Minimizing Energy Consumption of Onboard Battery Light Rail Transit Using Dynamic Programming

Chalita Jobsoongnern[1] and Tosaphol Ratniyomchai[1,*]

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ABSTRACT

Aims to minimize the energy consumption of the light rail transit using an onboard battery as the primary power source by finding the speed profile optimization, using a dynamic programming approach. By generating the speed candidate from the reference speed profile and comparing the result of each optimum speed profile, the different resolutions of the speed candidate in each position can be the better result. The result shows the maximum saving energy of up track direction and down track direction as 16.10% and 18.99% respectively. For the 22 km round-trip route of the Korat Light Rail Transit Green Line project in Nakhon Ratchasima, Thailand, this study recommends the adoption of an optimal speed profile as a driver's guide. The route has a shared traffic section and the light rail vehicle specific path. In addition, the running time is limited to be most effective in accordance with energy-saving.

1. INTRODUCTION

Currently one of the important public transportations is the electric railway. The people can choose by their needs to use any of transportation such as high-speed train, metro, and tram. In addition, another choice for using public transportation is the light rail transit (LRT) even in the big city but in any small city has a service of LRT such as scenery town. The LRT is closer to railroads and more environmentally and gaining attention in an aging society. Nowadays the technological advancements in energy storage devices and fast charging at each station are well known [1]. An example such as the LRT in Nanjing, China which is 90% of the route uses onboard energy storage in the catenary free operation that the cost-effective both for construction and easy to accept by the local citizen [2].

During the catenary free operation, an onboard energy storage has a significant role to provide the energy. The limitation of using only onboard energy storage as the main power source such as battery is challenge. The required sizing to provide enough energy for travel must take into consideration [3]. For enough energy to arrive at the final terminal, the way to reduce the energy consumed from the onboard energy storage, the battery charging at the passenger station is proposed. Besides catenary along with a platform that can reduce the peak demand power in accelerating range [4].

Moreover, the solution to reduce the energy consumption is to optimize the transportation system by taking into account the relative factors such as the timetable and the specific passenger needs [5]. Other factors to reduce the energy consumption can be the train motive power by finding the optimal speed profile or torque profile that the vehicle should follow to minimize its energy consumption [6]. Besides, there can also be a time schedule for managing the energy of the train using a grid system with energy storage devices that can solve both tight power supply and temporary capacity constraints [7, 8].

Furthermore, the state of charge (SOC) is one of the factors that can consider while using the catenary free operation [9]. The factor that can consider for the city train because of frequent stops. The regenerative braking energy recharges the onboard energy storage as a result that the total energy consumption is also reduced [10].

For the reduction of the traction of motor, the strategy is allowing trains to coast within the available amount of time to conserve energy, by using the algorithm with Genetic Algorithm (GA), Ant Colony Optimization (ACO), and Dynamic Programming (DP), the authors from [11] compared the result of the 3 algorithms that the dynamic programming was given the minimum energy consumption and the speed profile with no disturbance for a problem of too many discrete points [12]. Neither the ant colony optimization algorithm nor the genetic algorithm yielded a sufficiently smooth optimal speed profile with a distinct cruising phase. Besides, the authors from [13] compared the gradient method and dynamic programming that slow computation, and the Sequential Quadratic Equation Program (SQP) is faster computing, but in the speed

[1]*School of Electrical Engineering, Institute of Engineering, Suranaree University of Technology, 30000, Thailand.*
**Corresponding author:* Tosaphol Ratniyomchai; Email: tosaphol@sut.ac.th

application, the dynamic programming is easy to limit the state of variables such as speed and acceleration. Furthermore, Particle Swarm Optimization (PSO) is a widely recognized optimization algorithm inspired by the collective movement of birds in a flock. [14], which has also proven its efficacy in identifying optimal solutions within the context of speed profile optimization. [15, 16]. In light of these findings, the present investigation explores the application of dynamic programming as a viable approach for identifying the optimal speed profile that minimizes energy consumption.

The simulation presented in this article is exemplified through the application to the specific case of the green line Korat LRT route in Nakhon Ratchasima, Thailand. That is the project to solve the traffic congestion and other policies, and the design of catenary free operation by using only onboard energy storage with recharging energy at the passenger station [17]. Following the lead of [18], this study proposes utilizing the onboard li-ion battery as the energy source for the LRV. Furthermore, it leverages dynamic programming to identify the optimal speed profile by discretizing a reference speed profile and iteratively evaluating various candidate speeds with the aim of maximizing energy savings. The simulation separates of the up-track direction route and down track direction route.

This article concludes with the problem formulation in 2 which are the LRV movement model, the battery model, and the dynamic programming approach. The reader can find a comprehensive description of the investigated route and LRV configuration in 3. Sections 4 and 5, respectively, present the in-depth simulation results and the overarching conclusions derived from the analysis.

2. PROBLEM FORMULATION

2.1. LRV movement model

The Newton's second law is employed to explain the motion of the LRV movement. The friction forces have considered as in Fig. 1.

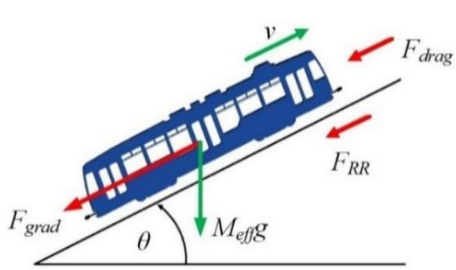

Fig. 1. Light rail vehicle movement model.

$$P_{Lrv} = \frac{F_T v}{\eta_g \eta_m \eta_i} + P_{aux} \tag{1}$$

$$F_T = M_{eff}\, a + F_R \tag{2}$$

$$F_R = F_{RR} + F_{grad} + F_{drag} \tag{3}$$

From Eq. (1) and Eq. (2), P_{Lrv} denotes the electrical power consumed during vehicle propulsion, F_T represents the tractive effort exerted by the locomotive, v is the LRV current velocity, η_g is the efficiency of the gearbox, η_m is the efficiency of the motor, η_i is the efficiency of the inverter, P_{aux} is power consumed by auxiliary systems, M_{eff} is the LRV equivalent mass and a is the train acceleration. F_R is the total resistance force calculated by Eq. (3). Where, F_{RR} is friction force, F_{grad} is gradient force and F_{drag} is aerodynamic resistance force. The calculation of all aforementioned parameters is presented as follows:

$$F_{RR} = f_{RR} W \tag{4}$$

$$F_{grad} = \pm M_{ff}\, g \sin\theta \tag{5}$$

$$F_{drag} = 0.5 \rho_{air} c_d A_F v_{air}^2 \tag{6}$$

From Eq. (4) and Eq. (5), f_{RR} represents the coefficient of rolling friction, W is the driven axis load, g is the gravity force and θ is the slope angle.

From Eq. (6), ρ_{air} is the atmospheric density, c_d is the coefficient of atmospheric drag, A_F is the projected frontal area of the LRV oriented normal to the direction of the oncoming airflow. and v_{air} is the velocity of the airflow.

$$E = \int_0^t P_{Lrv}\, dt \tag{7}$$

Eq. (7) provides the means to determine the LRV's energy consumption (E), expressed as a function of its running time (t).

2.2. The Model of Battery

An evaluation of both the generic and the simplest battery models reveals comparable levels of accuracy in their predictions. [19]. Thus, an onboard battery is used as the main power source of LRV by considered as a current source in parallel with the internal resistance as the simplest model shown in Fig. 2 [20].

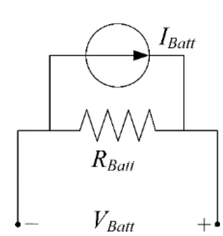

Fig. 2. Simplest battery model.

$$I_{Batt} = \frac{C_{Batt}}{3600 \times V_{Batt}} \tag{8}$$

$$R_{Batt} = \frac{V_{Batt}}{I_{Batt}} \tag{9}$$

$$V_{Batt} = R_{Batt} \times (I_{Batt} - I_{Lrv}) \tag{10}$$

The battery current, I_{Batt}, the internal resistance, R_{Batt}, and the battery voltage, V_{Batt}, are obtained the calculation in Eq. (8)-(10). Where, C_{Batt} is the battery capacity and I_{Lrv} is the LRV load current that consume from the battery calculated as in Eq. (11).

$$I_{Lrv} = \frac{P_{Lrv}}{V_{Batt}} \tag{11}$$

$$\%SOC(t+\Delta t) = \%SOC(t) + \int_{t}^{\Delta t} \frac{I_{Lrv}}{C_{Batt}} dt \times 100 \tag{12}$$

Equation (12) provides the mathematical framework for calculating the battery's percent state of charge (*%SOC*).

2.3. Dynamic programming

This study employs dynamic programming to optimize the speed profile of an electric train during catenary-free operation, minimizing energy consumption while utilizing onboard energy storage as proposed in [21]. In additions, the potential of optimized speed profiles extends beyond minimizing energy consumption, finding further application in the design of driver advisory systems [22]. This article leverages dynamic programming to determine the optimal speed profile for train journeys between two passenger stations, with the objective function prioritizing minimal energy consumption throughout the travel distance.

To facilitate the solution of the overall optimization problem, the train journey between the two passenger stations is decomposed into smaller sub-problems, each focused on determining the optimal speed profile for the travel segment between consecutive stations. Before the dynamic programming process must generate the state of the speed profile to receive the speed candidate for the decision in each stage of the optimization. The fixed speed profile reference of the LRV must be divided into a range within the limits of maximum and minimum speed shown in Fig. 3 [23].

The train speed is represented as v^k and K is the resolution of state generation that can be any value depending on the width of speed range and suitability. This simulation K is the number of 6, 10, 15, 20, 25 and 30. Where, s is the position during the passenger station, n is the station number whereas $n = 1, 2, \ldots, M$.

The divided factor Δv_d calculates from Eq. (13). $v_{\max}$ is the maximum speed and $v_{\min}$ is the minimum speed.

$$\Delta v_d = \frac{v_{\max}(s_{i+1}) - v_{\min}(s_{i+1})}{K - 1} \tag{13}$$

To create the state generation by the speed range dividing, the speed in traction mode and cruising mode are calculated by Eq. (14). While the speed in coasting mode and braking mode are calculated by Eq. (15), where $k = 1, 2, \ldots, K$.

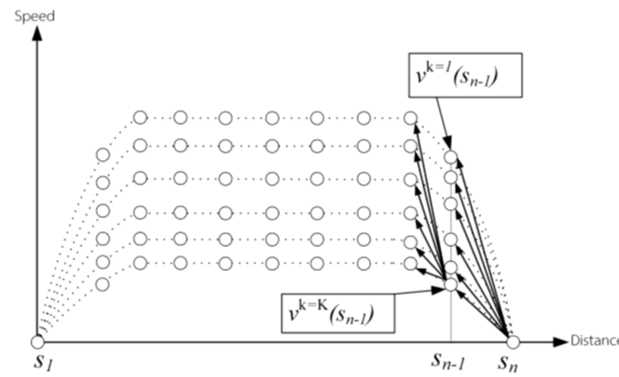

Fig. 3. State generation of speed profile.

$$v^k = \max\{v_{\max}(s_{i+1}) - \Delta v_d \times (k-1), 0\} \tag{14}$$

$$v^k = \max\{v_{\max}(s_{i+1}) + \Delta v_d \times (k-1), 0\} \tag{15}$$

The acceleration, $a(s_{i+1})$ can be calculated by Eq. (16). The model incorporates crucial factors such as the LRV's power requirements, travel duration, and energy expenditure to arrive at the optimal outcome.

$$a(s_{i+1}) = \frac{\left(v^k(s_{i+1})\right)^2 - \left(v^k(s_i)\right)^2}{2\left((s_{i+1}) - (s_i)\right)} \tag{16}$$

Eq. (17) defines the objective function for minimizing energy consumption that E represented the total energy consumption from Eq. (7) within the constrain in Eq. (18) which is the acceleration and deceleration range limited at 1 m/s^2. For Eq. (19)-(21), the total energy should calculate in each position s_i while s_i is the sub-position during the distance of passenger station as S^n and the limited running time as T. Besides, the running time is limited within 10% delay that allow to influence service schedule [24] and %SOC in must the range of 20% to 100%.

$$\min J = \sum_{n=1}^{M} E_{total}^{n} \tag{17}$$

$$|a(s_i)| \le a_{\max}, \forall i \tag{18}$$

$$s_i \in [S^n], n = 1, \ldots, M, \forall i \tag{19}$$

$$T \le T_{\max}, \forall i \tag{20}$$

$$20\% \le \%SOC \le 100\% \tag{21}$$

Fig. 4 shows the algorithm flow chart of the dynamic programming approach. From the start, choosing the parameters from the state generation at the first position as s_1. Secondly, checking the constrain feasibility. If the constrain is not satisfied, then back to the first step and choosing the new parameters. Leveraging the LRV and battery models, the next step calculates energy consumption for each speed profile, enabling efficiency evaluation. After that, collecting the best solution in each position and calculating the next position s_{i+1} until the final position. At the final position at S^n, choosing the final minimum solution and stop.

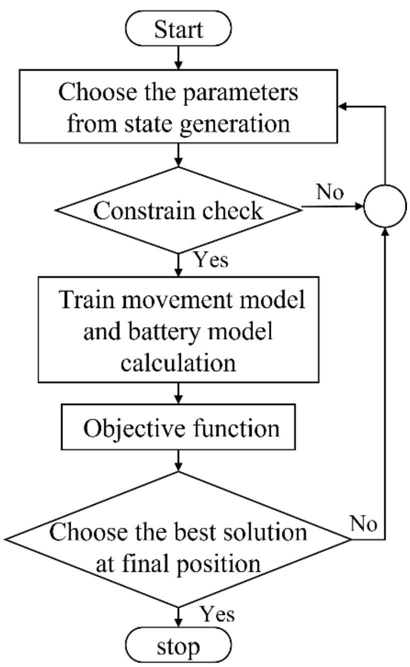

Fig. 4. Dynamic programming algorithm flow chart.

3. ROUTE AND LRV PARAMETERS

Fig. 5 illustrates the Green Line Korat LRT route in Nakhon Ratchasima, Thailand, chosen as the central case study for the simulation. The round-trip distance is 22 km with 20 passenger platforms, and no gradient changes are present within the route.

The normal speed profile of the LRV is shown in Fig. 6. The up-track direction, traveling from passenger station 1 to passenger station 17 and that the shared traffic section is from passenger station 7 to passenger 11. For the down track direction, traveling from passenger station 17 to passenger station 1 and the shared traffic section is passenger station 18 to passenger station 20. The shared traffic section has low speed caused by the crowded traffic.

The parameters of LRV used in the simulation presented in Table 1 [18]. The battery capacity of 70 kWh, 600 V and its model is considered an ideal source. The permissible range for the state of charge (SOC) is defined by lower and upper bounds of 0% and 100%, respectively.

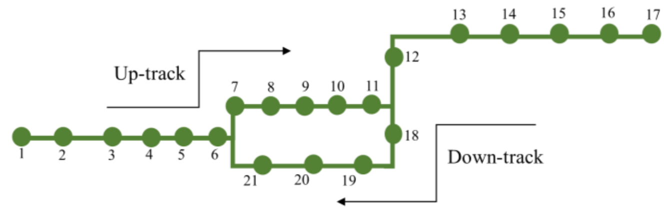

Fig. 5. Green line Route of Korat LRT.

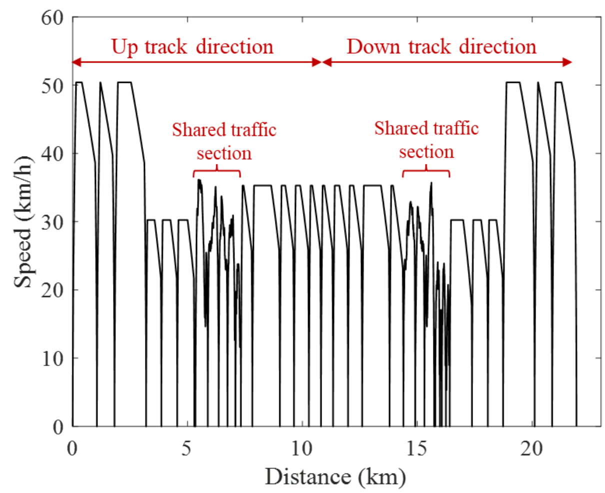

Fig. 6. Normal speed profile of round-trip.

Table 1. Light rail vehicle parameters [18]

Parameter	Value
Design speed limit (v)	40 km/h
Peak acceleration rate (a_{max})	0.7 m/s^2
Equivalent mass (M_{eff})	44000 kg
Coefficient of rolling friction (f_{RR})	0.006
Coefficient of aerodynamic drag (c_d)	0.6
Atmospheric density (ρ_{air})	1.225 kg/m^3
Cross-sectional area (A_F)	8.4 m^2
Gearbox eff. (η_g)	0.93
Motors eff. (η_m)	0.9
Inverter eff. (η_i)	0.9
power consumed by auxiliary systems (P_{aux})	20 kW

4. SIMULATION RESULT

The simulation results consider separately in up track and down track journeys. The distance of each direction route is around 11 km. Table 2 shows the result in each traveling section between two passenger stations of up track direction. By using K = 6, The delay time is within the limit of 10%. In the same maximum speed limit of traveling from passenger station 1 to passenger station 4, the obvious maximum saving

Table 2. Energy consumption comparison of normal speed and optimal speed of up track direction

Station		Distance between station	Energy consumption (kWh)		Energy saving (%)	Time running (s)		Delay time (%)
From	To		Normal speed	Optimal speed		Normal speed	Optimal speed	
1	2	1055	3.3451	2.841	15.07	114	125	9.65
2	3	762	2.8667	2.414	15.79	94	103	9.57
3	4	1375	3.8678	3.223	16.67	132	143	8.33
4	5	665	1.8917	1.599	15.47	111	120	8.11
5	6	690	1.9348	1.673	13.53	114	124	8.77
6	7	745	2.0354	1.764	13.33	118	129	9.32
7	8	585	2.6464	2.6464	-	138	138	-
8	9	475	1.7549	1.7549	-	89	89	-
9	10	385	1.4703	1.4703	-	79	79	-
10	11	340	1.6798	1.6798	-	107	107	-
11	12	240	1.1742	1.1742	-	80	80	-
12	13	500	1.7857	1.4532	18.62	85	93	9.41
13	14	1195	2.9305	2.5241	13.87	154	169	9.74
14	15	615	1.9631	1.6213	17.41	96	104	8.33
15	16	650	2.0437	1.6634	18.61	101	110	8.91
16	17	525	1.8341	1.4722	19.73	88	96	9.09

energy is 16.67% which is traveling from passenger station 3 to passenger station 4. Among all segments with constant speed restrictions, the connection between these two stations stands out as the longest, extending for 1375 meters. Considering the traveling from passenger station 4 to passenger station 6, the maximum energy saving is 15.47% which is traveling from passenger station 4 to passenger station 5 and a distance of 665 m. In the section of traveling from passenger station 12 to passenger station 17, Another maximum saving energy is 19.73%, traveling from passenger station 16 to passenger station 17 which is a distance of 525 m.

The optimal speed profile of up track direction with each K value from 6, 10, 15, 20, 25, and 30 is shown in Fig. 7. If zoom in the specific point, that can be seen the difference in the acceleration of traction mode, and the difference is also in constant speed mode, coasting mode, and braking mode. This is the result of the decision of each K value which is an effect of the resolution of state generation. If K has increased the frequency of the speed division and the resolution in the optimization will also increase.

Note that the section of shared traffic with the roadside did not consider the optimal speed profile as mentioned above.

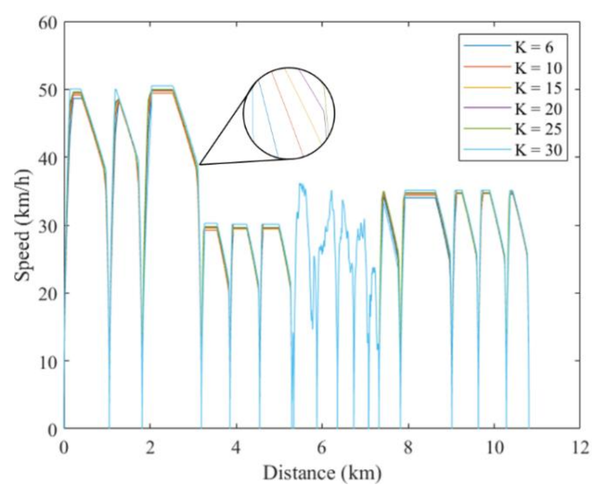

Fig. 7. Optimal speed profile of up track direction.

Figure 8 illustrates an intriguing trend: within the up track direction, for all explored resolution parameter (K) values, the total energy consumption rises steadily along the optimal speed profile, reaching a interval around 30 kWh.

The energy consumption is related to the reduction of state of charge of the onboard battery. The LRV consume energy from the battery almost 50% as shown in Fig. 9. that all case of K is reduced from 100% to around 50% dropping almost 50% from the initial. The state of charge when the LRV arrived at the final terminal is 54.53%, 54.14%, 54.26%, 54.40%, 54.52%, and 55.04% respectively from K = 6 to K = 30. The maximum state of charge is the case when K = 30.

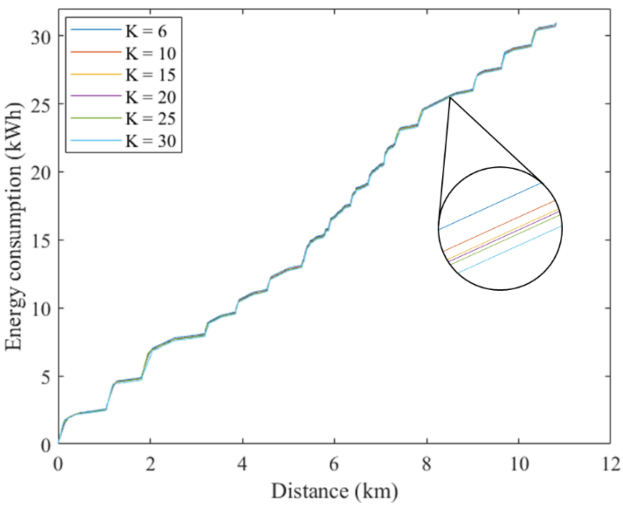

Fig. 8. Energy consumption of up track direction.

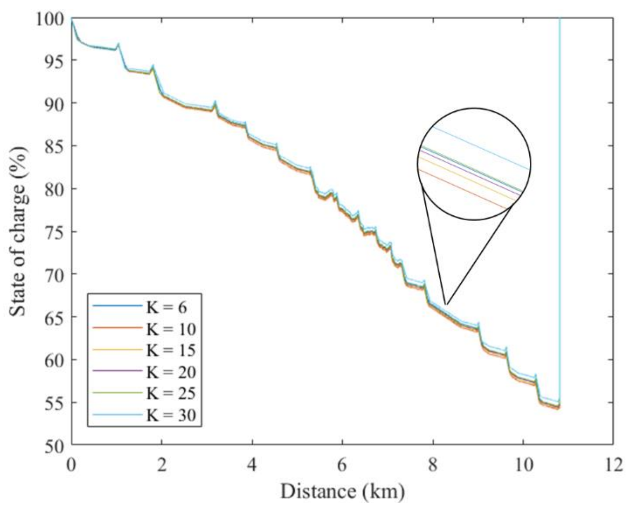

Fig. 9. State of charge of onboard battery of up track direction.

Table 3. Optimization result of up track direction

Case	Energy consumption (kWh)	Saving energy (%)	Time (s)	Delay time (%)
Before optimization	35.23	-	1700	-
K = 6	30.97	12.07	1809	6.41
K = 10	30.90	12.29	1805	6.18
K = 15	30.86	12.40	1802	6.00
K = 20	30.84	12.46	1796	5.65
K = 25	30.83	12.48	1791	5.35
K = 30	30.82	12.52	1804	6.12

Table 3 shows the comparison of the optimization result of energy saving with finding the optimal speed profile traveling up track direction. The saving energy according to K = 6, 10, 15, 20, 25, and 30 in comparison with the based case before optimization are 12.07%, 12.29%, 12.40%, 12.46%, 12.48%, and 12.52% respectively, that are all in the available time limitation. The maximum saving energy is 12.52% obtained from the energy consumption before optimization of 35.23 kWh reduce to 30.82 kWh when K = 30. For the result of running time, the delay time is 6.41%, 6.18%, 6.00%, 5.65% 5.35%, and 6.12% following the K values respectively. The delay time of all cases are within the available time limitation. The result shows some signs of K value in this simulation, and the optimal K value for the up track direction in this simulation is K = 30.

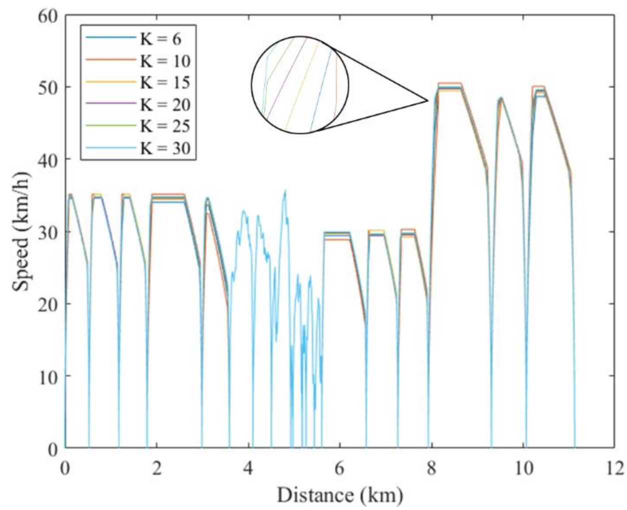

Fig. 10. Optimal speed profile of down track direction.

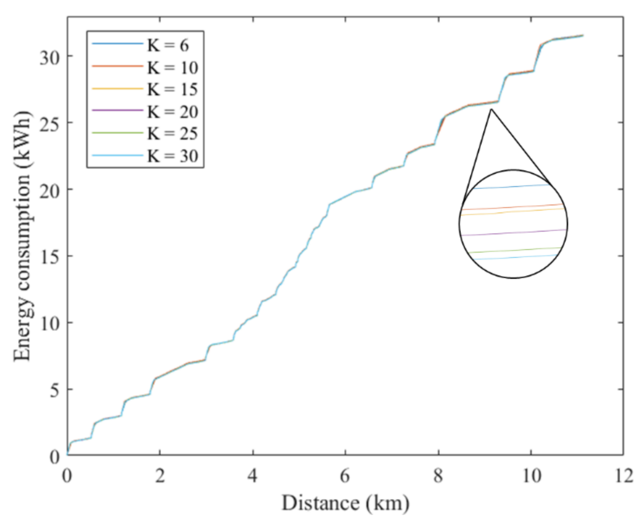

Fig. 11. Energy consumption of down track direction.

The same with up track direction, the optimal speed profile of down track direction in each K value as shown in Fig. 10. There is a difference of the length of the passenger station interval which is close to the separated path of the shared traffic section, and the number of passenger stations is different. The variations in the optimization results are further evident in the distinct speed profiles for each operating mode: traction, constant speed, coasting, and braking.

As shown in Fig. 11, the energy consumption of each K value increases following the optimal speed profile in Fig. 10. Cause of the similar distance with up track direction, the energy consumption of down track direction is also around 30 kW. However, the percentage of state of charge of onboard battery of each K value is reduced following the increase of energy consumption as shown in Fig. 12. The state of charge when the LRV arrived at the final terminal is 39.99%, 40.13%, 40.04%, 40.01%, 40.07%, and 40.10% respectively from K = 6 to K = 30. The maximum state of charge is the case when K = 10.

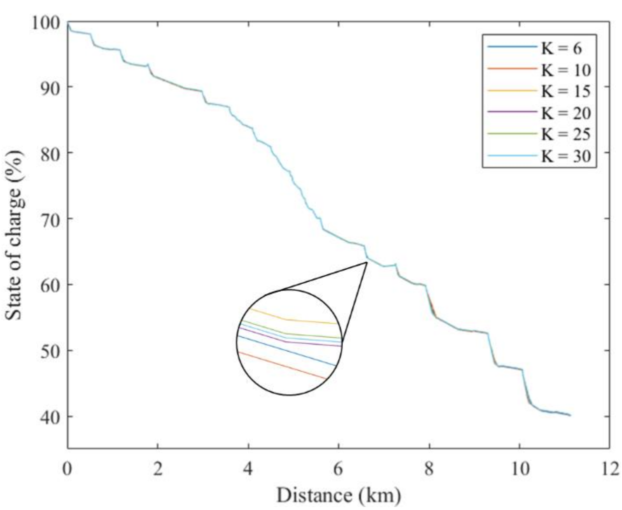

Fig. 12. State of charge onboard battery of down track direction.

Table 4. Optimization result of down track direction

Case	Energy consumption (kWh)	Saving energy (%)	Time (s)	Delay time (%)
Before optimization	36.03	-	1754	-
K = 6	31.63	12.21	1870	6.82
K = 10	31.60	12.30	1874	7.06
K = 15	31.61	12.31	1865	6.53
K = 20	31.58	12.35	1858	6.12
K = 25	31.56	12.41	1852	5.76
K = 30	31.55	12.43	1848	5.53

Table 4 presents the optimization outcomes for minimizing energy consumption during down track travel, achieved for various resolution parameter K values: 6, 10, 15, 20, 25, and 30. The saving energy in comparison with the energy consumption of the based case before optimization according to each K value are 12.21%, 12.30%, 12.31%, 12.35%, 12.41%, and 12.43% respectively. In addition, the delay times of different time running are 6.82%, 7.06%, 6.53%, 6.12% 5.76%, and 5.53% respectively, that are all in the available time limitation. The maximum saving energy is 12.43% compared to the energy consumption before optimization of 36.03 kWh reduced to 31.55 kWh when K value is 30. The optimal K value of 30 is also obtained from down track direction.

From the simulation result of both up track and down track direction, the optimal value of K in this simulation is 15 and 20 respectively. which can be seen clearly from Table 2 and Table 3. By the different resolution of speed candidates from K = 6 to K = 30, their effect on the optimal speed profile and the saving energy that the difference of up track direction is 0.45%. The difference down track direction is 0.22%. For both directions, the maximum saving energy is increased when K is more than 6 to K = 30. Notably, the optimization of speed profiles, as revealed by the simulation results, demonstrates that the minimum energy consumption associated with each optimal speed is dependent on the chosen resolution parameter K.

5. CONCLUSION

LRV energy consumption directly impacts both battery depletion and recharge duration, thereby influencing service scheduling, operational efficiency, and system reliability. Therefore, minimizing energy consumption is necessary. To minimize the energy consumption by using a dynamic programming approach, the minimum total energy consumption at each step position is used as an objective function. The optimization process yielded the optimal speed profile, minimizing energy consumption within the defined speed range. This led to a maximum energy saving of 12.52% for the up track direction and 12.43% for the down track direction. There is a possibility to vary the resolution of state generation or known as speed candidate to find the optimum value that affects more saving energy. The other objective function might be considered to find the different results with the same system and comparison such as running time limit, regenerative braking energy, or state of charge of onboard battery for catenary free system.

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