observability conditions is satisfied, it will be observed.

More specifically, according to these laws above, the constraints for the 5-bus system (Fig. 3) are determined as follows:

In this 5-bus system, bus 1 is zero injection bus.

The vector representing the possibility of placing PMU at buses:

$$X = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5] \tag{6}$$

Establish the connection matrix A of the 5-bus grid as follows:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \tag{7}$$

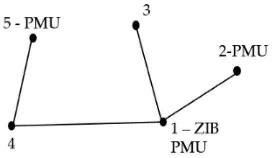

Fig. 3. Connection diagram of 5-bus system.

We consider one PMU set refer to X = [1 1 0 0 1].

When do not consider the effect of ZIB, the bus observability index are as in (8).

$$
\begin{aligned}
&f(1) = x_1 + x_2 + x_3 + x_4 = 2\\
&f(2) = x_1 + x_2 = 2\\
&f(3) = x_1 + x_3 = 1\\
&f(4) = x_1 + x_4 + x_5 = 2\\
&f(5) = x_4 + x_5 = 1
\end{aligned}
\tag{8}
$$

In Fig. 3, bus 1 connects to buses 2, 3, and 4. If we know three voltage values of four buses (1, 2, 3, 4), the voltage value of the fourth bus will be calculated according to Kirchhoff's current law applied at bus 1. For bus 3, its observability index when considering the effect of ZIB is formed as in (9):

$$f(3) = x_1 + x_3 + f(1).f(2).f(4) = 3 \tag{9}$$

In which, the sign "+/." represents the logical function "OR/AND" respectively.

If we calculate the observability index of bus 3 as in (9), we find that f(3) = 3. That is incorrect and may lead to a wrong decision. Because bus 3 is observed just two times, once by PMU and once through Kirchhoff's current law at bus 1. Setting PMU at buses 2 and 5 makes one observer at buses 1, 2, and 4, resulting in that bus 3 can be observed one time. To avoid the repetition of the observability index when the PMU is located at the ZIB 1, the subtraction by one unit is made. Then the observability index of bus 3 is calculated as follows:

$$f(3) = x1 + x3 + (f(1).f(2).f(4) - x1) = 2 \tag{10}$$

To see more influence of the ZIB on the observability of the system, we consider one part of the IEEE 30-bus power grid as in Fig. 4.

Consider a PMU set in which has PMU at buses 2, 4, 10, and 27. This placement can observe buses 2, 4, 6, 9, 10, 27, and 28. Applying Kirchhoff's current law at ZIB 28, because buses 6, 27, and 28 are all observed, then bus 8 is also observed. After that, when considering ZIB 6, bus 7 will also become observable. In this case, the observability of bus 8 through bus ZIB 28 is transferred to group ZIB 6 to make bus 7 observable.

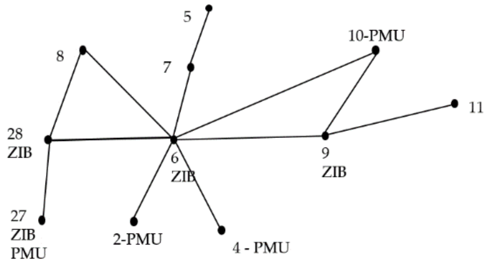

Fig. 4. Partial diagram of IEEE 30-bus system.

When establishing the constraint function for the OPP problem solving by integer programming, the authors need to transform and simplify the constraint using logic operators as in [7] or add the extra variables as in [19]. However, when using the GA algorithm, the constraint function value is used to check the observability of the system, so there is no need to transform or simplify it. With the above analysis, in this paper, the proposed process of calculating bus i-th observability index is as follows:

- Step 1: Calculate the bus observability index with no ZIBs effect:

$$f(i)_1 = \sum_{j=1}^{N} A_{ij} x_{ij} \quad , \quad i = 1 \ldots N \tag{11}$$

- Step 2: Considering the effect of ZIBs

$$f(i)_2 = f(i)_1 + \sum_{k=1}^{Nzib} \left(\prod_{\substack{n \neq i \\ n \in Gzib(k)}} f(n)_1 - x_k \right) \tag{12}$$

- Step 3: Consider effect of ZIBs observability to another ZIB:

$$
\begin{aligned}
f(i)_3 = f(i)_2 + \sum_{k=1}^{Nzib} \Bigg(& \prod_{\substack{n \neq i \\ n \in Gzib(k)}} f(n)_2 \\
& - \prod_{\substack{n \neq i \\ n \in Gzib(k)}} f(n)_1 \Bigg)
\end{aligned}
\tag{13}
$$

where, *N* is the total number of buses on the system; N_{zib} is the number of ZIB connected to bus *i*; Gzib(k) is a group of buses adjacent to bus ZIB k-th; x_k refers to the decision of yes/no putting PMU at bus *i*; *Π* refers to the logical function AND.

The buses observability index calculating process above first uses the BOI value with no ZIBs, then considers the effect of the ZIBs. The consideration of ZIBs observability effect to another ZIB in step 3 is only taken for buses with unsatisfactory BOI values. Therefore, this method can guarantee the reliability observability of buses. Besides, it can eliminate the duplicate calculation of the observability index by taking into account the case of putting PMU at ZIBs. This inspection gets valuable in cases that require observing a bus more than once. In addition, we neither need to transform the observability functions nor add extra variables.

The proposed steps for solving the OPP problem by GA are as follows:

- Determining the objective function: use the objective function model based on the topology method and the integer model as described in Section 2.

- Select parameters for the OPP program: maximum number of iterations, the number of individuals, crossover rate, and mutation rate.

- Initialize the population of individuals: the gene values in each individual are taking random values of 0 or 1.
- The value of the objective function is the total number of PMUs needed to satisfy the system observable condition.
- Perform selection operator according to the roulette wheel method.
- Perform mutation operations on a variable value by randomly changing it from 0 to 1 and vice versa. This will increase the search space and avoid the problem of premature convergence. In this operator, the mutation position is also randomly selected.
- Check the system observability conditions for each individual through the three steps process mentioned above (equation (11) to (13)). If the individual does not satisfy the observability condition, the PMU will be added to ensure the observability of the system. Then will update the objective function value and the observability of the unsatisfied individual.
- Updating the new set of individuals: keep several good individuals of the previous generation, the rest is created through crossover operator. The mutation is per-formed on the individuals except the best one;
- Checking the stopping condition and giving results: the selection of the optimal solutions are based on numbers of PMU and the SORI index. The program stops when it reaches the maximum number of iterations or when 30 consecutive SORI values do not change.

4. RESULTS AND DISCUSSION

To study the OPP problem, the simulations were run for IEEE 14-bus, 30-bus, 118-bus systems, and the results were compared with other existing studies. In this paper, the result presented corresponding to three different cases:

- Case 1: Find the optimal number and location of PMU without bus type (ZIB and RB).
- Case 2: Solve the OPP problem considering bus type (ZIB and RB).
- Case 3: Solve the OPP problem consider ZIB in case of a single PMU loss.

In each case study, the simulation accounts for ten times per one network and gets the best results.

The connection diagrams and connection data of three IEEE systems (14-bus, 30-bus, and 118-bus) can access from [28]-[30] respectively. Data of ZIB and RB location and branch numbers of the three simulation grids are shown in Table 2.

In this paper, the binary GA parameters are set as:

- Maximum iteration: 1000
- Population size: 3000
- Crossover rate: 0.7
- Mutation rate: 0.001

Table 2. Location of ZIB, RB, and the number of branches

System	ZIB location	RB location	No. of branches
14-bus	7	8	20
30-bus	6, 9, 22, 25, 27, 28	11, 13, 26	41
118-bus	5, 9, 30, 37, 38, 63, 64, 68, 71, 81	10, 73, 87, 111 112, 116, 117	186

4.1 Simulation result for three IEEE systems in case 1

The first case to be studied is to solve the OPP problem in which all buses on the grid are considered capable of placing PMUs. The optimal placement solutions of 14, 30, and 118-bus grids, ensuring each bus is observed at least once, are shown in Table 3. The BOI values of each system are represented from Table 4 to Table 6.

For the IEEE 14-bus system, bus 4 has a maximum BOI with a value of four; two is the observer times of buses 5, 7, and 9. The remaining buses are observed by one PMU.

The SORI of the 30-bus system is 52 with three placement options. These options differ in the location of only one PMU at bus 18 or bus 19, or bus 20. With all three options, PMU obser bus 6 the most with BOI value is five.

For the IEEE 118-bus system, there are several options with the same optimal number of 32 PMUs with different values of SORI. The results in Table 3 show the best option with the SORI equal to 164. With this placing PMUs option, the maximum BOI is three at buses 3, 37, 69, 85, 89, 92, 94, and 103.

Table 3. Optimal solutions of OPP problem in case 1

Test system	No. of PMU	PMU location	SORI
14-bus	4	2, 6, 7, 9	19
30-bus	10	2, 4, 6, 9, 10, 12, 15, 18, 25, 27	52
		2, 4, 6, 9, 10, 12, 15, 19, 25, 27	
		2, 4, 6, 9, 10, 12, 15, 20, 25, 27	
118-bus	32	3, 5, 9, 12, 15, 17, 21, 25, 29, 34, 37, 40,45, 49, 52, 56, 62, 64, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114	164

Table 4. BOI values of IEEE 14-bus system in case 1

Bus no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
BOI	1	1	1	3	2	1	2	1	2	1	1	1	1	1

Table 5. BOI values of IEEE 30-bus system in case 1

PMU	**[2,4,6,9,10,12,15,18,25,27]**									
Bus no.	1	2	3	4	5	6	7	8	9	10
BOI	1	3	1	4	1	5	1	1	3	3
Bus no.	11	12	13	14	15	16	17	18	19	20
BOI	1	3	1	2	3	1	1	2	1	1
Bus no.	21	22	23	24	25	26	27	28	29	30
BOI	1	1	1	1	2	1	2	2	1	1
PMU	**[2,4,6,9,10,12,15,19,25,27]**									
Bus no.	1	2	3	4	5	6	7	8	9	10
BOI	1	3	1	4	1	5	1	1	3	3
Bus no.	11	12	13	14	15	16	17	18	19	20
BOI	1	3	1	2	2	1	1	2	1	2
Bus no.	21	22	23	24	25	26	27	28	29	30
BOI	1	1	1	1	2	1	2	2	1	1
PMU	**[2,4,6,9,10,12,15,20,25,27]**									
Bus no.	1	2	3	4	5	6	7	8	9	10
BOI	1	3	1	4	1	5	1	1	3	4
Bus no.	11	12	13	14	15	16	17	18	19	20
BOI	1	3	1	2	2	1	1	1	1	2
Bus no.	21	22	23	24	25	26	27	28	29	30
BOI	1	1	1	1	2	1	2	2	1	1

4.2 Simulation result for three IEEE systems in case 2

The second case to be studied also has two objects: minimize the number of PMUs and ensure the system's observability. However, in this case, to reduce the number of search spaces, two types of buses, which are ZIB and RB, are considered as follows:

- The ZIB bus will affect the system's observability when calculating the bus observability index, as described in Section 3.
- The RB bus will be omitted in the PMU set. In every individual, the value of the gene corresponding to RB will be set to zero. The removal of the RB bus from the search set makes the search space narrower. For example, in the 118 buses system, seven buses are radial buses. Removing the RB bus from the search space helps reduce 118 buses to 111 buses that need to be considered for placing PMU.

The location results of PMUs on the IEEE 14-bus, 30-bus, and 118-bus grids are shown in Table 7. The results show that 3, 7, and 29 PMUs are enough to observe each bus once for the IEEE 14, 30, and 118 buses, respectively. In this case, the number of PMUs needed to cover the system fully decrease by one to three PMUs compare to Case 1. The PMU placement of the 14-bus system has only one option located at buses 2, 6, and 9.

Table 6. BOI values of IEEE 118-bus system in case 1

Bus	**BOI**	**Bus**	**BOI**	**Bus**	**BOI**	**Bus**	**BOI**
1	1	16	2	31	2	46	1
2	1	17	2	32	1	47	1
3	3	18	1	33	2	48	1
4	1	19	2	34	2	49	2
5	2	20	1	35	1	50	1
6	1	21	1	36	1	51	2
7	1	22	1	37	3	52	1
8	2	23	1	38	1	53	1
9	1	24	1	39	2	54	2
10	1	25	1	40	2	55	1
11	2	26	1	41	1	56	1
12	2	27	1	42	2	57	1
13	1	28	1	43	1	58	1
14	2	29	1	44	1	59	1
15	2	30	1	45	2	60	1
Bus	**BOI**	**Bus**	**BOI**	**Bus**	**BOI**	**Bus**	**BOI**
61	2	76	1	91	1	106	2
62	1	77	1	92	3	107	1
63	1	78	1	93	1	108	1
64	1	79	1	94	3	109	1
65	2	80	2	95	1	110	1
66	2	81	1	96	1	111	1
67	1	82	1	97	1	112	1
68	1	83	1	98	1	113	1
69	3	84	1	99	1	114	1
70	2	85	3	100	2	115	1
71	2	86	2	101	1	116	1
72	1	87	1	102	1	117	1
73	1	88	2	103	3	118	1
74	1	89	3	104	2		
75	1	90	1	105	1		

The algorithm gives six options for allocating PMUs on the 30-bus grid. Looking at Table 8, we see that each references [22], [23] and [25] give a similar output compared with three of these getting from this work. However, this simulation finds better results because of three more PMUs locations sets with a similar index of the system observability.

Table 7. Optimal solutions of OPP problem in case 2

Test system	PMU locations	SORI
14-bus	2, 6, 9	16
30-bus	3, 5, 10, 12, 18, 24, 27	39
	3, 5, 10, 12, 19, 24, 27	
	1, 7, 10, 12, 19, 24, 27	
	1, 7, 10, 12, 18, 24, 27	
	1, 5, 10, 12, 19, 24, 27	
	1, 5, 10, 12, 18, 24, 27	
118-bus	3, 8, 11, 12, 17, 20, 23, 29, 36, 40, 44, 47, 49, 53, 56, 62, 65, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 115	156
	3, 8, 11, 12, 17, 20, 23, 29, 36, 40, 43, 47, 49, 53, 56, 62, 65, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110, 115	
	3, 8, 11, 12, 17, 20, 23, 28, 36, 40, 44, 47, 49, 52, 56, 62, 65, 71, 75, 77, 80, 85, 86, 91, 94, 102, 105, 110, 115	
	3, 8, 11, 12, 17, 20, 23, 28, 36, 40, 43, 47, 49, 52, 56, 62, 65, 71, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110, 114	
	3, 8, 11, 12, 17, 20, 23, 28, 36, 40, 44, 47, 49, 52, 56, 62, 65, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 115	
	3, 8, 11, 12, 17, 20, 23, 28, 36, 40, 44, 47, 49, 52, 56, 62, 65, 71, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110, 114	
	3, 8, 11, 12, 17, 20, 23, 28, 35, 40, 43, 47, 49, 52, 56, 62, 65, 72, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110, 115	
	3, 8, 11, 12, 17, 20, 23, 28, 35, 40, 43, 47, 49, 52, 56, 62, 65, 71, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110, 115	

It is noticeable that all the output given in Table 7 for the IEEE 118-bus system meet the system's observability requirement and fix all the problems mentioned in section 2.2. With 29 PMUs, eight PMU sets with the best observable index value are obtained for this system. In the problem of buses 33 and 35, one PMU is placed at 35 or 36 to make bus 35 observable and then bus 33. There are also three PMUs at buses 56, 62, and 65 to observe buses 63 and 64.

In this case, the PMUs location of the IEEE 30-bus system is looking in more detail. The calculation of the bus observability index follows three steps in section 4 and presents the values in Table 1 refer to F1, F2, and F3, respectively. We also apply this process to check several published PMU allocations given in Table 8. Table 1 show the result for our PMU set X = [1, 5, 10, 12, 19, 24, 27] in comparison with that in references [9], [17], [21], [26] and [27]. It is noticeable that the result from this work outperforms the others. In literature [21], [26], and [27], they need the transmit observability of ZIB to obtain buses 7 and 26. With the PMU set from reference [9] and [17], there is a weak observability at buses 5 and 7.

Table 8. The reference optimal solutions of the OPP problem in case 2 and case 3

Test system	Case	Reference	PMU locations
30-bus	2	[25]	1, 5, 10, 12, 19, 24, 27
		[23]	1, 7, 10, 12, 18, 24, 27
		[22]	1, 5, 10, 12, 18, 24, 27
		[17]	2, 3, 10, 12, 18, 24, 30
		[9]	1, 6, 10, 12, 19, 24, 27
		[21],[27]	2, 4, 10, 12, 15, 18, 27
		[26]	2, 4, 10, 12, 15, 20, 27
14-bus	3	[12],[17], [22],[25]	1, 2, 4, 6, 9, 10, 13
		[21],[26]	2, 4, 5, 6, 9, 11, 13

4.3 Simulation result for three IEEE systems in case 3

This case aims to find the minimum location of the PMUs, ensuring the system observability when a single PMU fails. In this study, the requirement observability index of each bus is at least two. The difference from Case 2 is the necessity of considering radial buses as a capable option of putting PMU. Besides, there also consider the ZIBs effects. Table 10 illustrates information about optimal PMUs placement sets for three IEEE grids obtained from the GA algorithm.

For the IEEE 14-bus grid, there are two outputs with seven PMUs giving 35 times total observability. These two options differ in the location of one PMU putting at bus number 10 or 11. For this system, along with the second option matching with the one in reference [21] and [26], we also found another PMU set equivalent. Table 10 shows the comparison of observability index value calculated through three steps process between this work and the articles mentioned in Table 8. Looking at the results, we see that, with the same number of seven PMUs, the results of this paper obtain better observability at buses 4 and 6 with 35 times total compared to 33 in the others. As the results in 0, after step 2, the value of F2 is at least two, so there is no need to perform step 3.

Table 9. The bus observability index of the IEEE 30-bus system in case 2

	This paper			**[21],[27]**			**[26]**			**[9]**			**[17]**		
Bus	**F1**	**F2**	**F3**	**F1**	**F2**	**F3**	**F1**	**F2**	**F3**	**F1**	**F2**	**F3**	**F1**	**F2**	**F3**
1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	3	3	3	3	3	3	2	2	2	3	3	3
5	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1
6	1	1	1	3	3	3	3	3	3	2	3	3	2	2	2
7	1	1	1	0	0	1	0	0	1	1	1	1	0	0	0
8	0	2	2	0	1	1	0	1	1	1	2	2	0	0	1
9	1	1	1	1	1	1	1	1	1	2	2	2	1	1	1
10	1	2	2	1	1	1	2	2	2	2	3	3	1	2	2
11	0	1	1	0	1	1	0	1	1	0	2	2	0	1	1
12	1	1	1	3	3	3	3	3	3	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1	1	2	2	2	2	2	2	1	1	1	1	1	1
15	1	1	1	3	3	3	2	2	2	1	1	1	2	2	2
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	1	1	1	2	2	2	1	1	1	1	1	1	1	1	1
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	2	2	2	1	1	1	2	2	2	2	2	2	1	1	1
21	1	2	2	1	1	1	1	1	1	1	2	2	1	2	2
22	2	3	3	1	1	1	1	1	1	2	3	3	2	3	3
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24	1	2	2	0	1	1	0	1	1	1	2	2	1	2	2
25	2	2	2	1	1	1	1	1	1	2	2	2	1	1	1
26	0	1	1	0	0	1	0	0	1	0	1	1	0	1	1
27	1	1	1	1	1	1	1	1	1	1	2	2	1	1	1
28	1	1	1	1	1	1	1	1	1	2	3	3	0	1	1
29	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

The results for the IEEE 30-bus grid are eight PMUs location sets with fourteen PMU and the total number of observability equal to 80. It is possible to divide into two groups of options with a similar set of buses as [2, 3, 4, 7, 8, 10, 12, 13, 15, 18, 24] and [1, 2, 4, 7, 8, 10, 12, 13, 15, 24]. Within each group, there can be one or two differences in placement between options.

For IEEE 118 buses, the two best results after ten simulations are these obtained 60 PMUs with the total number of observability equals 310. By using these two placement sets, bus 49 is observed the most with seven times.

From these results shown in Table 10, it is clear that the number of PMUs needed when one PMU loss increased more than twice compared to Case 2 and nearly doubled in comparison to Case 1.

Table 10. The optimal solution of OPP problem in case 3

Test system	PMU locations	SORI
14-bus	2, 4, 5, 6, 9, 10, 13	35
	2, 4, 5, 6, 9, 11, 13	
30-bus	2, 3, 4, 7, 8, 10, 12, 13, 15, 16, 18, 19, 24, 29	80
	2, 3, 4, 7, 8, 10, 12, 13, 15, 17, 18, 20, 24, 29	
	2, 3, 4, 7, 8, 10, 12, 13, 15, 17, 18, 20, 24, 30	
	2, 3, 4, 7, 8, 10, 12, 13, 15, 17, 18, 19, 24, 30	
	1, 2, 4, 7, 8, 10, 12, 13, 15, 16, 19, 20, 24, 30	
	1, 2, 4, 7, 8, 10, 12, 13, 15, 16, 19, 20, 24, 29	
	1, 2, 4, 7, 8, 10, 12, 13, 15, 16, 18, 19, 24, 29	
	1, 2, 4, 7, 8, 10, 12, 13, 15, 17, 18, 19, 24, 29	
118-bus	2, 3, 7, 10, 11, 12, 15, 17, 19, 21, 22, 24, 26, 27, 29, 31, 32, 34, 35, 40, 42, 43, 45, 46, 49, 50, 51, 52, 54, 56, 59, 62, 65, 66, 70, 73, 75, 77, 79, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 102, 105, 106, 108, 110, 111, 112, 115, 117, 118	310
	1, 2, 7, 10, 11, 12, 15, 17, 19, 21, 22, 24, 26, 27, 29, 31, 32, 34, 35, 40, 42, 44, 45, 46, 49, 50, 51, 53, 54, 56, 59, 62, 65, 66, 70, 73, 75, 77, 79, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 102, 105, 106, 109, 110, 111, 112, 114, 117, 118	

4.4 Comparison of result for three IEEE system

The simulation results of IEEE 14-bus, 30-bus, and 118-bus systems in the three cases mentioned above are compared with those published and showed comparable numbers of PMUs in Table 12 through Table 14. In addition, to be more specific with other articles also using the GA, the comparisons are illustrated in Table 15.

From Table 12 and Table 13, the number of PMU obtained in Case 1 and Case 2 are the same as in almost prior publishes for 14-bus, 30-bus, and 118-bus systems. In case 1, it requires 4, 10, and 32 PMUs for three networks, respectively. Then, when considering zero injection buses and radial buses, the number of PMU in Case 2 sequence down to 3, 7, and 29 while the value of BOI still met the constrain of greater than or equal to one.

Table 11.The bus observability index of IEEE 14-bus system in case 3

Reference	This paper (Option 1)			This paper (Option 2), [21],[26]			[12],[17],[22],[25]		
Bus	F1	F2	F3	F1	F2	F3	F1	F2	F3
1	2	2	2	2	2	2	2	2	2
2	3	3	3	3	3	3	3	3	3
3	2	2	2	2	2	2	2	2	2
4	4	4	4	4	4	4	3	3	3
5	4	4	4	4	4	4	4	4	4
6	3	3	3	4	4	4	2	2	2
7	2	2	2	2	2	2	2	2	2
8	0	2	2	0	2	2	0	2	2
9	3	3	3	2	2	2	3	3	3
10	2	2	2	2	2	2	2	2	2
11	2	2	2	2	2	2	2	2	2
12	2	2	2	2	2	2	2	2	2
13	2	2	2	2	2	2	2	2	2
14	2	2	2	2	2	2	2	2	2
SORI	33	35	35	33	35	35	31	33	33

In Case 3, when considering the loss of data from one PMU, the result getting from this research for the 14-bus and 30-bus systems are equal to prior published. Looking through Table 14, the output of the IEEE 118-bus in this research is better than in several published papers as it just required 60 PMUs while 61, 62, 64, or 68 are the number needed for mentioned references.

In Table 15, in comparison with other papers using the genetic algorithm, it is clear that the number of PMU required in this work is similar to or better than in other published. There is just 2/7 reference papers ([10], [14]) solving the problem in case 3. Although there are 68 and 61 PMUs needed in [10] and [14] respectively, this research only required 60 PMUs.

Table 12. The comparison of different results was published for the OPP problem in case 1

Reference	This paper	[9]	[17]	[31]	[32]
14-bus	4	4	4	4	-
30-bus	10	10	10	-	10
118-bus	32	32	32	32	32

Table 13.The comparison of different results published for the OPP problem in case 2

Reference	**This paper**	**[9]**	**[12]**	**[16]**	**[17]**	**[21]**
14-bus	3	3	3	3	3	3
30-bus	7	7	7	7	7	7
118-bus	29	28	28	28	28	28
Reference		**[23]**	**[25]**	**[26]**	**[27]**	**[22]**
14-bus		3	3	3	3	3
30-bus		7	7	7	7	7
118-bus		29	29	29	29	28

Table 14. The comparison of different results was published for the OPP problem in case 3

Reference	**This paper**	**[9]**	**[12]**	**[17]**	**[31]**
14-bus	7	7	7	7	9
30-bus	14	14	15	15	-
118-bus	60	57	62	64	72
Reference	**[32]**	**[25]**	**[26]**	**[22]**	**[21]**
14-bus	-	7	7	7	7
30-bus	21	16	14	14	14
118-bus	68	62	61	61	61

Table 15. The comparison of different results was published using GA for the OPP problem in three cases

Reference	**This paper**			**[3]**			**[4]**		
Case	1	2	3	1	2	3	1	2	3
14-bus	4	3	7	-	-	-	4	-	-
30-bus	10	7	14	10	7	-	10	-	-
118-bus	32	28	60	37	32	-	32	-	-
Reference	**[10]**			**[13]**			**[14]**		
Case	1	2	3	1	2	3	1	2	3
14-bus	4	3	9	4	3	-	-	3	7
30-bus	10	7	21	10	7	-	-	7	14
118-bus	32	28	68	32	29	-	-	29	61
Reference	**[6]**			**[15]**					
Case	1	2	3	1	2	3			
14-bus	4	3	-	3	3	-			
30-bus	10	7	-	7	7	-			
118-bus	33	29	-	28	28	-			

5. CONCLUSION

In this paper, a binary genetic algorithm with a proposed process of calculating the bus observability index is applied to obtain the optimal location of placing PMUs, which ensures the complete observed system. The calculation is through a simple three-step process. With this proposed calculation, the evaluation of the system's observability becomes simpler than before because of no extra variable requirement, no system reconfiguration requirement, and no logical transformation requirement. In addition, the consideration of the ZIB observability transmission to another group is only for those buses that have not yet met the observability conditions. Hence, the observability reliability of the PMU placement will also be guaranteed.

The proposed technique is validated for three different cases of IEEE 14-bus, 30-bus, and 118-bus systems. The results obtained from the program were relatively good compared with the published results in terms of the total number of PMU requirements. In the concern of observability, this research solves unobservable problems of several buses in the IEEE 118-bus grid exit in previous publishes, especially fixing the unobserved issue of bus 47. The simulation from this work can give more than one PMU location choice with the same number of PMUs and an equal value of observability. Therefore, we can choose any of them because the results only differ in the observed values of some individual buses with an increase or decrease value of one.

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